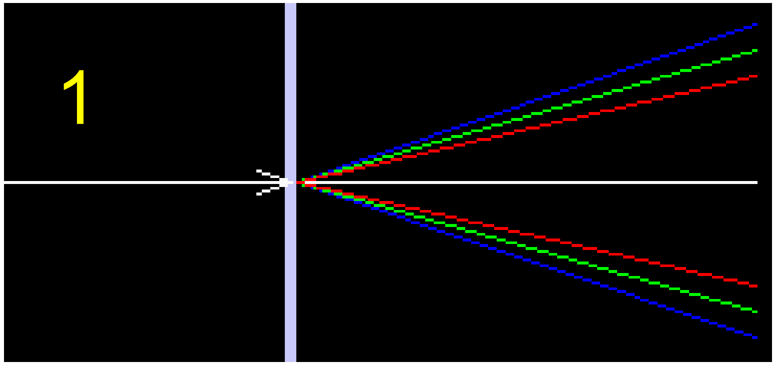
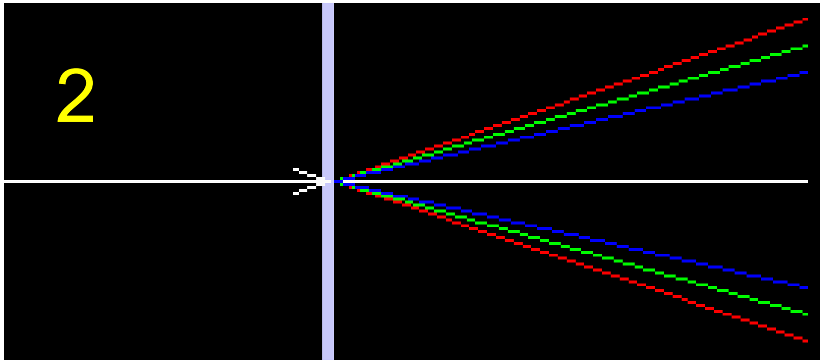
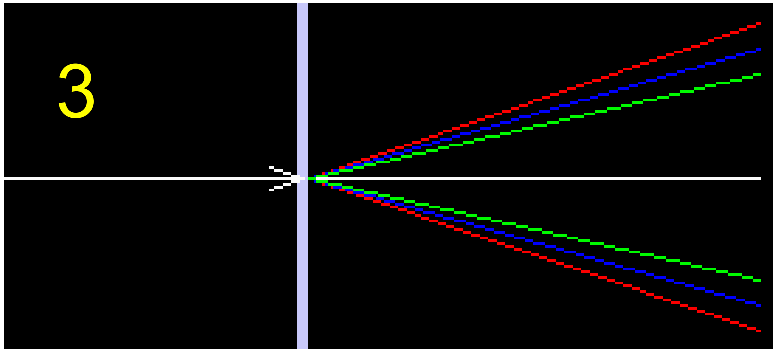
Diffraction grating, first order

Which picture shows correctly the first-order spectrum (m = 1) for a beam of light consisting of a single red wavelength, a single blue wavelength, and a single green wavelength?





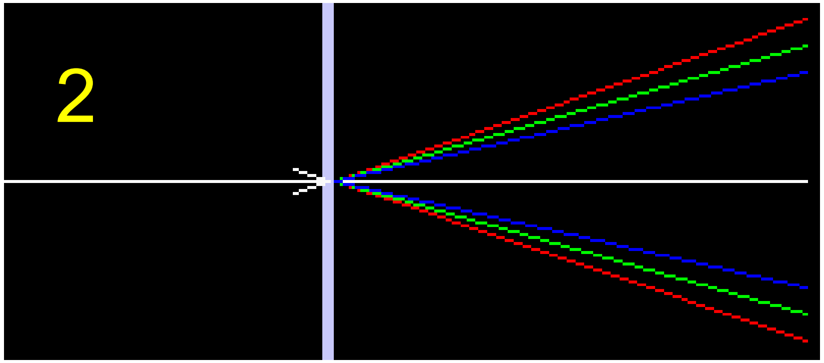


Diffraction grating, first order

For the diffraction grating, $d \sin(\theta) = m\lambda$.

Ranking the colors by increasing wavelength, we have blue, green, red. The longer the wavelength, the larger the angle.

Is this the same as what happens with a prism?



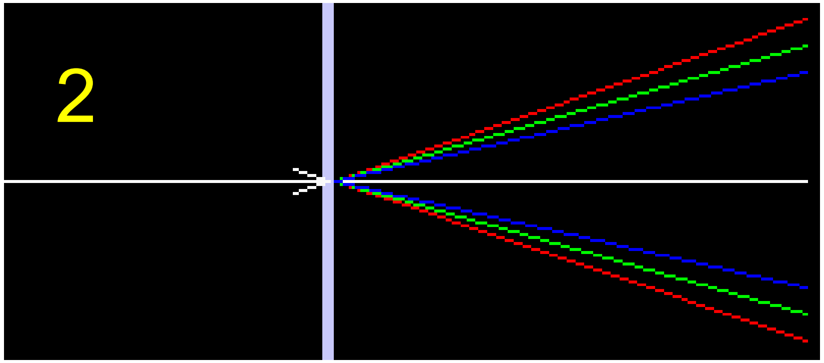
Diffraction grating, first order

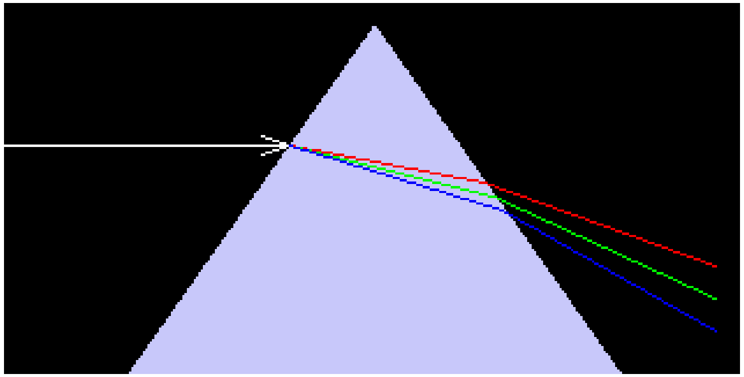
For the diffraction grating, $d \sin(\theta) = m\lambda$.

Ranking the colors by increasing wavelength, we have blue, green, red. The longer the wavelength, the larger the angle.

Is this the same as what happens with a prism?

This is opposite to what happens with a prism.

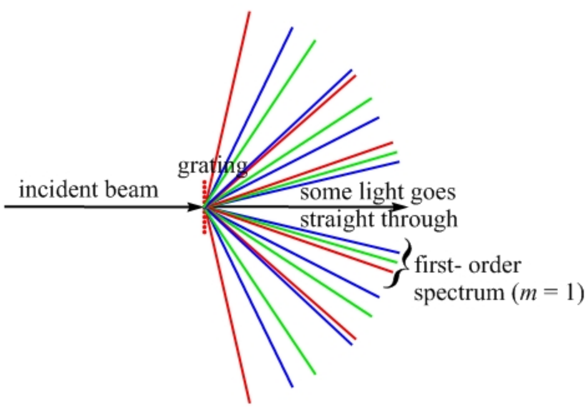




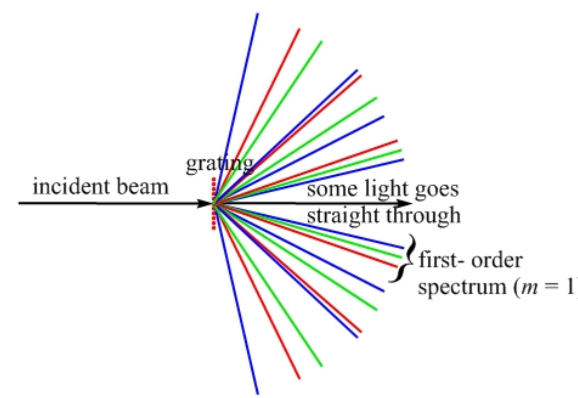
Diffraction grating, higher orders

This diffraction grating has a grating spacing of d = 2000 nm. The wavelengths in the beam of light are 450 nm (blue), 550 nm (green), and 650 nm (red). Which picture correctly shows all the observed lines?

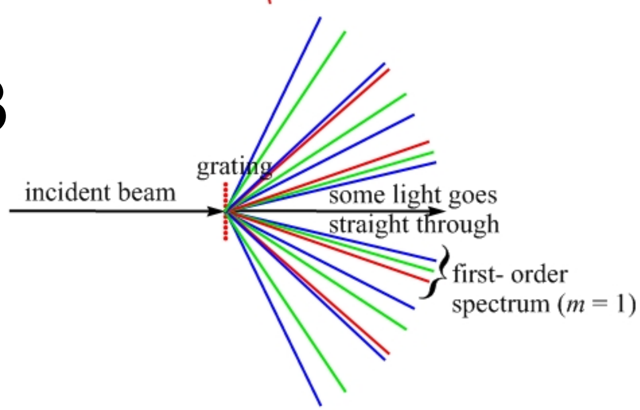
1



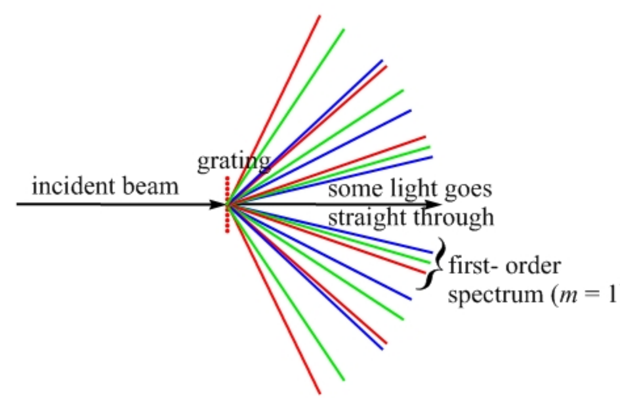
2



3



4



Diffraction grating, higher orders

For the diffraction grating, $\sin\theta = \frac{m\lambda}{d}$

Order	Blue (450 nm)	Green (550 nm)	Red (650 nm)
m = 1			
m = 2			
m = 3			
m = 4			

Diffraction grating, higher orders

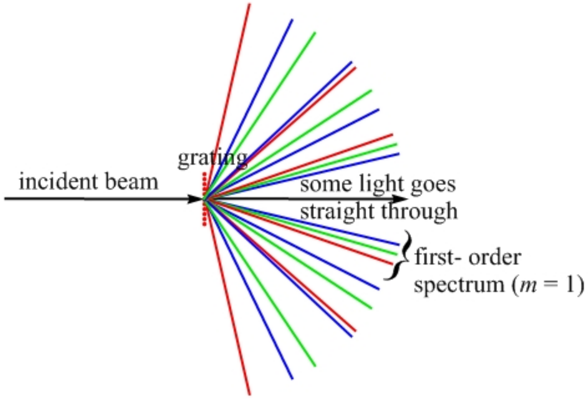
For the diffraction grating, $\sin\theta = \frac{m\lambda}{d}$

Order	Blue (450 nm)	Green (550 nm)	Red (650 nm)
$m = 1$	$\sin\theta = 450/2000$	$\sin\theta = 550/2000$	$\sin\theta = 650/2000$
$m = 2$	$\sin\theta = 900/2000$	$\sin\theta = 1100/2000$	$\sin\theta = 1300/2000$
$m = 3$	$\sin\theta = 1350/2000$	$\sin\theta = 1650/2000$	$\sin\theta = 1950/2000$
$m = 4$	$\sin\theta = 1800/2000$	$\sin\theta = 2200/2000$	$\sin\theta = 2600/2000$

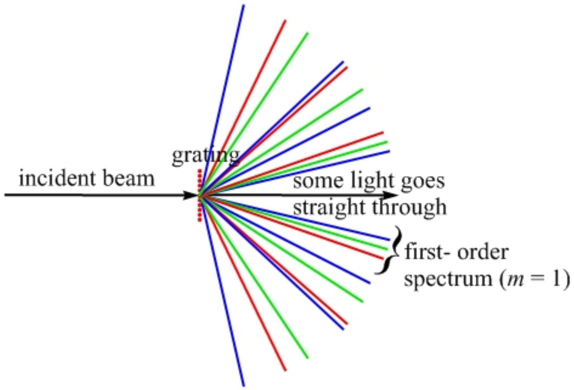
Diffraction grating, higher orders

This diffraction grating has a grating spacing of d = 2000 nm. The wavelengths in the beam of light are 450 nm (blue), 550 nm (green), and 650 nm (red). Which picture correctly shows all the observed lines?

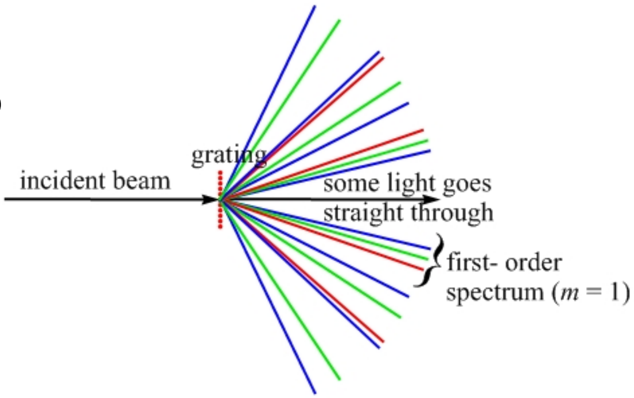
1



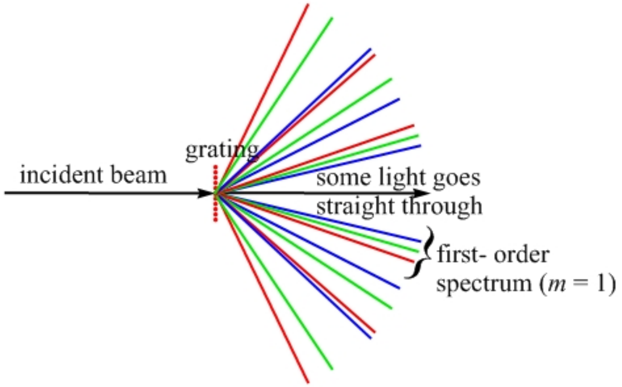
2



3

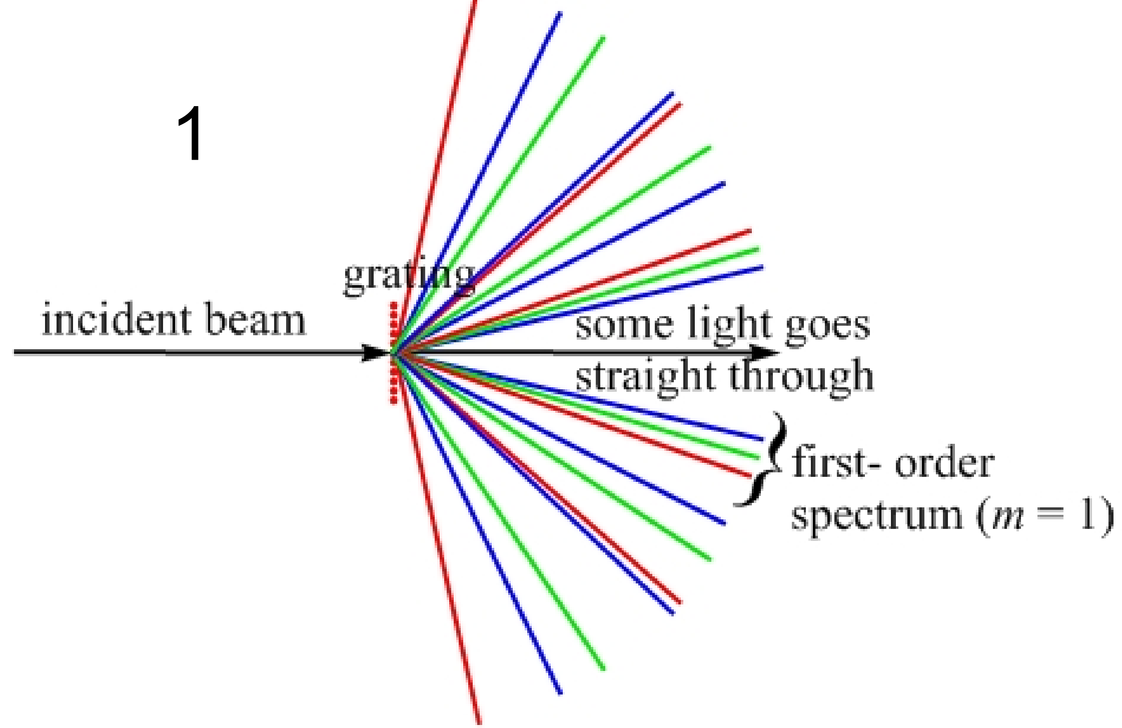


4



1

Diffraction grating higher orders



Order	Blue (450 nm)	Green (550 nm)	Red (650 nm)
m = 1	sinθ = 450/2000	sinθ = 550/2000	sinθ = 650/2000
m = 2	sinθ = 900/2000	sinθ = 1100/2000	sinθ = 1300/2000
m = 3	sinθ = 1350/2000	sinθ = 1650/2000	sinθ = 1950/2000
m = 4	sinθ = 1800/2000	~~sinθ = 2200/2000~~	~~sinθ = 2600/2000~~

Diffraction

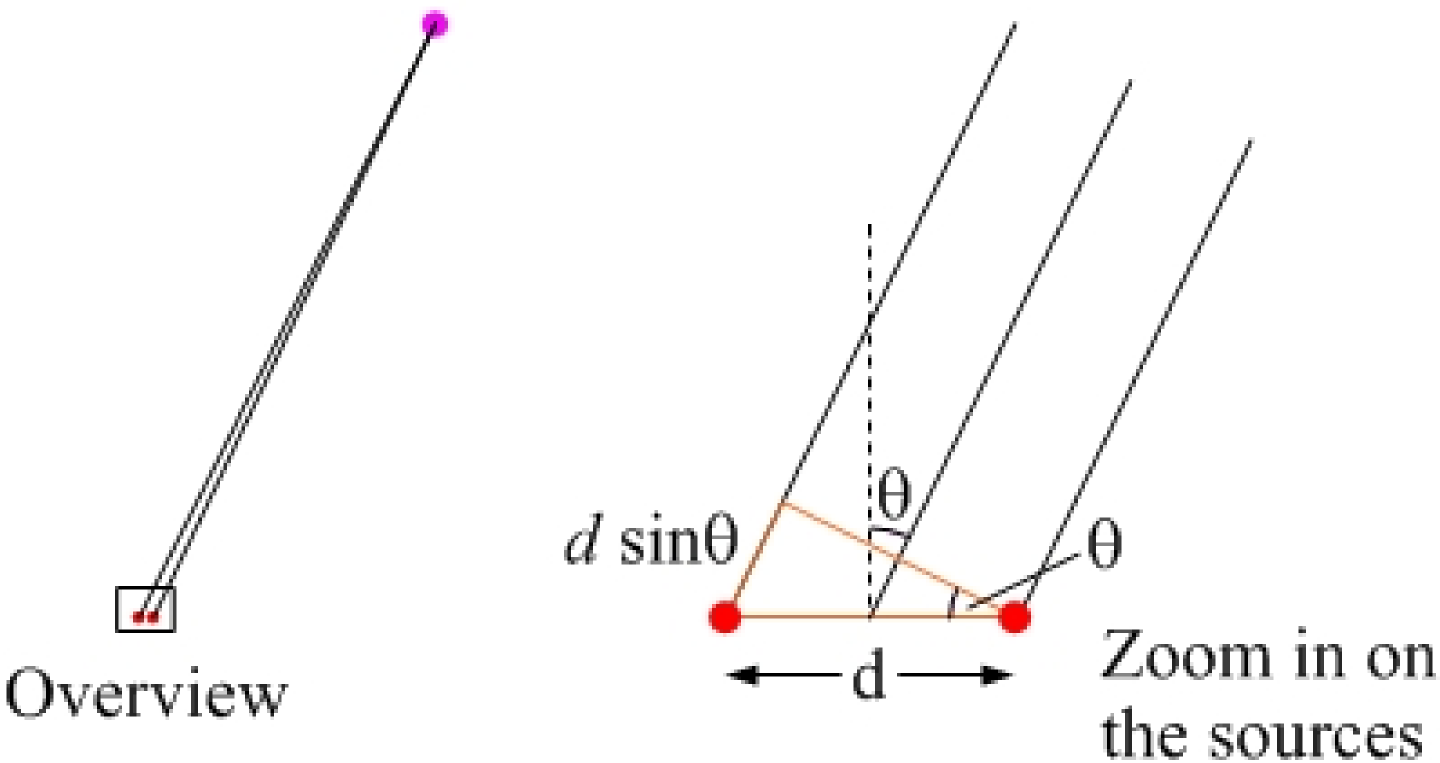
Diffraction is the spreading out of a wave when it encounters a single object or opening.

Simulation

The double-source equation

For two sources a distance d apart, constructive interference occurs when

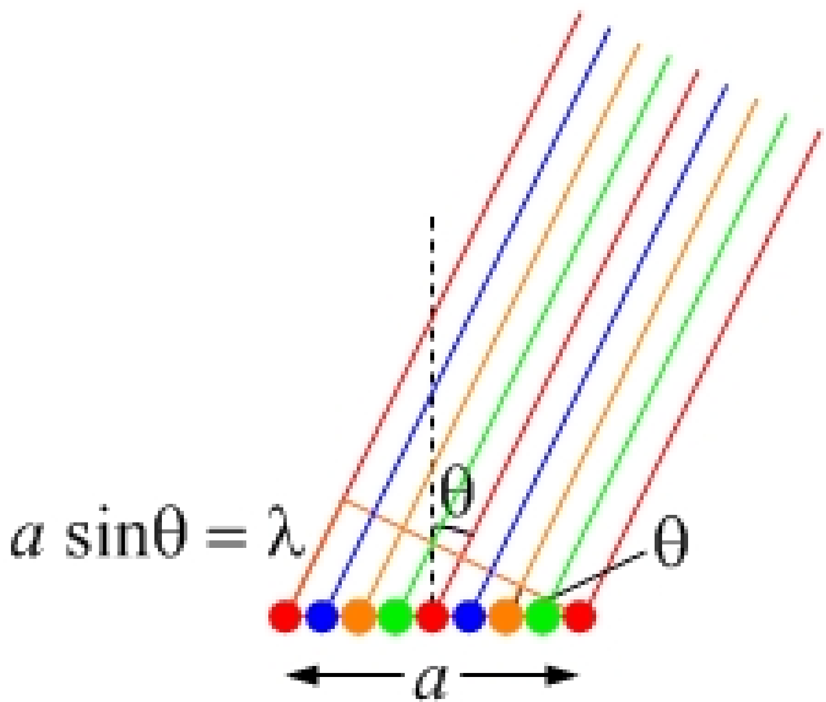
$$d \sin(\theta) = m\lambda$$



The single-slit equation

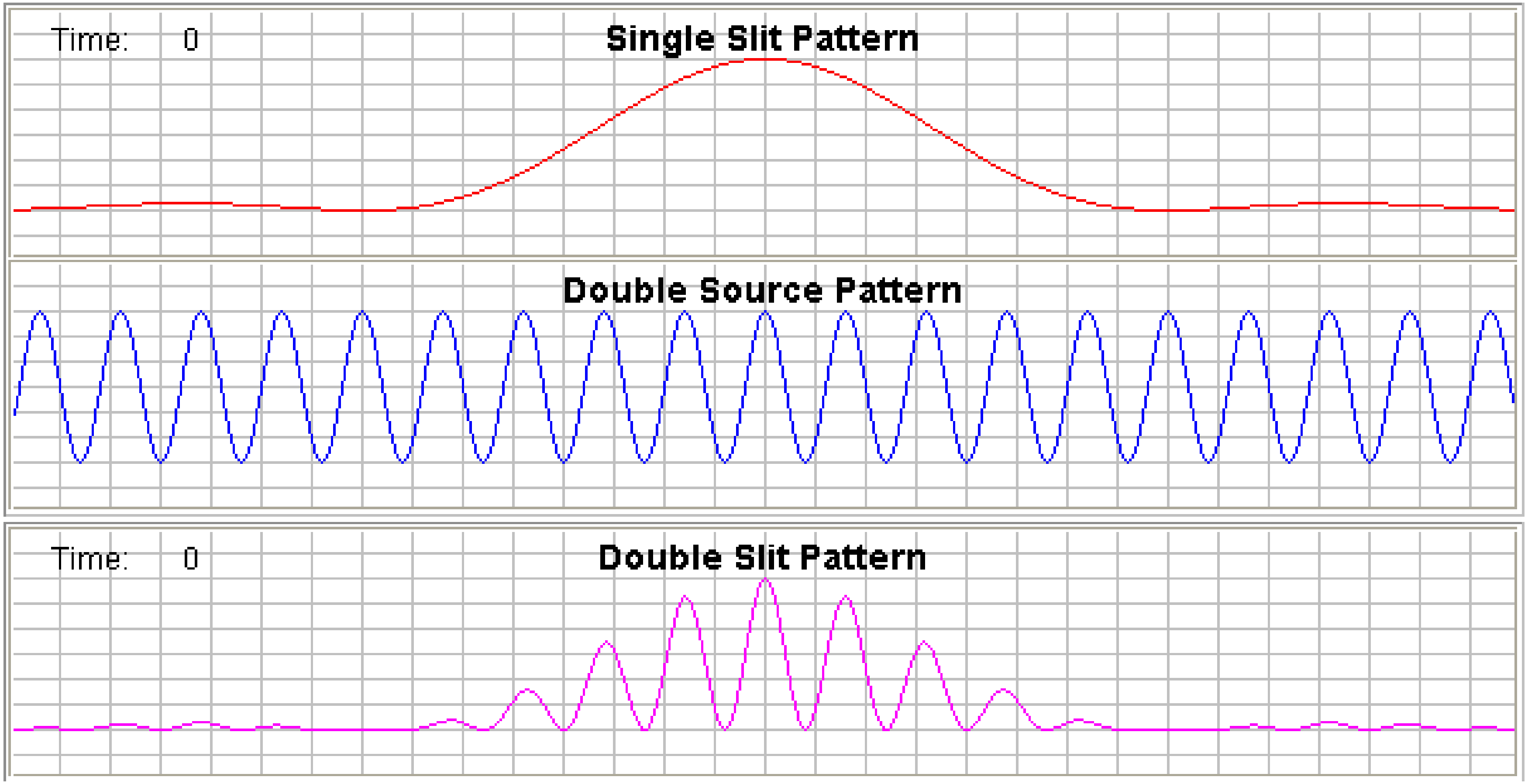
Let's call the width of the slit *a*. Each point on the slit acts as a source of waves. For a point a long way from the sources, destructive interference is given by the equation

$$a \sin(\theta) = m\lambda.$$



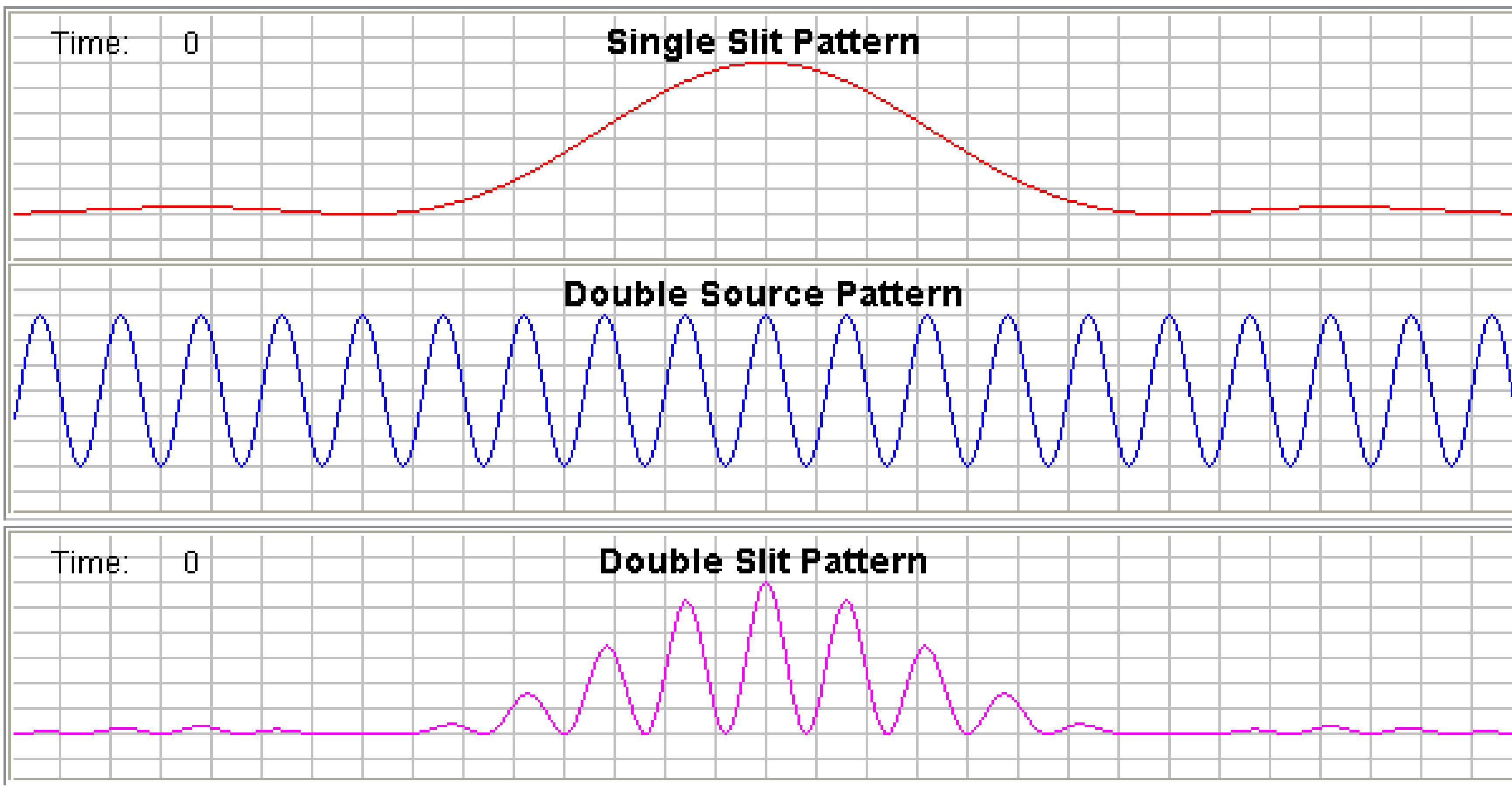
The double slit

The double slit is a combination of the single slit pattern and the double source pattern.



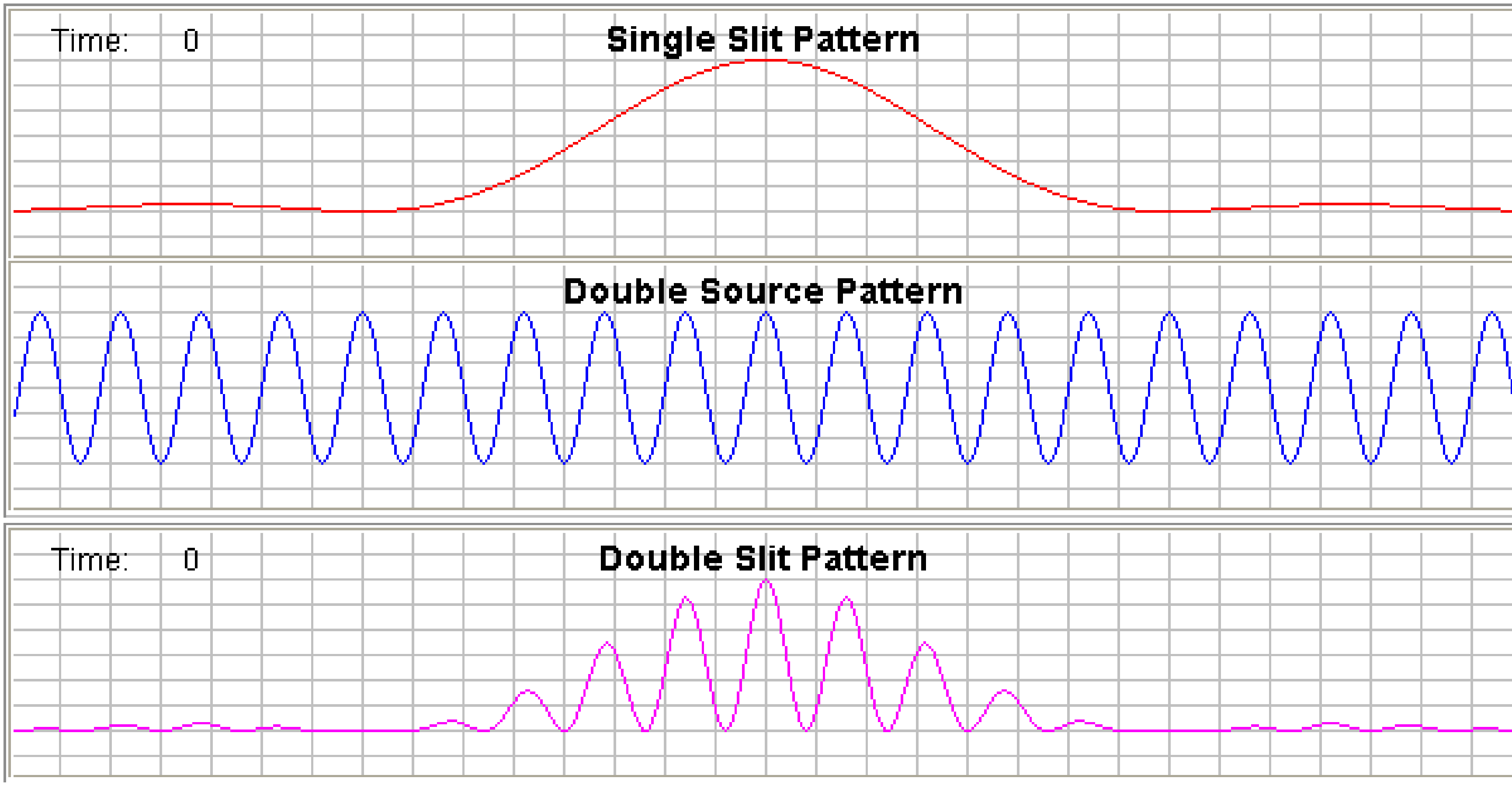
The double slit

If each slit sent out light uniformly in all directions, the peaks in the pattern would be equally bright, as in the "Double Source" picture.



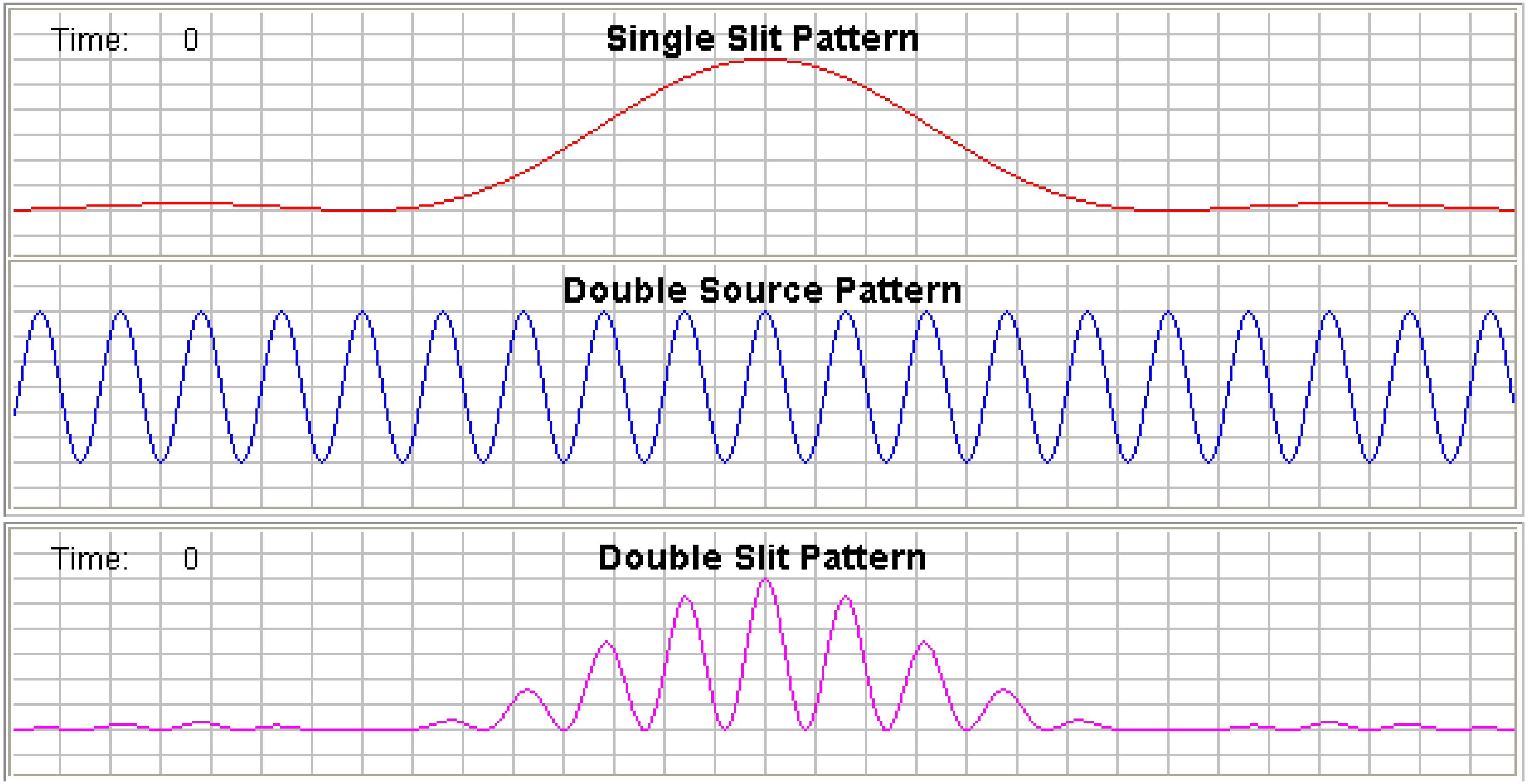
The double slit

Instead, each slit sends out a diffraction pattern, with most of the light in the central peak, as in the "Single Slit" picture.



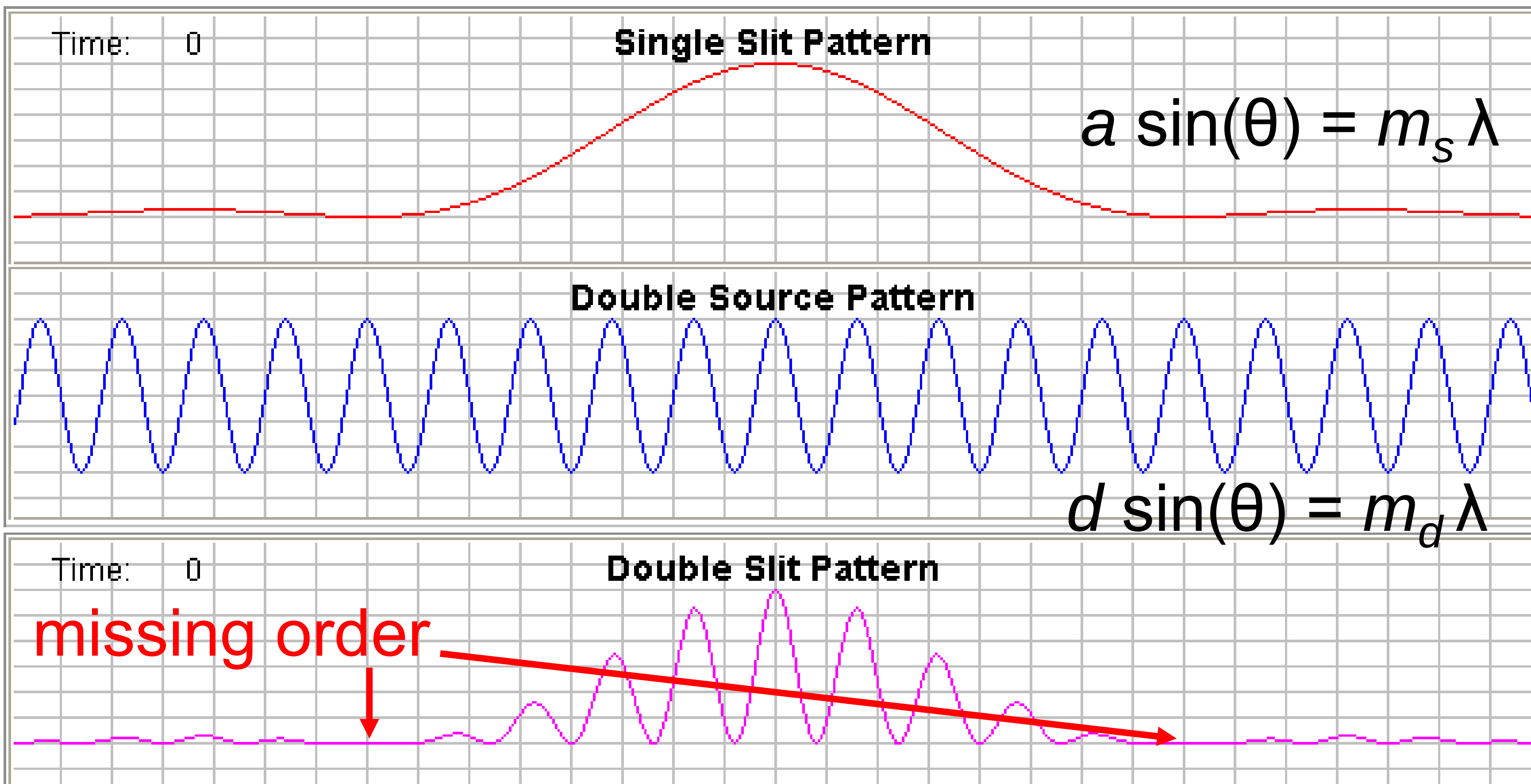
The double slit

Interference between the two diffraction patterns produces the “Double Slit” pattern shown at the bottom.



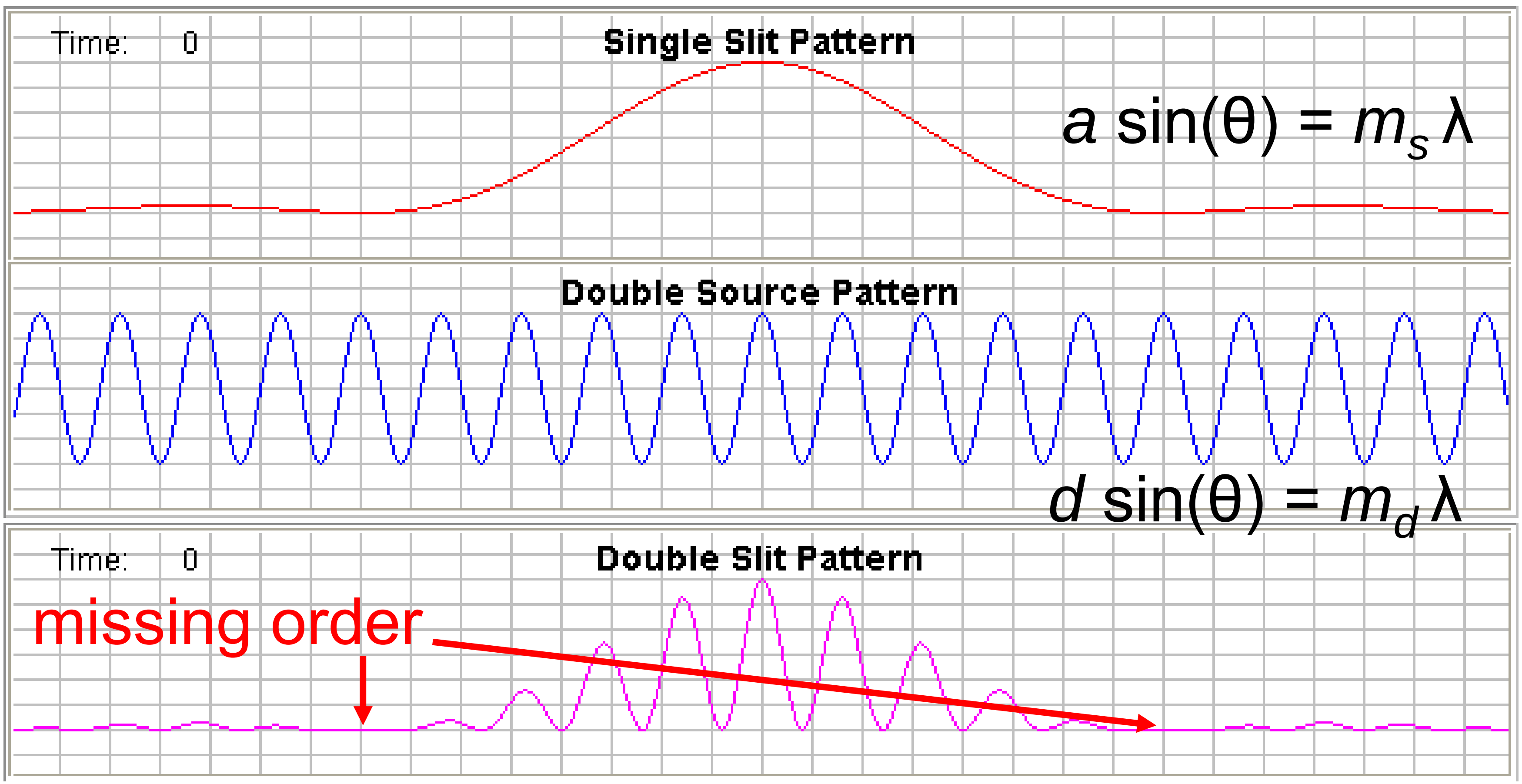
The double slit

The "Double Slit" pattern shows missing orders. Peaks predicted by the double-source equation are not present, because they coincide with zeros in the single slit pattern.



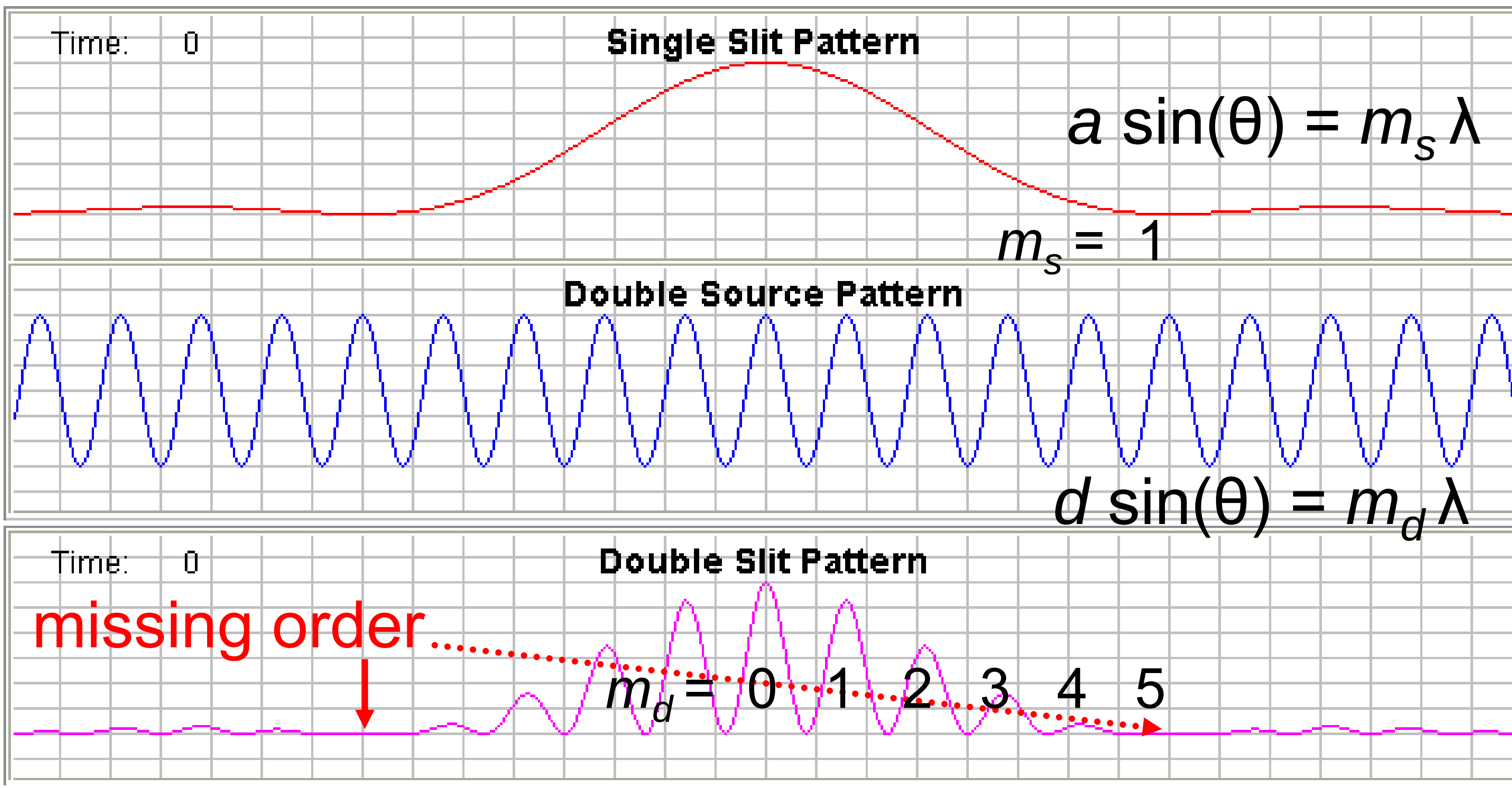
The double slit

What is the ratio of *d* to *a* in this double slit? It's an integer less than 10 – enter the number on your clicker.



The double slit

$$\frac{d\sin\theta}{a\sin\theta} = \frac{m_d\lambda}{m_s\lambda} \quad \Rightarrow \quad \frac{d}{a} = \frac{m_d}{m_s} = \frac{5}{1}$$



A bit of history

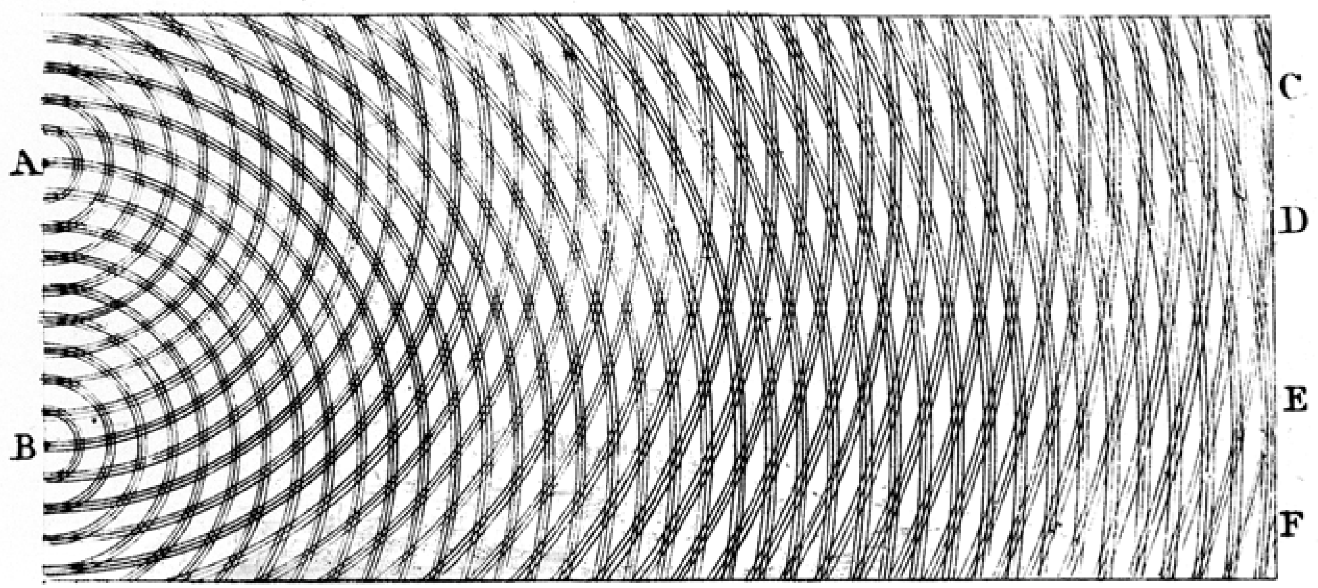
Prior to 1800, the two competing theories of light were:

1. Light acts as if it is made up of particles, as explained by Sir Isaac Newton (1643 – 1727) via his corpuscular theory.
2. Light acts as a wave, as promoted by the Dutch scientist Christiaan Huygens (1629 – 1695).

Given the stature of Newton, the particle model dominated.

A bit of history

Then, in 1801, along came Thomas Young with his double slit, an experiment that could only be explained in terms of light acting as a wave.

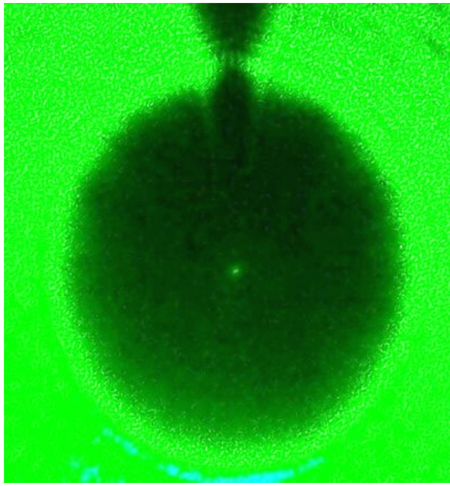


Thomas Young’s own diagram of double-slit interference. *Diagram from Wikipedia*.

A bit of history

The French scientist Augustin Fresnel presented his work on diffraction to the French Academy in 1818. Siméon Poisson, who did not believe the wave theory, was there. Poisson realized that, if the wave theory was correct, there should be a bright spot at the center of the shadow of a round object. Waves leaving the edge of the object would all be the same distance from the center of the shadow, and would thus interfere constructively – what nonsense!

Dominique Arago showed that there is such a bright spot, providing compelling evidence for the wave theory.



Circular apertures

When light passes through a circular aperture (opening), such as through the pupil in each of your eyes, a diffraction pattern of concentric circles is created. This has implications for resolving two objects.

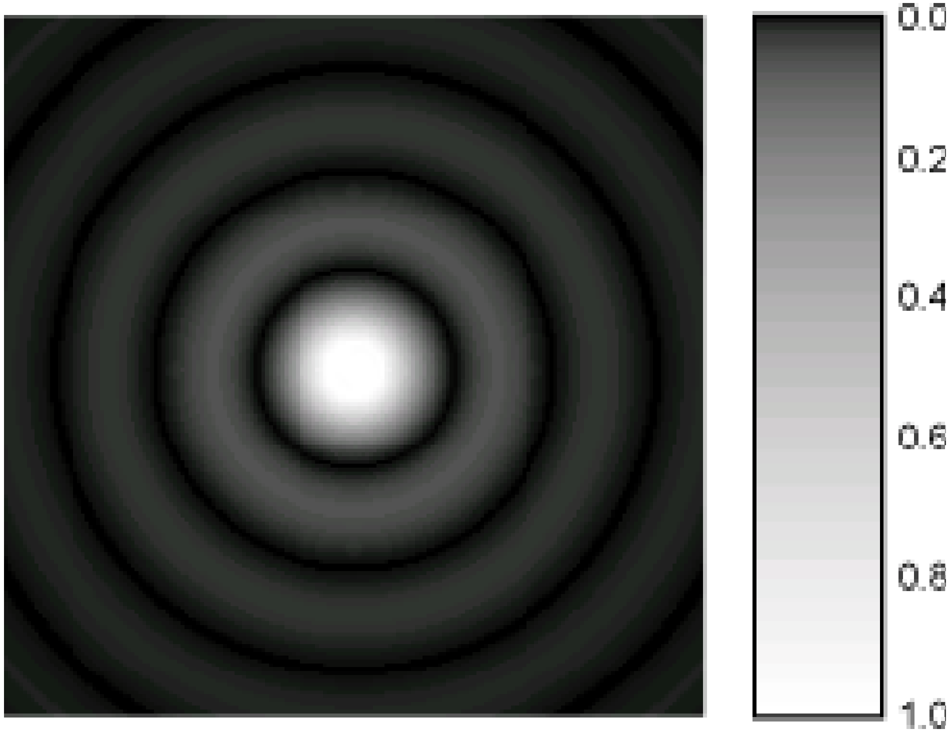
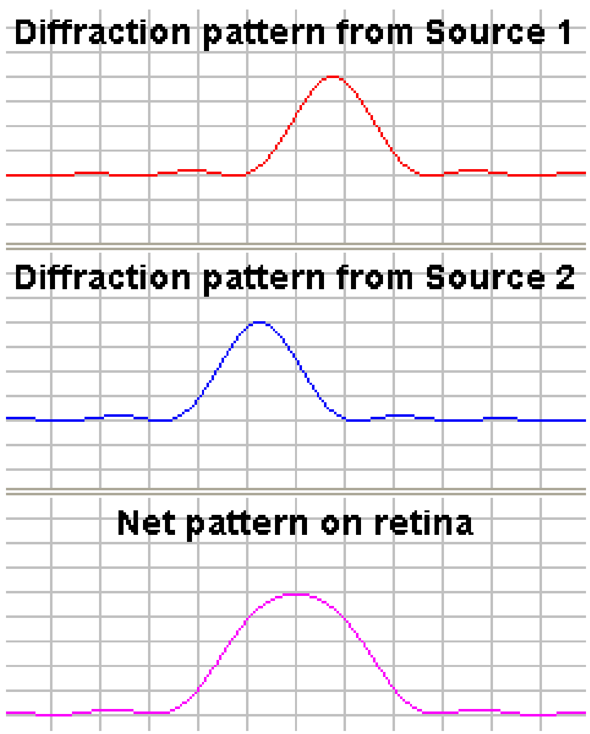


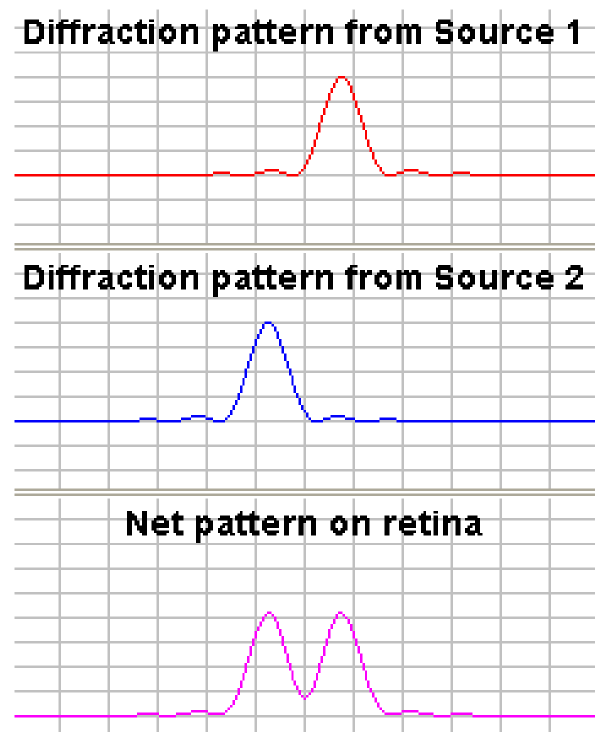
Diagram from Wikipedia

Circular apertures

You can tell that two objects are two (rather than one) as long as the peak in the diffraction pattern of one object is no closer to the other peak than the first zero in the other pattern.



Your eye in bright sunlight. Small pupil = big spread.



Your eye in the dark. Larger pupil = less spreading.

Circular apertures

The minimum angular separation between two objects for them to be resolved is:

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

where *D* is the diameter of the opening.

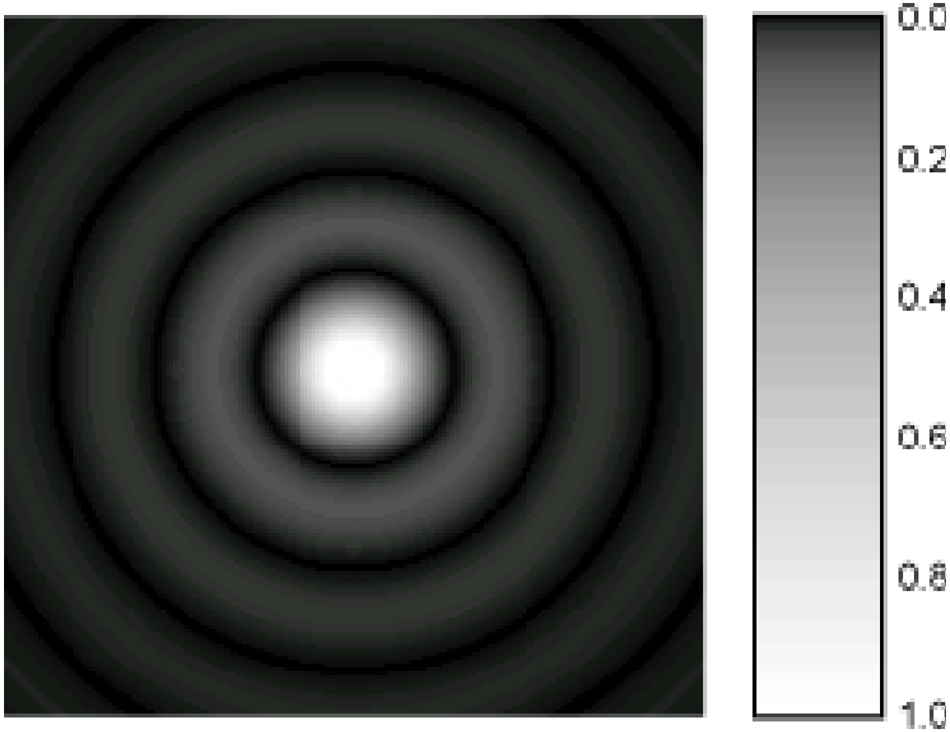


Diagram from Wikipedia

Thin-film interference

Interference between light waves is the reason that thin films, such as soap bubbles, show colorful patterns.

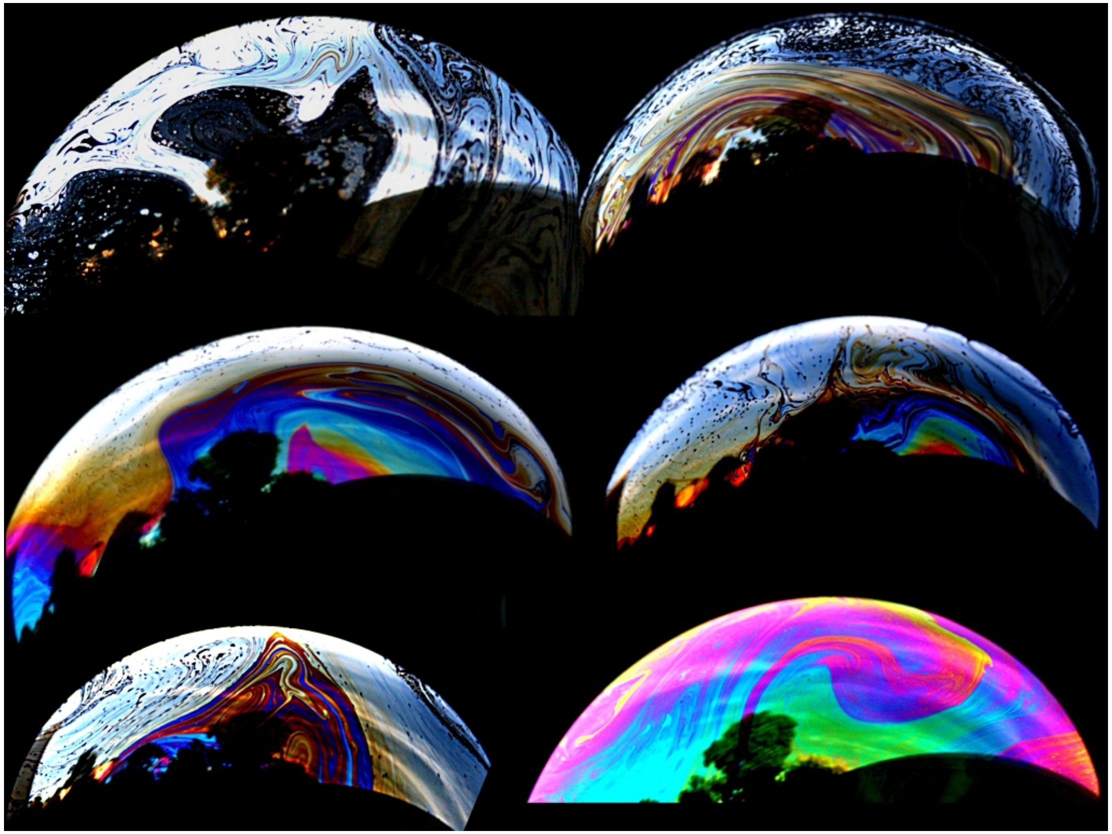


Photo credit: Mila Zinkova, via Wikipedia

Thin-film interference

This is known as thin-film interference - interference between light waves reflecting off the top surface of a film with waves reflecting from the bottom surface. To obtain a nice colored pattern, the thickness of the film has to be comparable to the wavelength of light.

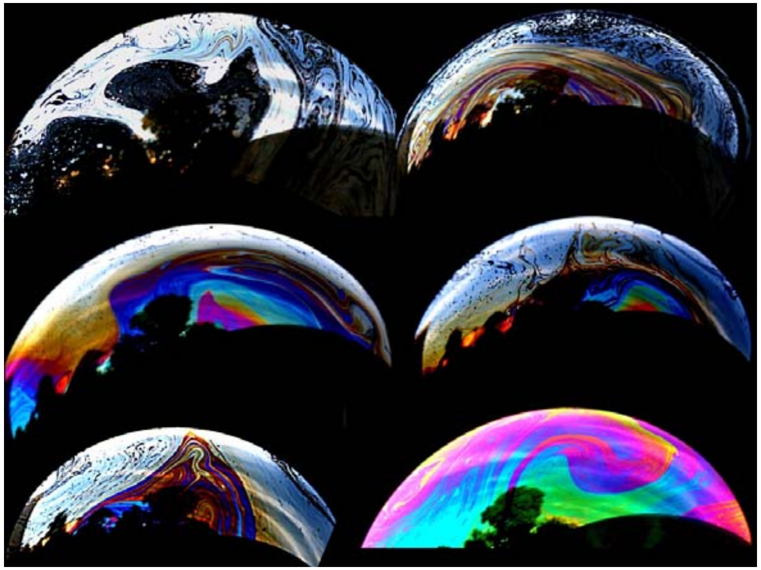
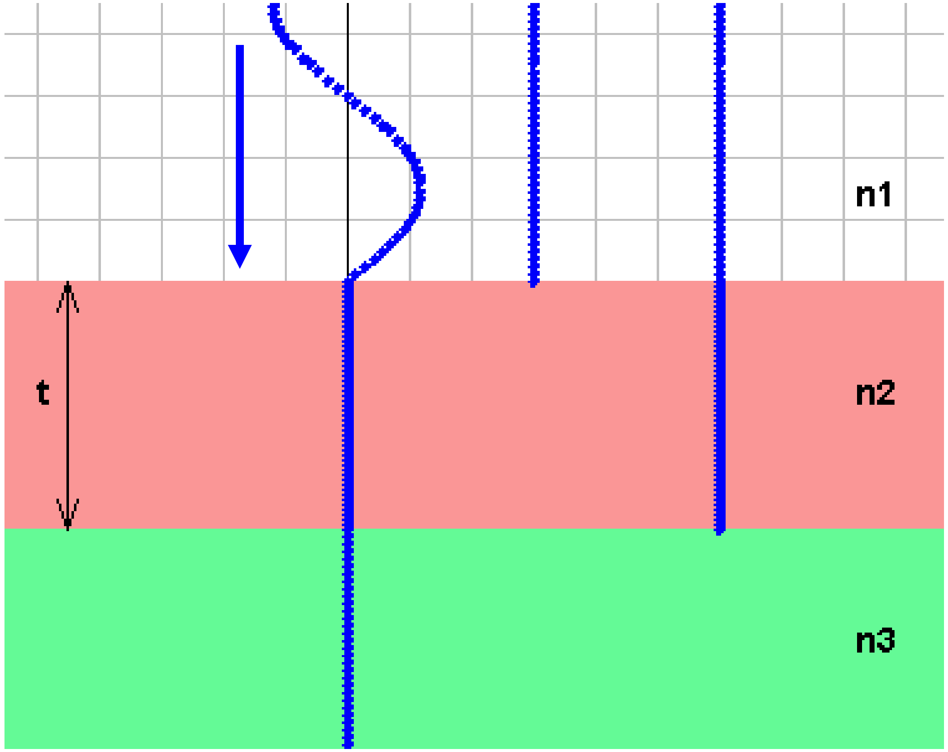


Photo credit: Mila Zinkova, via Wikipedia

Thin films – sequence of events

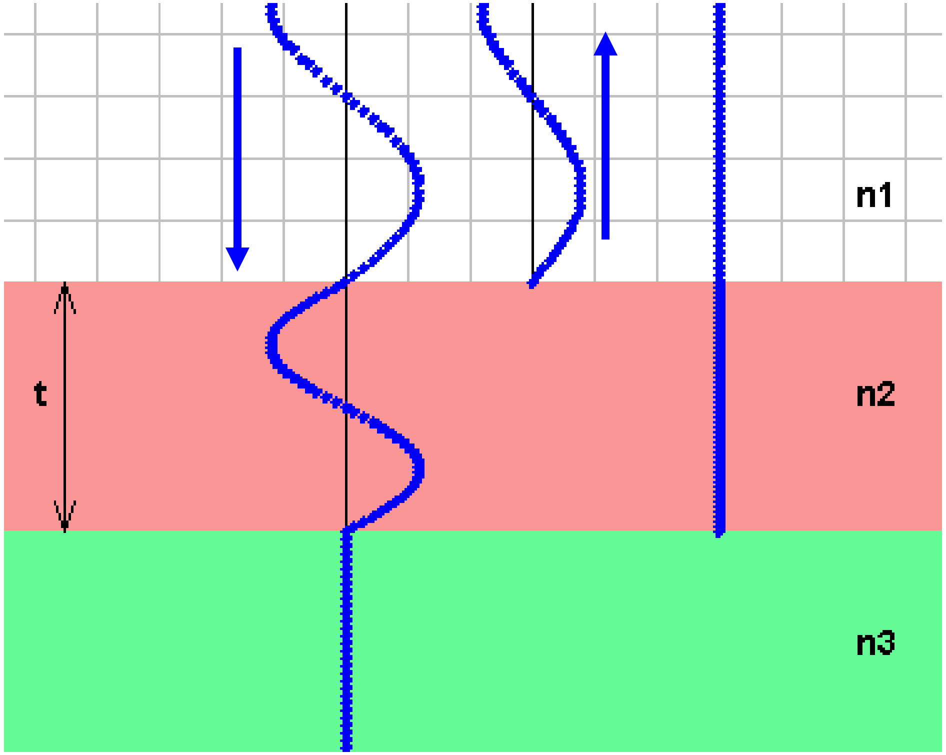
1. A wave is incident on the top surface of the film.

Simulation



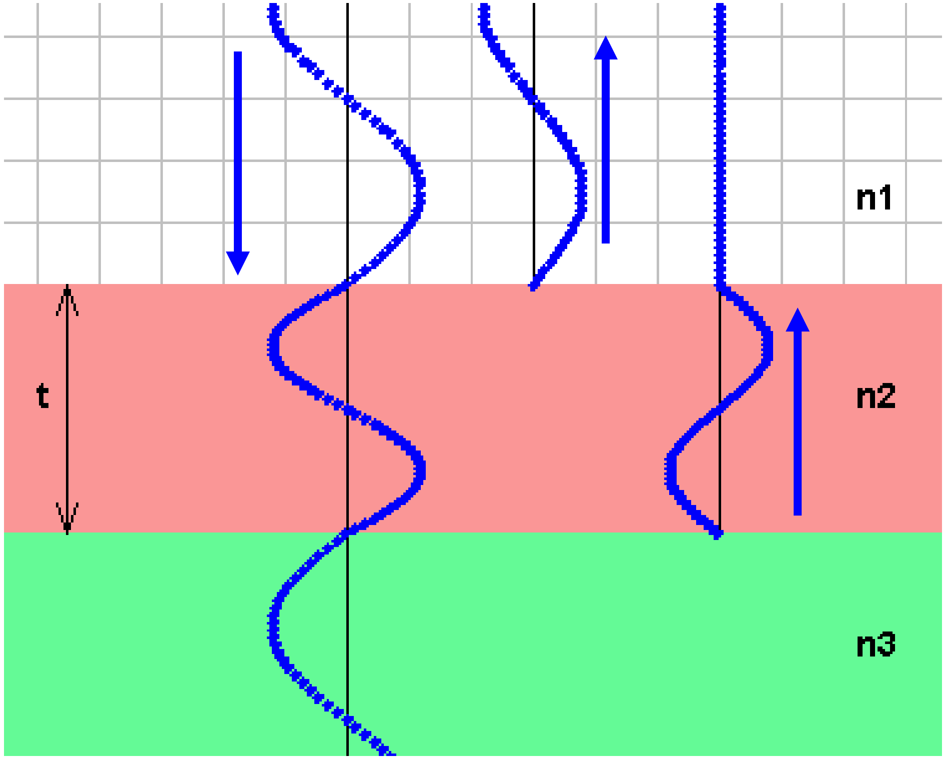
Thin films – sequence of events

2. The incident wave is partly reflected (possibly experiencing an inversion) and partly transmitted. The reflected wave is moved to the right here so we can see it.



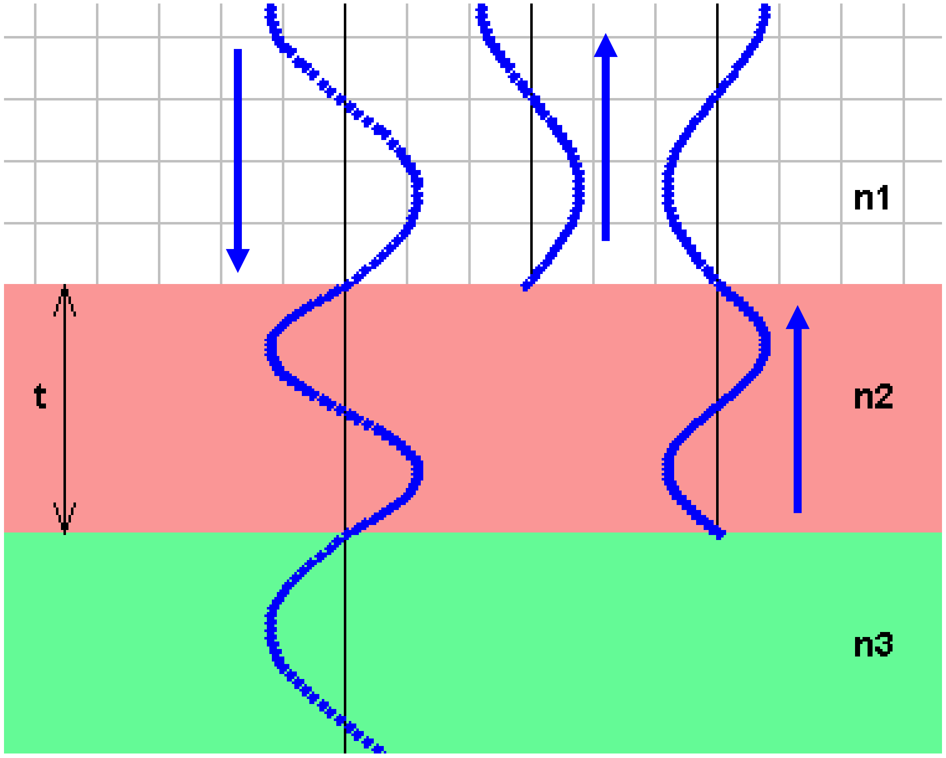
Thin films – sequence of events

3. The wave also partly reflects off the bottom surface of the film (the pink material), possibly experiencing an inversion. The reflected wave is moved to the far right.



Thin films – sequence of events

4. The two reflected waves interfere with one another. The film thickness needs to be just right if we want completely constructive or completely destructive interference.

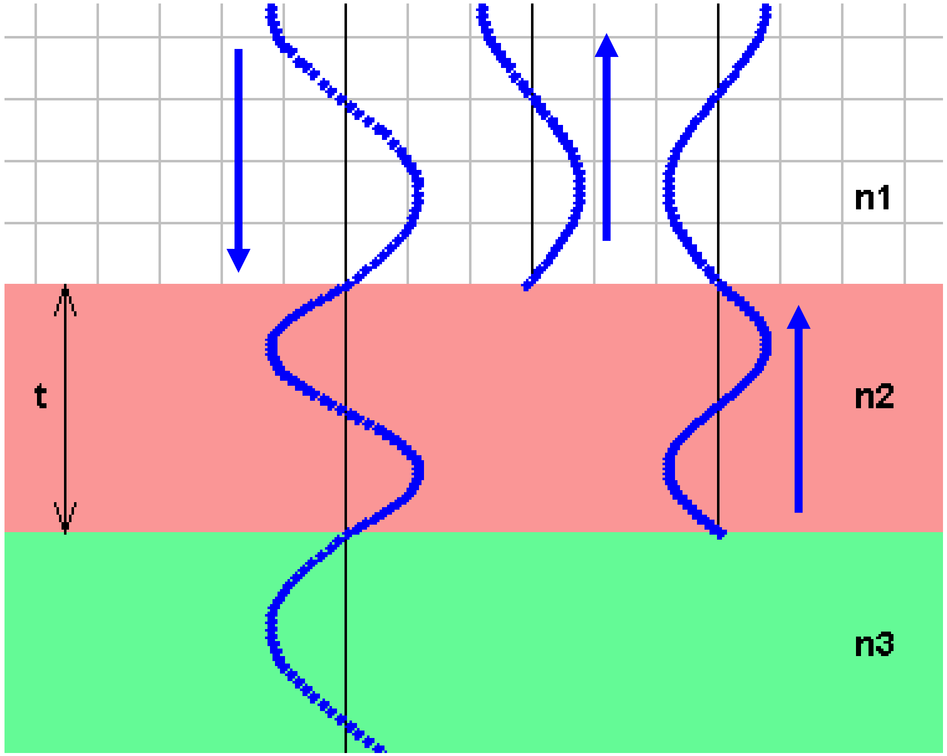


What kind of interference?

In this case, the film thickness is exactly one wavelength, so the wave that reflects off the bottom surface of the film travels a down-and-back extra distance of 2 wavelengths compared to the wave reflecting off the top surface.

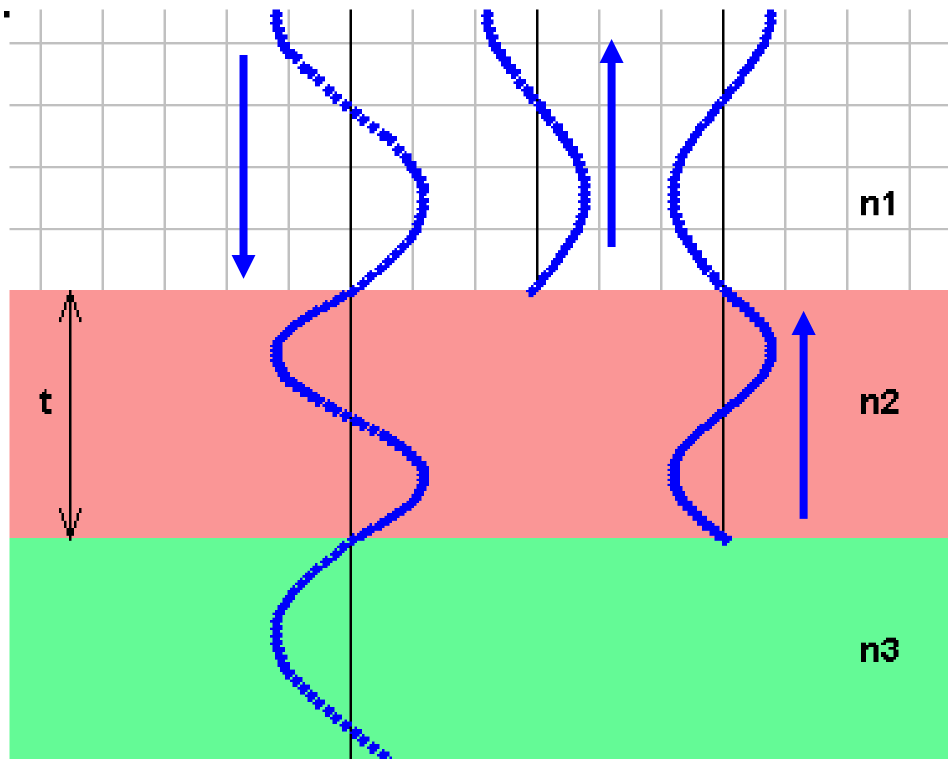
What kind of interference do we get between the two reflected waves?

1. Constructive
2. Destructive



What kind of interference?

Even though the extra distance traveled is an integer number of wavelengths, we can see that the reflected waves interfere destructively. This is because the wave reflecting off the top surface is inverted, which is like an extra half-wavelength shift.

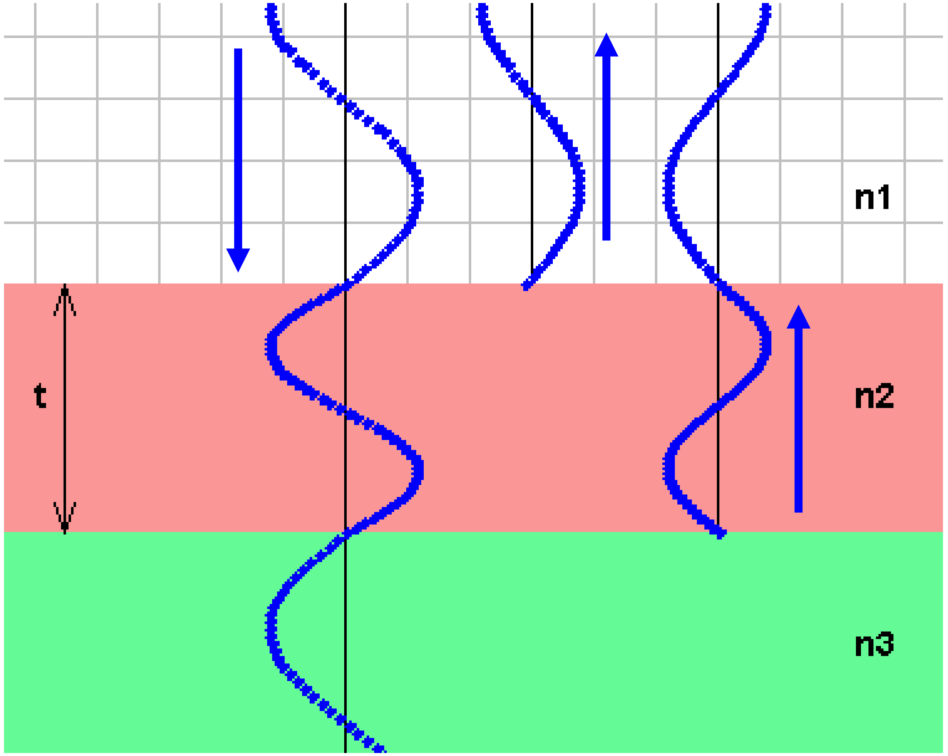


Thin films – a systematic approach

Let's use a five-step method to analyze thin films.

The basic idea is to determine the effective path-length difference between the wave reflecting from the top surface of the film and the wave reflecting from the bottom surface.

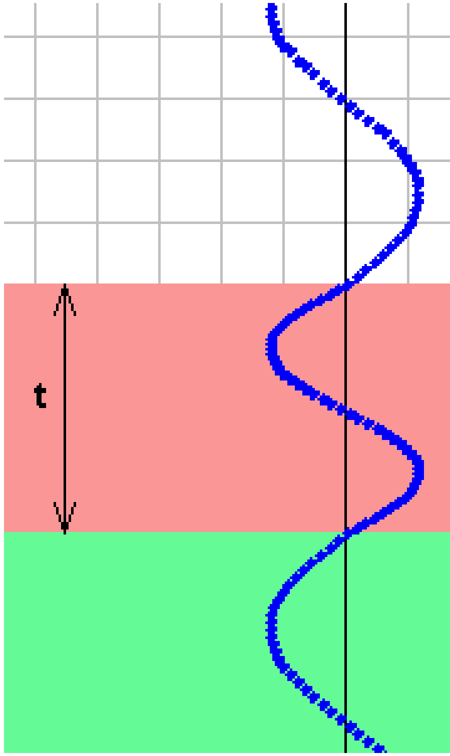
The effective path-length difference accounts for the extra distance of 2*t* traveled by the wave that reflects from the bottom surface, and any inversions upon reflection.



Thin films – a systematic approach

For a wave that gets inverted when it reflects, that is equivalent to a half-wavelength shift.

However, we have three media, and thus three different wavelengths! Because we're trying to match the wave that goes into the film with the wave bouncing off the top, it is the wavelength in the film, λ_{film} , that appears in the equations.

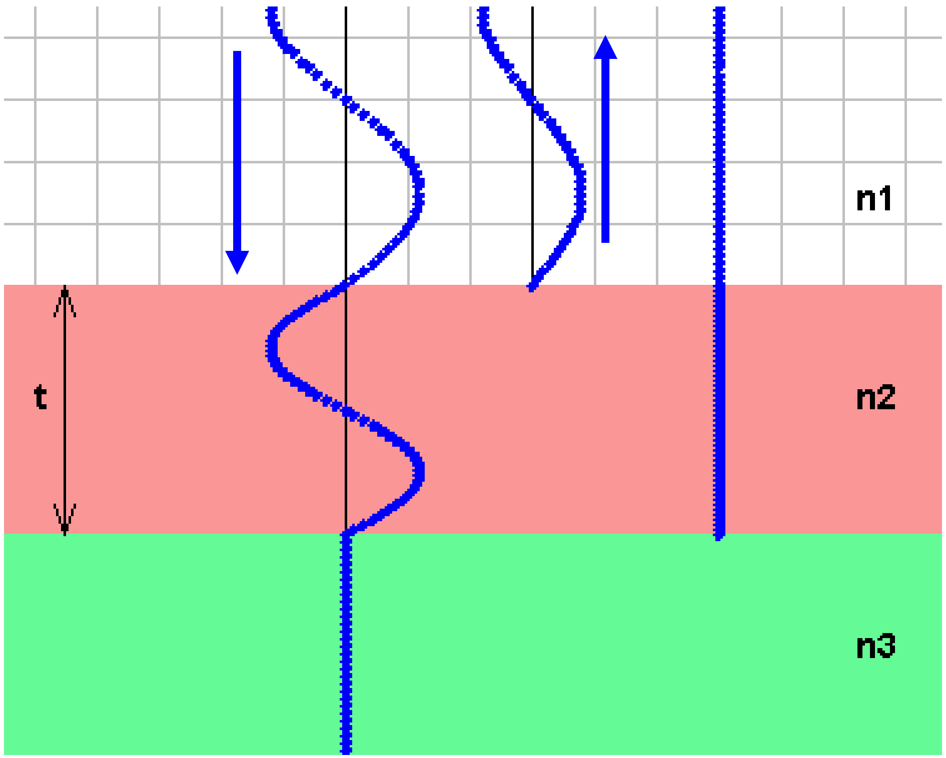


Thin films – the five-step method

Step 1 – Determine Δ_t , the shift for the wave reflecting from the top surface of the film.

If $n_2 > n_1$, $\Delta_t = \dfrac{\lambda_{film}}{2}$

If $n_2 < n_1$, $\Delta_t = 0$



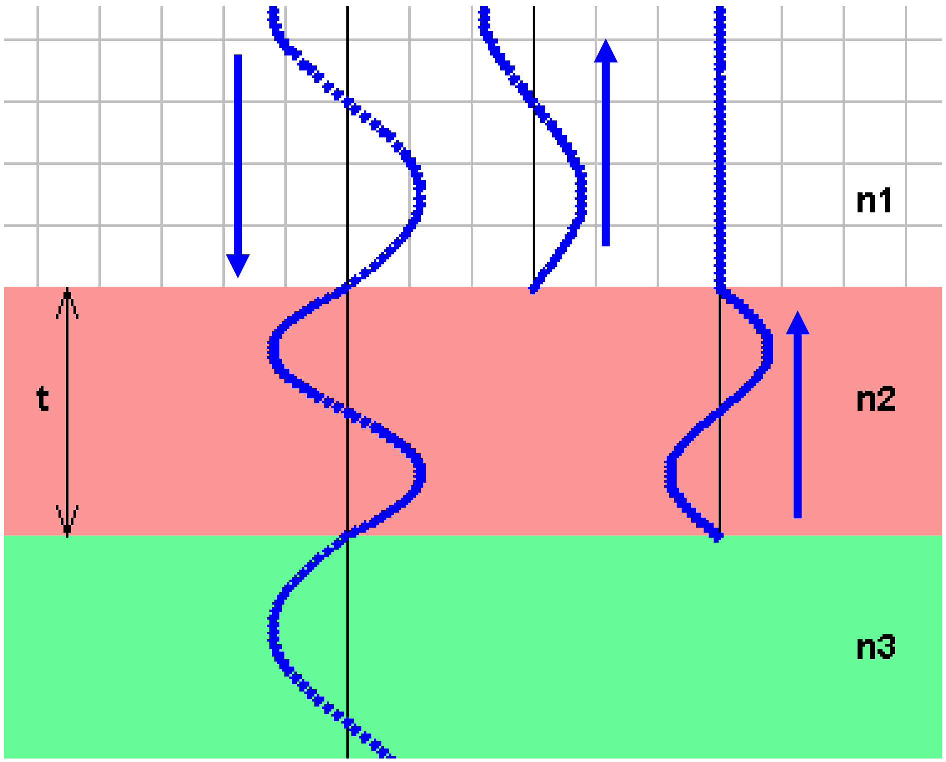
Thin films – the five-step method

Step 2 – Determine Δ_b , the shift for the wave reflecting from the bottom surface of the film. We have at least 2*t*, from the extra distance traveled.

If $n_3 > n_2$,

$$\Delta_b = 2t + \frac{\lambda_{film}}{2}$$

If $n_3 < n_2$, $\Delta_b = 2t$



Thin films – the five-step method

Step 3 – Find the effective path-length difference, Δ.

$$\Delta = \Delta_b - \Delta_t$$

Thin films – the five-step method

Step 4 – Bring in the appropriate interference condition, depending on the situation.

For constructive interference, $\Delta = m\lambda_{film}$

For destructive interference, $\Delta = (m + 1/2)\lambda_{film}$

Thin films – the five-step method

Step 5 – Solve the resulting equation. The equation generally connects the thickness of the film to the wavelength of the light in the film.

It is often useful to remember that $\lambda_{film} = \dfrac{\lambda_{vacuum}}{n_{film}}$

An example using the five-step method

White light in air shines on an oil film of thickness *t* that floats on water. The oil has an index of refraction of 1.50, while the refractive index of water is 1.33.

When looking straight down at the film, the reflected light looks orange, because the film thickness is just right to produce completely constructive interference for a wavelength, in air, of 600 nm.

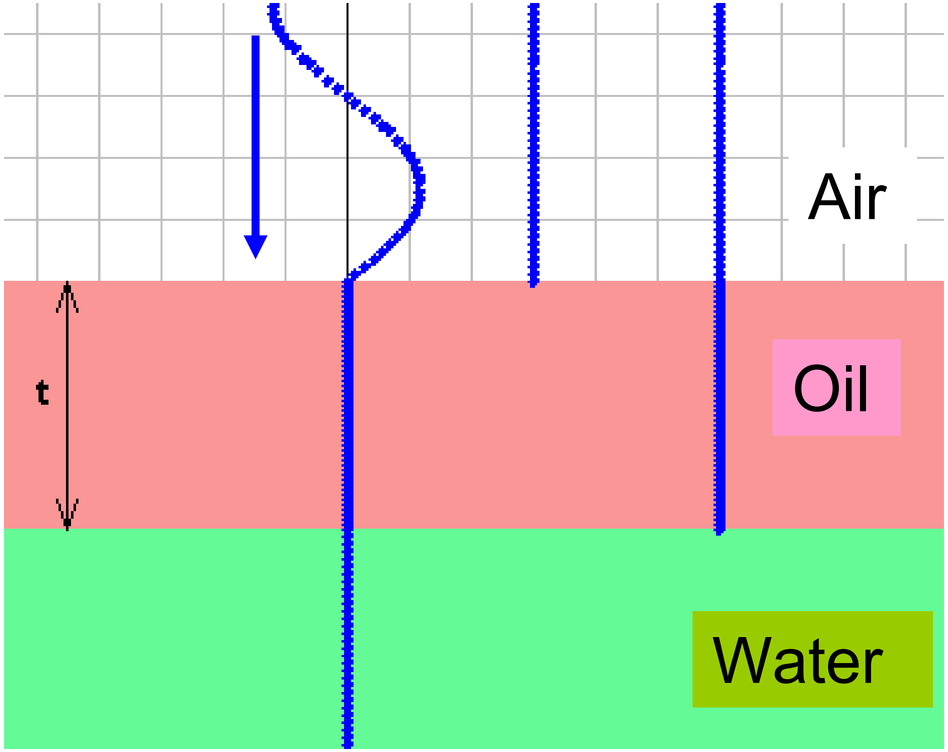
What is the minimum possible thickness of the film?

Step 1

Step 1 – Determine Δ_t , the shift for the wave reflecting from the top surface of the film.

1. $\Delta_t = \dfrac{\lambda_{film}}{2}$

2. $\Delta_t = 0$

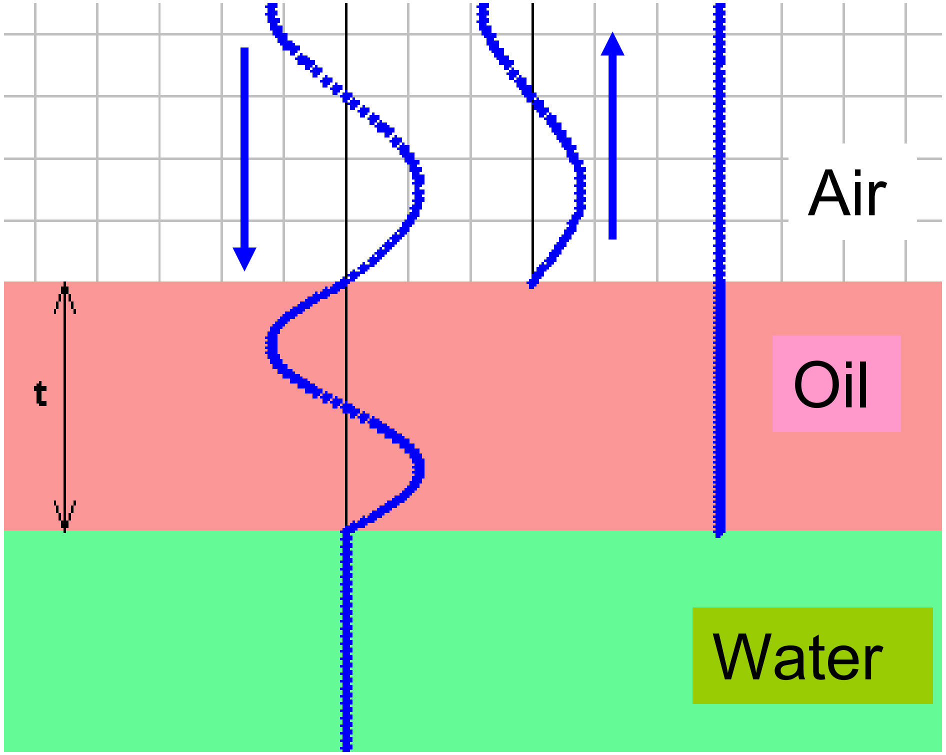


Step 2

Step 2 – Determine Δ_b , the shift for the wave reflecting from the top surface of the film.

1. $\Delta_b = 2t + \frac{\lambda_{film}}{2}$

2. $\Delta_b = 2t$



Step 3

Step 3 – Determine Δ , the effective path-length difference for the two reflected waves.

1. $\Delta = 2t + \frac{\lambda_{film}}{2}$

2. $\Delta = 2t$

3. $\Delta = 2t - \frac{\lambda_{film}}{2}$

Step 4

Step 4 – Bring in the appropriate interference condition.

1. $$2t - \frac{\lambda_{film}}{2} = m\lambda_{film}$$

2. $$2t - \frac{\lambda_{film}}{2} = (m + 1/2)\lambda_{film}$$

Step 4

In this situation, we were told that the film thickness was the minimum necessary to give constructive interference for a particular wavelength, so let's go with the first choice.

1. $$2t - \frac{\lambda_{film}}{2} = m\lambda_{film}$$

Re-arrange to get: $2t = (m + 1/2)\lambda_{film}$

This looks like destructive interference, but it is not!

Step 5

Step 5 – Solve for the minimum film thickness.

1. $t_{\text{min}} = 100\ \text{nm}$

2. $t_{\text{min}} = 150\ \text{nm}$

3. $t_{\text{min}} = 300\ \text{nm}$

4. $t_{\text{min}} = 450\ \text{nm}$

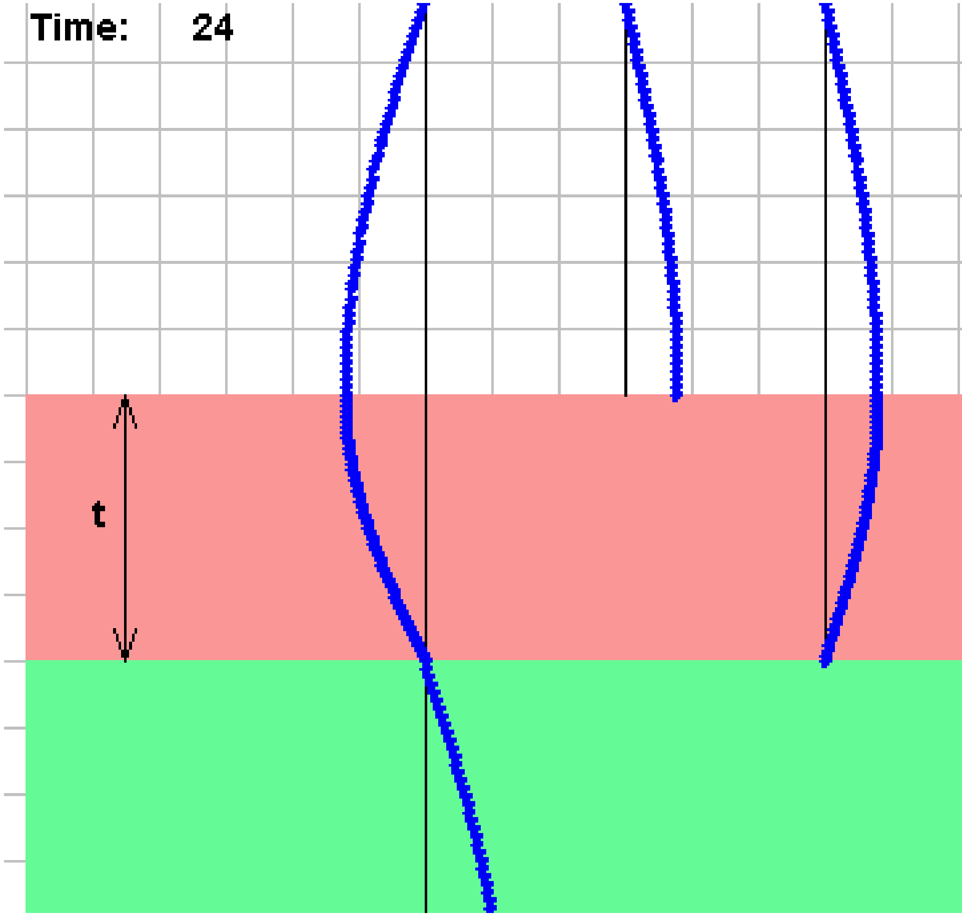
Step 5

$$2t = (m + 1/2)\lambda_{film}$$

$$= (m + 1/2)\frac{\lambda_{vacuum}}{n_{film}}$$

$$= (m + 1/2)\frac{600\text{ nm}}{1.5}$$

$$= (m + 1/2)(400\text{ nm})$$

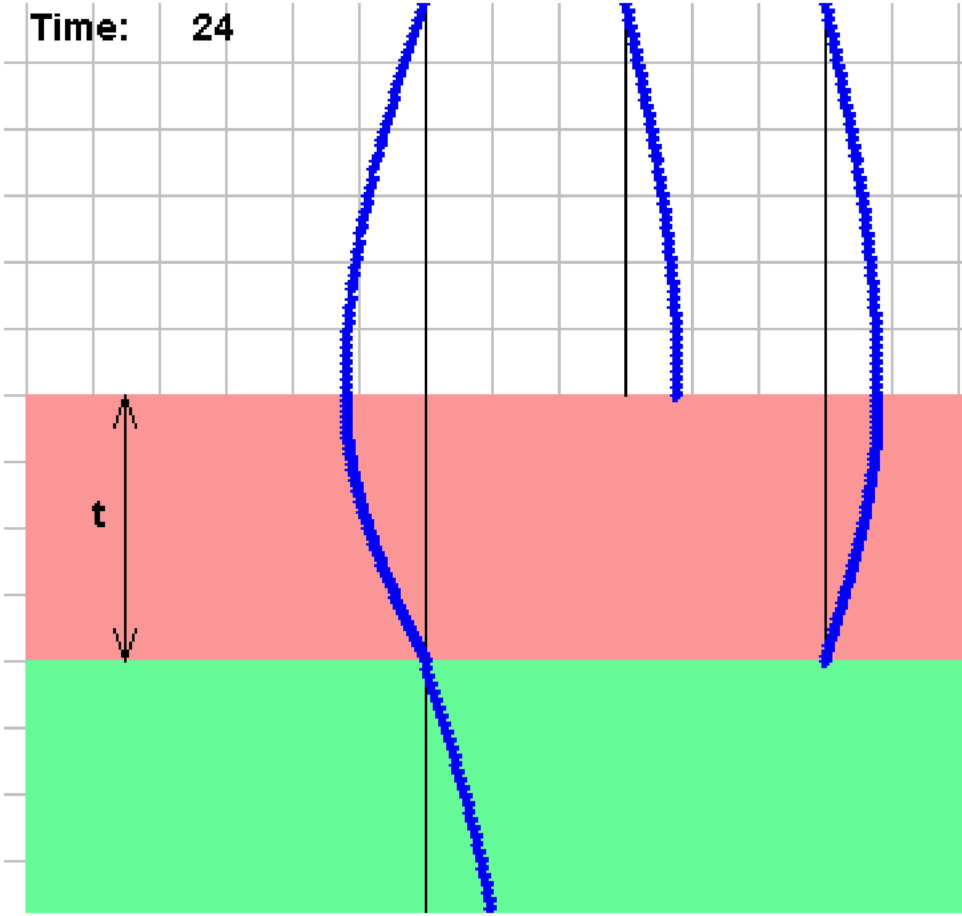


Step 5

$$2t = (m + 1/2)(400\ \text{nm})$$

To find the minimum *t*, use the smallest *m* that makes sense, which in this case is

m = __



Step 5

$$2t = (m + 1/2)(400\ \text{nm})$$

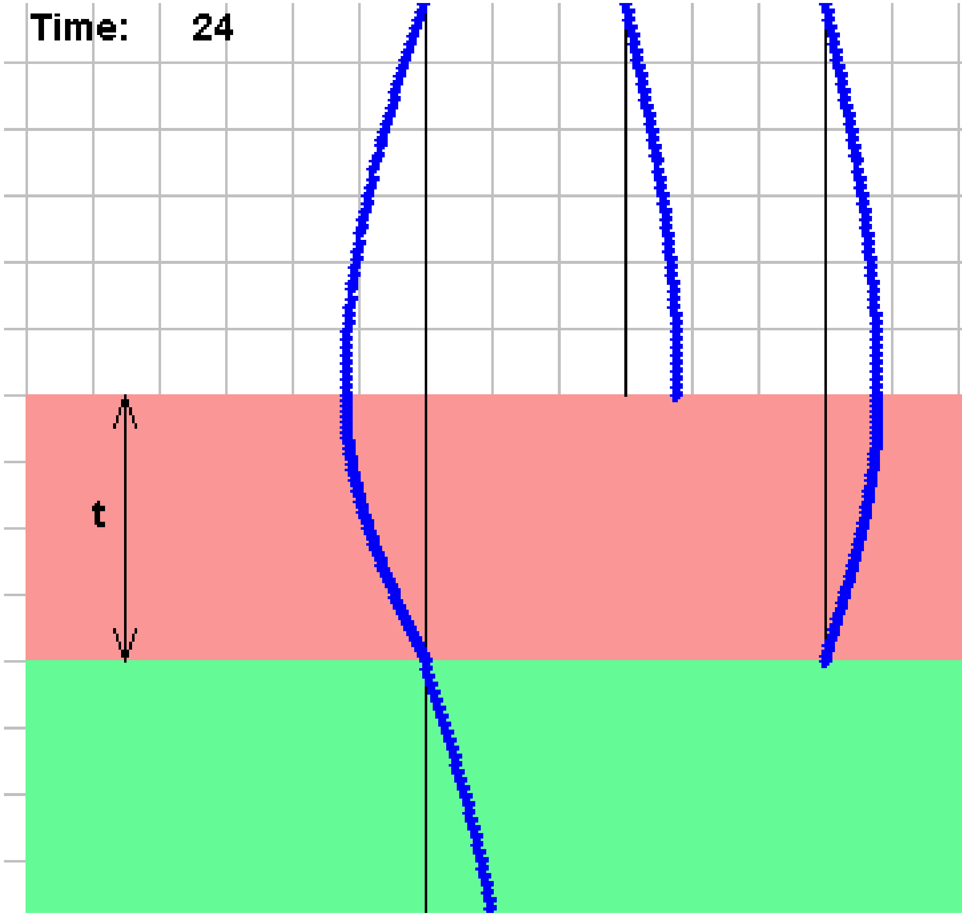
To find the minimum *t*, use the smallest *m* that makes sense, which in this case is

m = 0

$$2t_{\min} = (0 + 1/2)(400\ \text{nm})$$

$$= 200\ \text{nm}$$

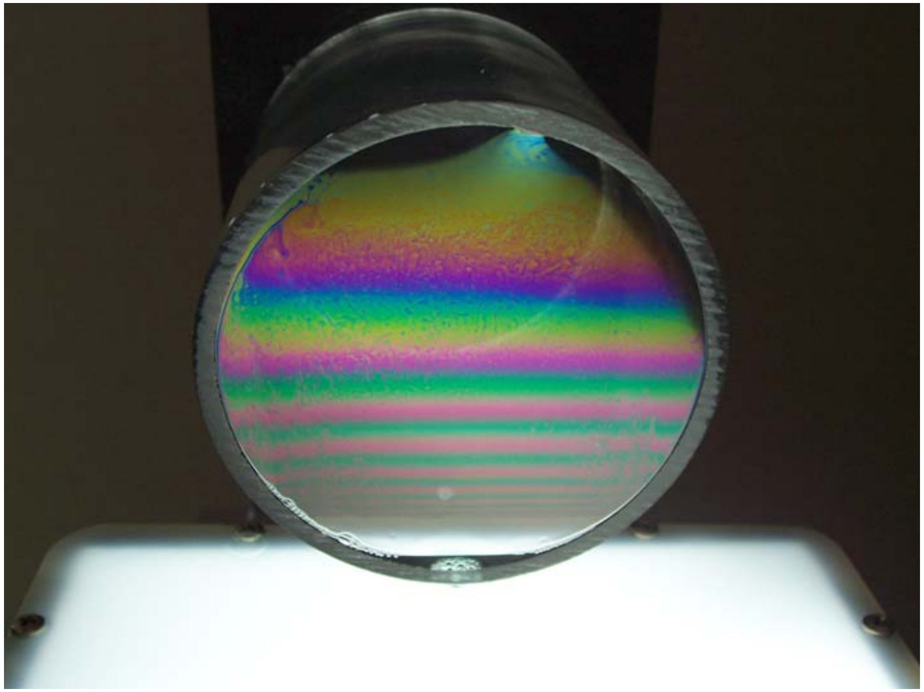
$$t_{\min} = 100\ \text{nm}$$



A soap film

We make a soap film by dipping a loop into soap solution, and then hold the loop so it is vertical.

Why do we get horizontal bands on the soap film?

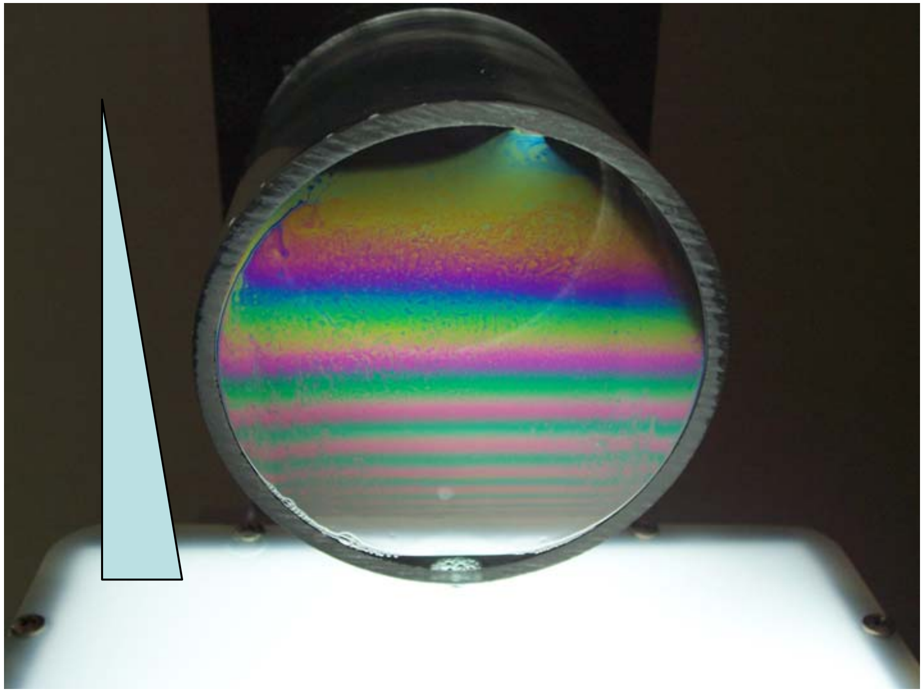


A soap film

We make a soap film by dipping a loop into soap solution, and then hold the loop so it is vertical.

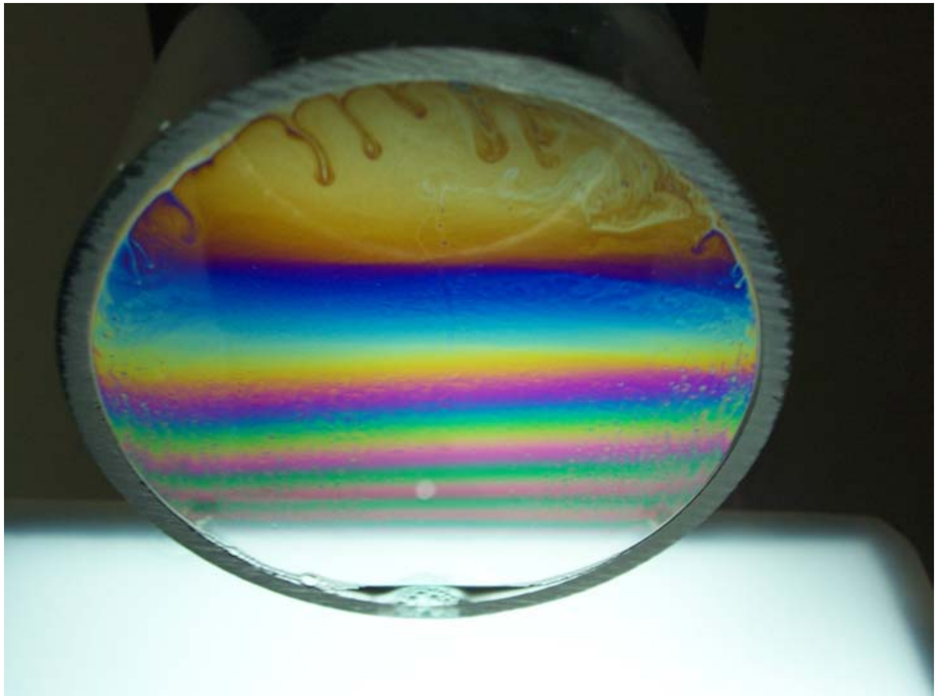
Why do we get horizontal bands on the soap film?

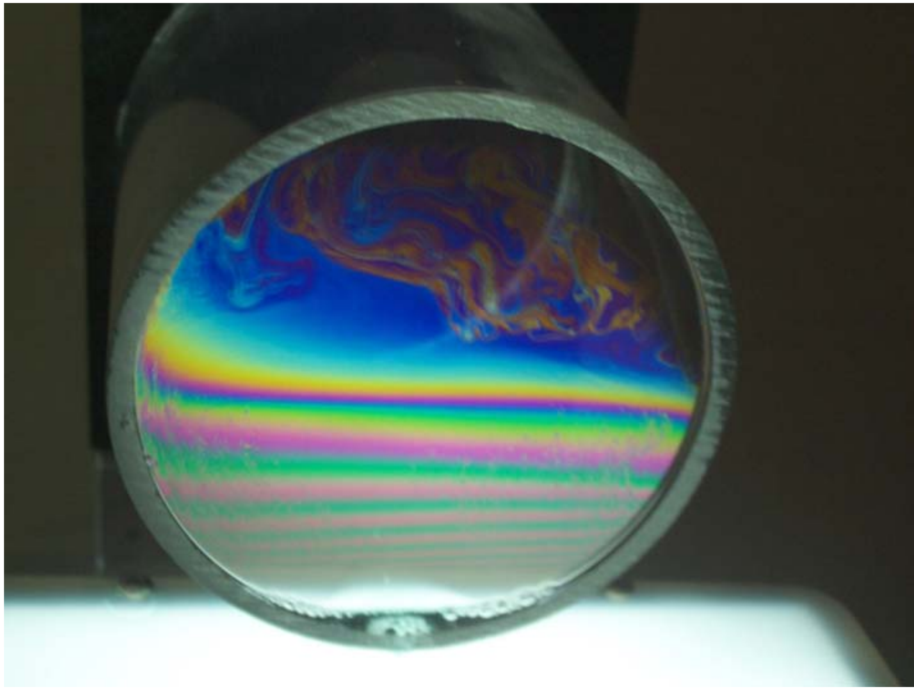
Gravity causes the film to be thicker at the bottom, with decreasing thickness as you move up. Different thicknesses correspond to different colors.



A soap film

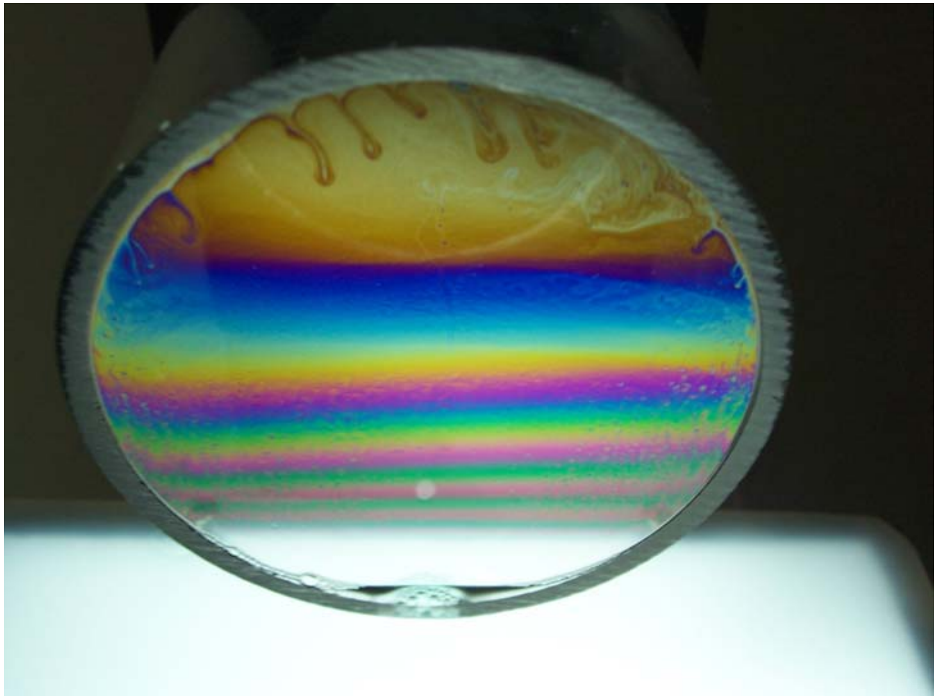
As time goes by, the film gets increasingly thin, with the top of the film first going white/gold, and then black (non-reflective for all colors). Why does the film go black at the top before popping?

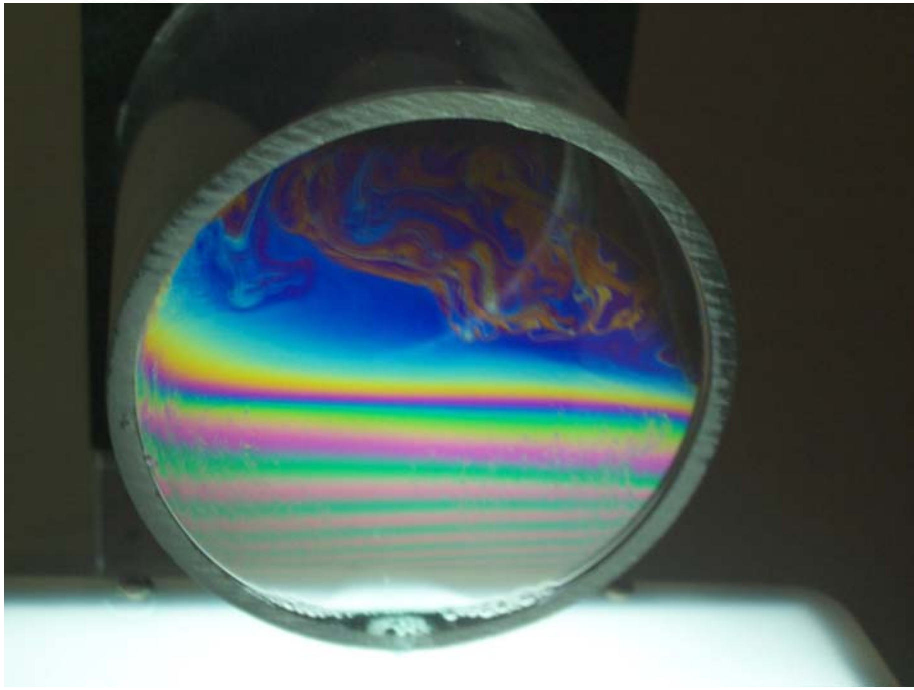




A soap film

Let’s start the five-step method, recognizing that we have a thin film of water surrounded by air.

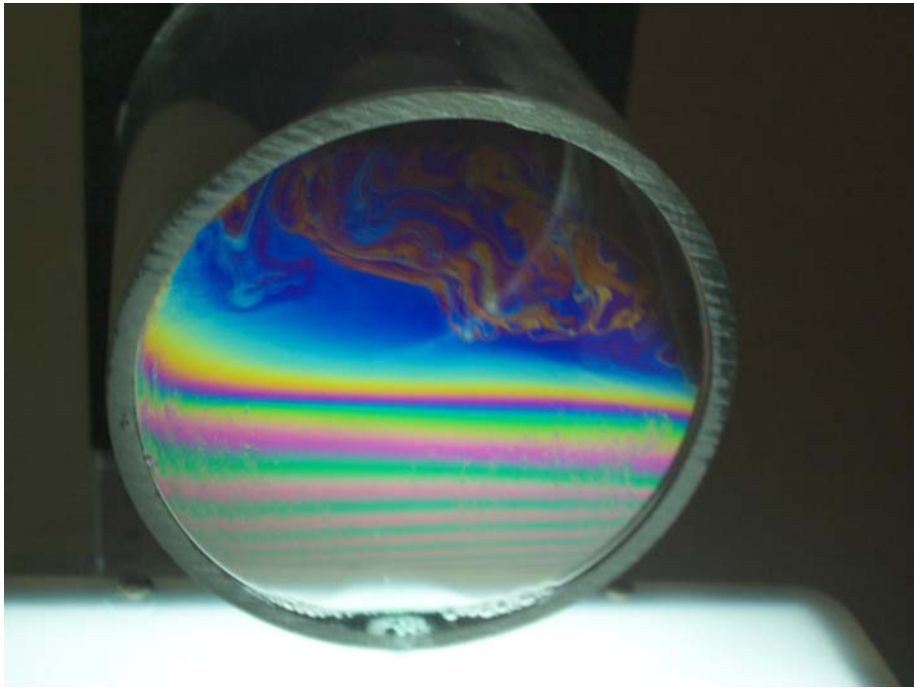




A soap film

Step 1 – Determine Δ_t , the shift for the wave reflecting from the top (or front) surface of the film.

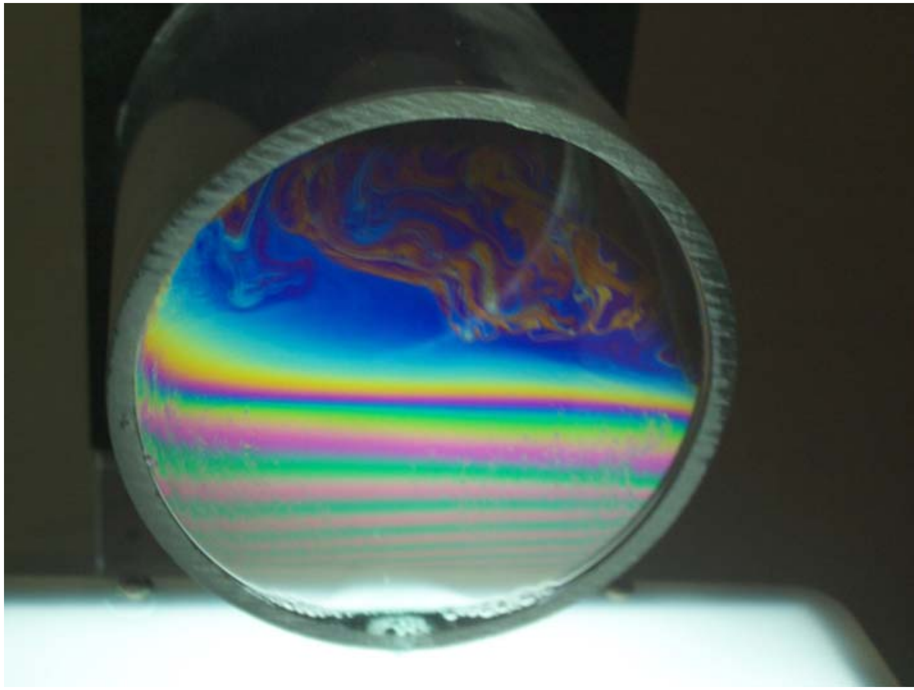
$n_2 > n_1$, so $\Delta_t = \dfrac{\lambda_{film}}{2}$



A soap film

Step 2 – Determine Δ_b , the shift for the wave reflecting from the bottom (or back) surface of the film.

$n_3 < n_2$, so $\Delta_b = 2t$

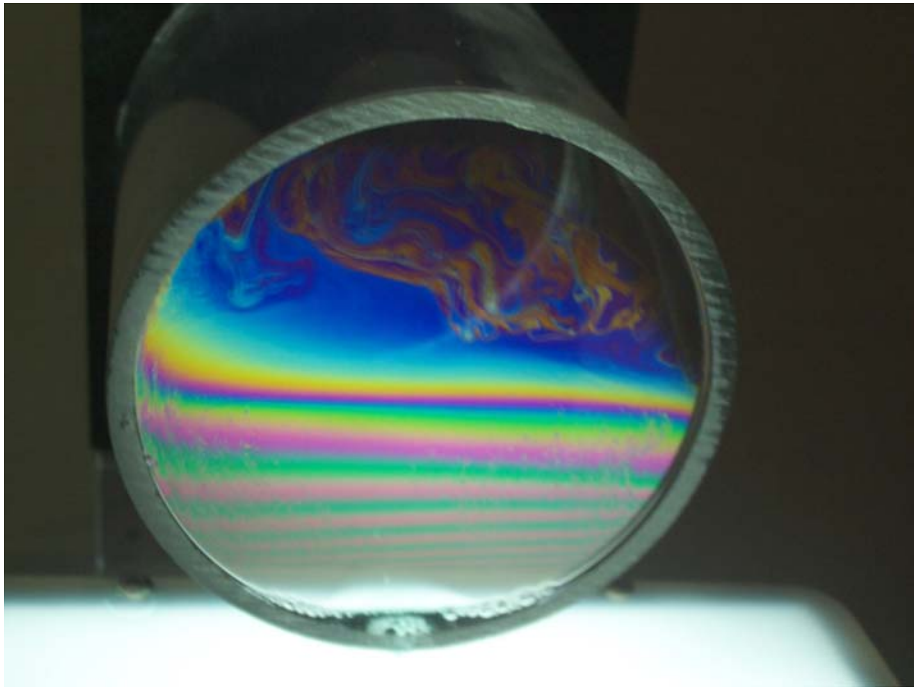


A soap film

Step 3 – Determine Δ , the effective path-length difference.

$$\Delta = \Delta_b - \Delta_t = 2t - \frac{\lambda_{film}}{2}$$

What happens in the limit that the film thickness, *t*, approaches zero?



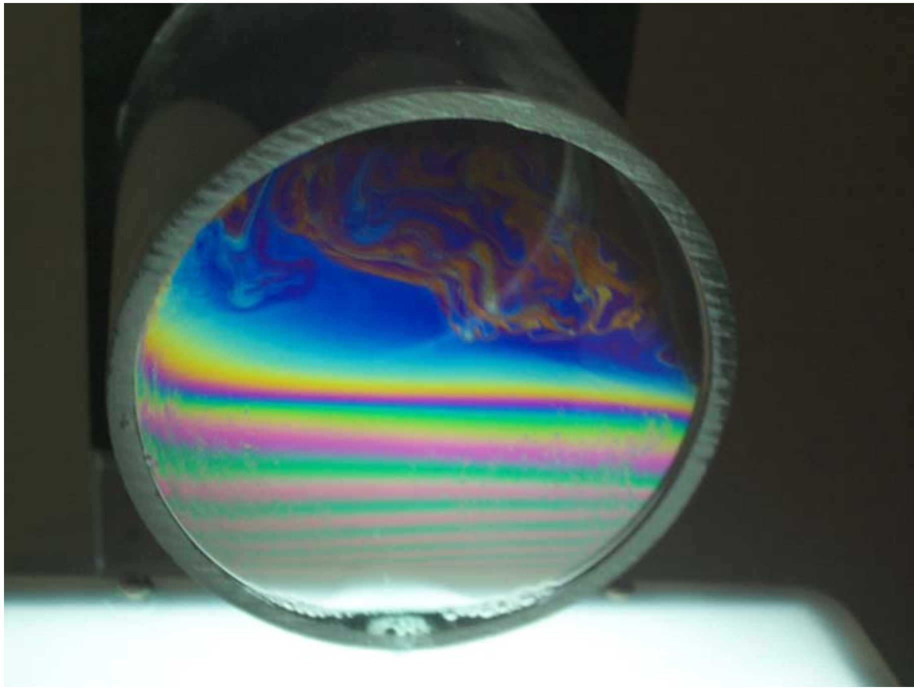
A soap film

Step 3 – Determine Δ , the effective path-length difference.

$$\Delta = \Delta_b - \Delta_t = 2t - \frac{\lambda_{film}}{2}$$

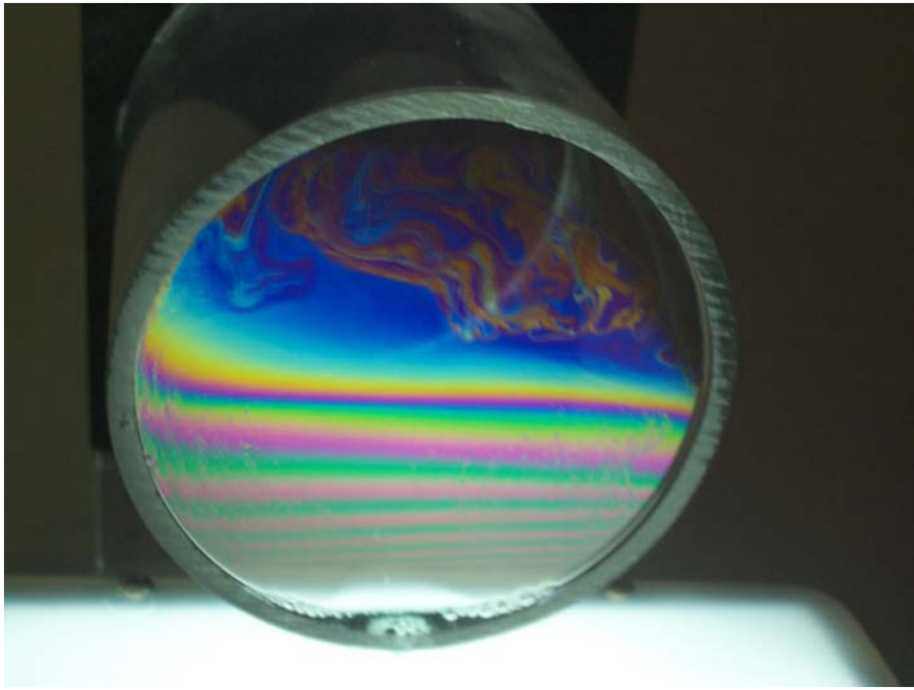
What happens in the limit that the film thickness, *t*, approaches zero?

The effective path-length difference is half a wavelength, giving destructive interference.



Waxing philosophical

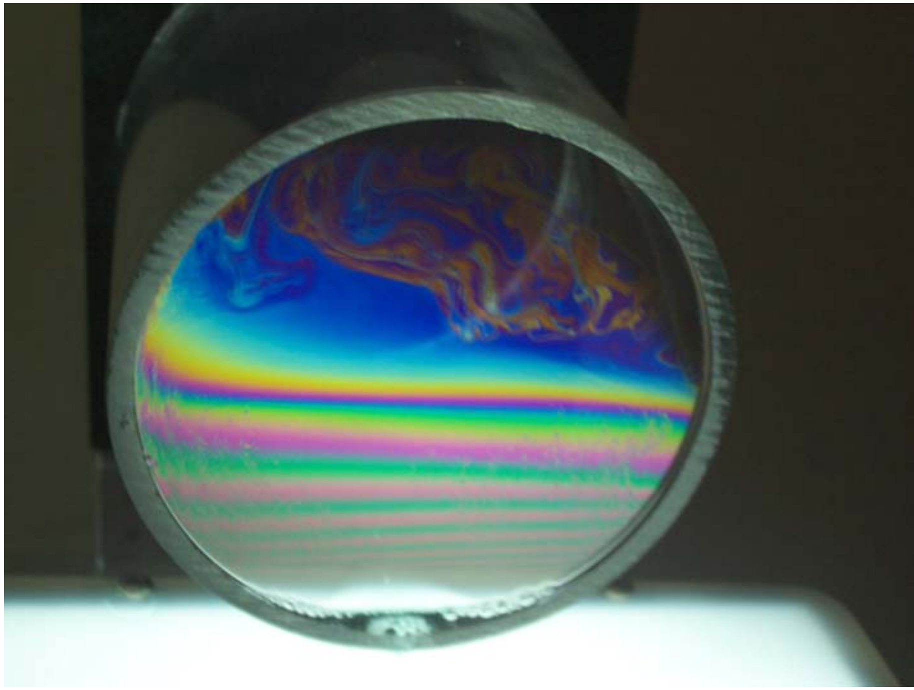
If the interference is destructive, we see no light reflecting from the film. Where does it go?



Waxing philosophical

If the interference is destructive, we see no light reflecting from the film. Where does it go?

It is all transmitted through the film, instead.

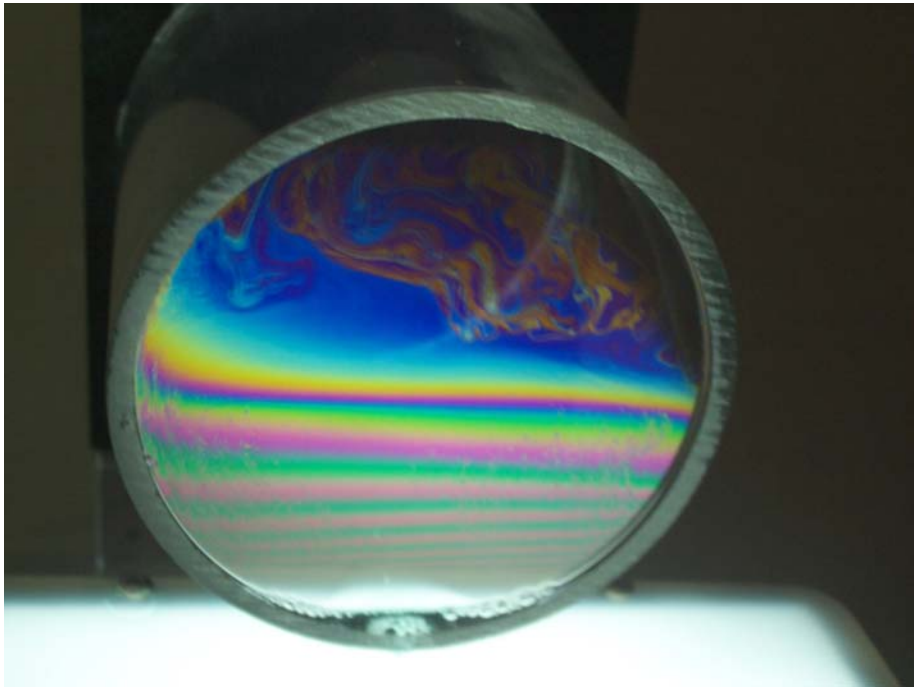


Waxing philosophical

If the interference is destructive, we see no light reflecting from the film. Where does it go?

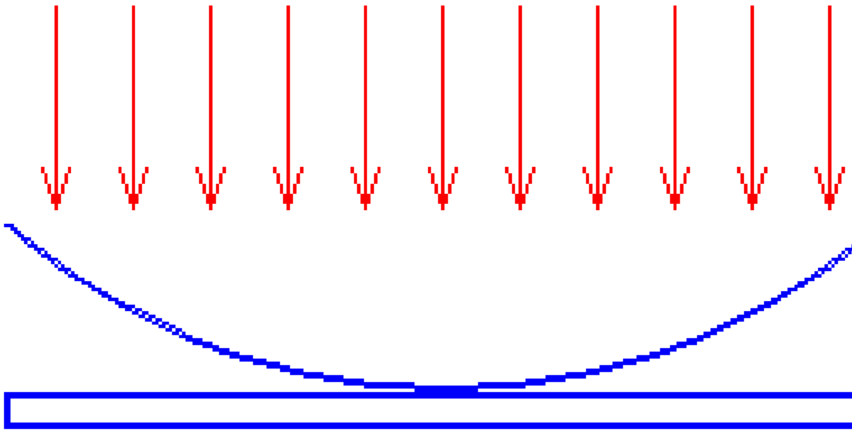
It is all transmitted through the film, instead.

How does it know not to reflect???



Newton's rings

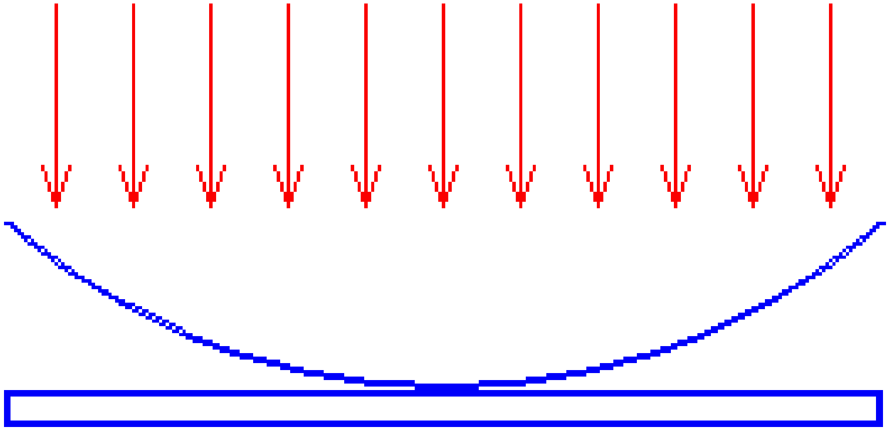
Newton's rings is the name given to the bulls-eye type interference pattern obtained when a spherical piece of glass (a watch glass, for instance) sits on a flat piece of glass. This creates a thin film between the two pieces of glass with a thickness depending on the distance from contact point. If you shine light down from above, you see a pattern of bright and dark rings from constructive and destructive interference.



Side View

Newton's rings

Newton, one of the main supporters of the particle theory of light, was not swayed by the fact that Newton's rings provide evidence to support the wave theory.

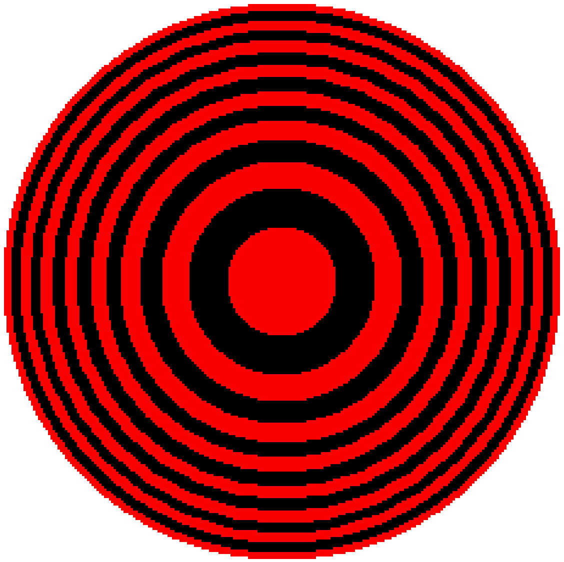


Side View

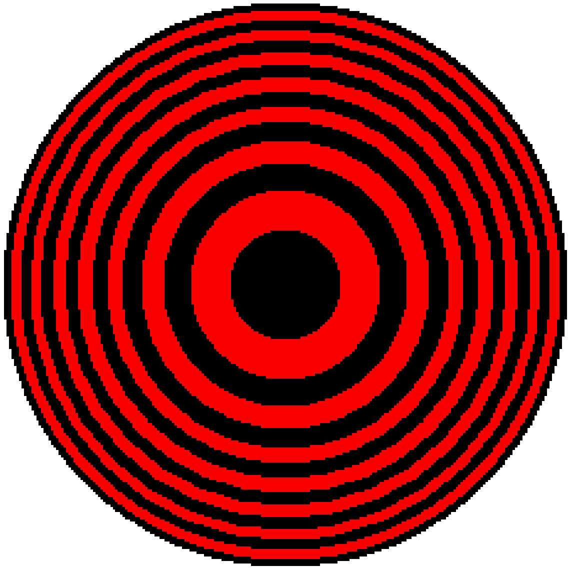
Understanding the pattern

Our thin film is a film of air between two pieces of glass. Is the center of the pattern, where the pieces of glass touch, bright due to constructive interference or dark due to destructive interference?

1. Bright
2. Dark



Top View



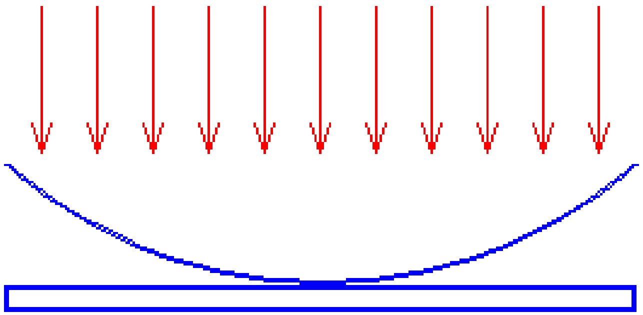
Top View

Understanding the pattern

The center of the pattern is dark. We can understand this by looking at the first three steps in the five-step method.

Step 1. Because the air film has a lower index of refraction than glass, the wave reflecting off the top of the film does not experience a phase shift.

$$\Delta_t = 0$$

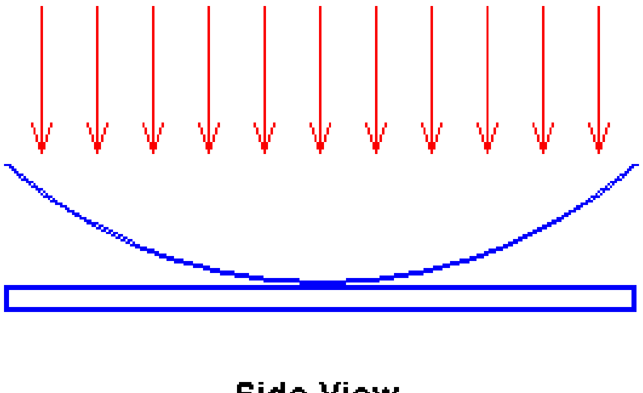


Understanding the pattern

Step 1. $\Delta_t = 0$

Step 2. Because the glass below the air film has a higher index of refraction than the air, the wave reflecting off the bottom surface of the film has a half-wavelength shift. It also travels an extra distance of 2t.

$$\Delta_b = 2t + \frac{\lambda_{film}}{2}$$



Understanding the pattern

Step 1. $\Delta_t = 0$

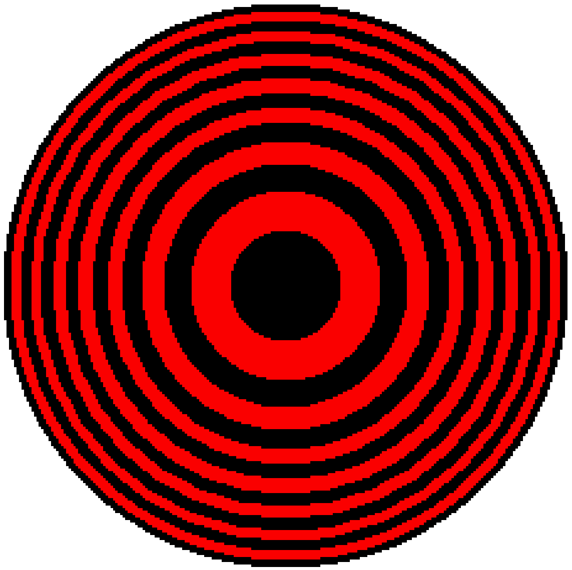
Step 2. $\Delta_b = 2t + \dfrac{\lambda_{film}}{2}$

Step 3. The relative shift is thus

$$\Delta = \Delta_b - \Delta_t = 2t + \frac{\lambda_{film}}{2}$$

Understanding the pattern

Step 3. The relative shift is thus



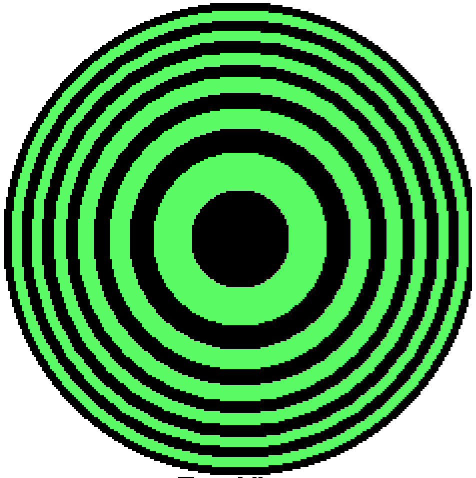
$$\Delta = 2t + \frac{\lambda_{film}}{2}$$

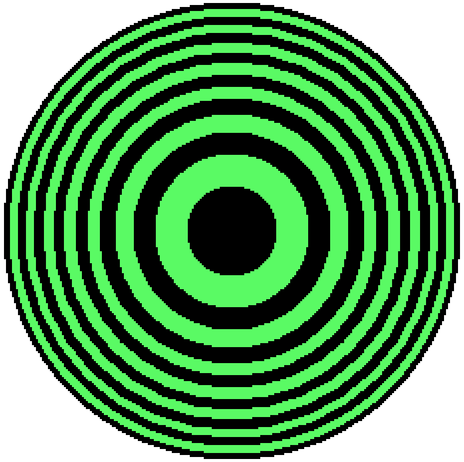
As *t* goes to zero, the relative shift approaches half a wavelength, so at the center, where the film is very thin, we get destructive interference – the center is dark.

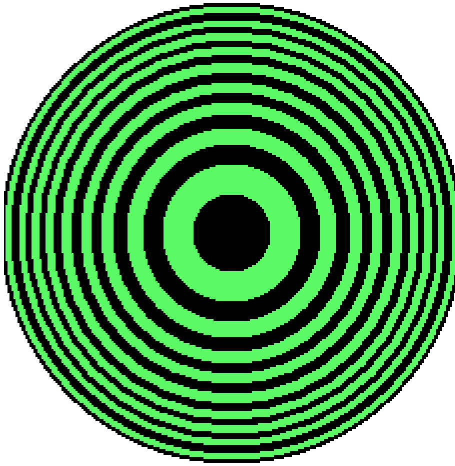
Changing color

Let's say you create a Newton's rings pattern with red light. When you switch to green light will the rings in the pattern be larger, the same size, or smaller than with red light?

1. Larger
2. Same size
3. Smaller


Top View


Top View


Top View

Changing color

Green light has a smaller wavelength than red light. With green light you don't have to go as far from the center to find the film thickness that gives constructive (or destructive) interference, so the rings are smaller with green light than with red.