22-1 Maxwell's Equations

In the 19th century, many scientists were making important contributions to our understanding of electricity, magnetism, and optics. For instance, the Danish scientist Hans Christian Ørsted and the French physicist André-Marie Ampère demonstrated that electricity and magnetism were related and could be considered part of one field, electromagnetism. A number of other physicists, including England's Thomas Young and France's Augustin-Jean Fresnel, showed how light behaved as a wave. For the most part, however, electromagnetism and optics were viewed as separate phenomena.

James Clerk Maxwell was a Scottish physicist who lived from 1831 – 1879. Maxwell advanced physics in a number of ways, but his crowning achievement was the manner in which he showed how electricity, magnetism, and optics are inextricably linked. Maxwell did this, in part, by writing out four deceptively simple equations.

Physicists love simplicity and symmetry. To a physicist, it is hard to beat the beauty of Maxwell's equations, shown in Figure 22.1. To us, they might appear to be somewhat imposing, because they require a knowledge of calculus to fully comprehend them. These four equations are immensely powerful, however. Together, they hold the key to understanding much of what is covered in this book in Chapters 16 – 20, as well as Chapters 22 and 25. Seven chapters boiled down to four equations. Think how much work we would have saved if we had just started with Maxwell's equations instead, assuming we understood all their implications immediately.

Equation 1: $\int \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

Equation 2: $\int \vec{B} \bullet d\vec{A} = 0$

Equation 3: $\int \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$

Equation 4: $\int \vec{B} \bullet d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

term added by Maxwell

Figure 22.1: Maxwell's equations.

Understanding Maxwell's equations.

Equation 1 is known as Gauss' Law for electric fields. It tells us that electric fields are produced by charges. From this equation, Coulomb's Law can be derived. Note that the constant ε_0 in Equation 1, the permittivity of free space, is inversely related to k, the constant in Coulomb's Law: $k = 1/(4\pi\, \varepsilon_0)$

Equation 2 tells us that magnetic field lines are continuous loops.

Equation 3 is Faraday's Law in disguise, telling us that electric fields can be generated by a magnetic flux that changes with time.

Consider Equation 4. Setting the left side equal to the first term on the right-hand side, we have an equation known as Ampère's law, which tells us that magnetic fields are produced by currents. Everything up to this point was known before Maxwell. However, when Maxwell examined the equations (the first three, plus Equation 4 with only the first term on the right) he noticed that there was a distinct lack of symmetry. The equations told us that there were two ways to create electric fields (from charges, or from changing magnetic flux), but they only had one way to create magnetic fields (from currents). One of Maxwell's major contributions, then, was to bring in the second term on the right in Equation 4. This gave a second way to generate magnetic fields, by electric flux that changed with time, making the equations much more symmetric.

Maxwell did not stop there. He then asked the interesting question, what do the equations predict if there are no charges and no currents? Equations 3 and 4 say that, even in the absence of charges and currents, electric and magnetic fields can be produced by changing magnetic and electric flux, respectively. Furthermore, Maxwell found that when he solved the equations, the solutions for the electric and magnetic fields had the form $E(x,t) = E_0 \cos(\omega t - kx)$ and $B(x,t) = B_0 \cos(\omega t - kx)$. We recognize these, based on what we learned in Chapter 21, as the equations for traveling waves.

Finally, Maxwell derived an equation for the speed of the traveling electric and magnetic waves,

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = \sqrt{\frac{1}{(4\pi \times 10^{-7} \text{ Tm/A})(8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^2))}} = 3.00 \times 10^8 \text{ m/s} .$$

(**Equation 22.1**: Maxwell's derivation of the speed of light)

Maxwell recognized that this speed was very close to what the French physicists Hippolyte Fizeau and Léon Foucault had measured for the speed of light in 1849. Thus, Maxwell proposed (in 1873) that light consists of oscillating electric and magnetic fields in what is known as an electromagnetic (EM) wave. It was 15 years later, in 1888, that Heinrich Hertz, from Germany, demonstrated the production and detection of such waves, proving that Maxwell was correct.

Despite Hertz's experimental success, he rather famously stated that he saw no application for electromagnetic waves. Only 120 or so years later, the world has been transformed by our use of electromagnetic waves, from the mobile phones (and other communication devices) that almost all of us carry around, to the radio, television, and wireless computer signals that provide us with entertainment and information, and to x-rays used in medical imaging.

Throughout the 19^{th} century, evidence for light behaving as a wave piled up very convincingly. However, all previous known types of waves required a medium through which to travel. Scientists spent considerable effort searching for evidence for the medium that light traveled through, the so-called luminiferous aether, which was thought to fill space. Such an aether could, for instance, explain how light could travel through space from the Sun to Earth. An elegant experiment in 1887, by the American physicists Albert Michelson and Edward Morley, showed no evidence of such a medium, and marked the death knell for the aether idea. Our modern understanding is that light, or any electromagnet wave, does not require a medium through which to travel. This view is consistent with Maxwell's equations. Electromagnetic waves consist of oscillating electric and magnetic fields, and by Maxwell's equations such time-varying fields produce oscillating magnetic and electric fields, respectively. Hence, an electromagnetic wave can be thought of as self-sustaining, with no medium required.

Finally, note how Equation 22.1 reinforces the idea that light, electricity, and magnetism are all linked. The equation brings together three constants, one associated with light (c, the speed of light), one associated with magnetism (μ_0, which appears in equations for magnetic field), and one associated with electricity (ε_0, which appears in equations for electric field).

Related End-of-Chapter Exercises: 36, 37, 38.

Essential Question 22.1 In Chapter 19, we used the equation $E/B = v$ when we discussed the velocity selector. Use this to help show how the units in Equation 22.1 work out.

Answer to Essential Question 22.1 In the denominator under the square root in Equation 22.1, units of (T m/A) × C^2/(N m^2) become T C^2/(A N m), canceling a factor of meters. 1 A = 1 C/s, so this leads to units of T C s/(N m). The fact that $E/B = v$ tells us that electric field units of N/C, divided by magnetic field units of T, can be replaced by m/s. This leads to units of s^2/m^2. Bringing these up to the numerator, we invert the units to m^2/s^2, and taking the square root gives the required units of the speed of light, m/s.

22-2 Electromagnetic Waves and the Electromagnetic Spectrum

Although we often focus on light, because we rely on light so much as we interact with the world around us, light is just one example of an electromagnetic wave. As shown in Figure 22.2, we classify electromagnetic waves into a variety of categories based on their frequency (or wavelength), as well as on how the waves are produced.

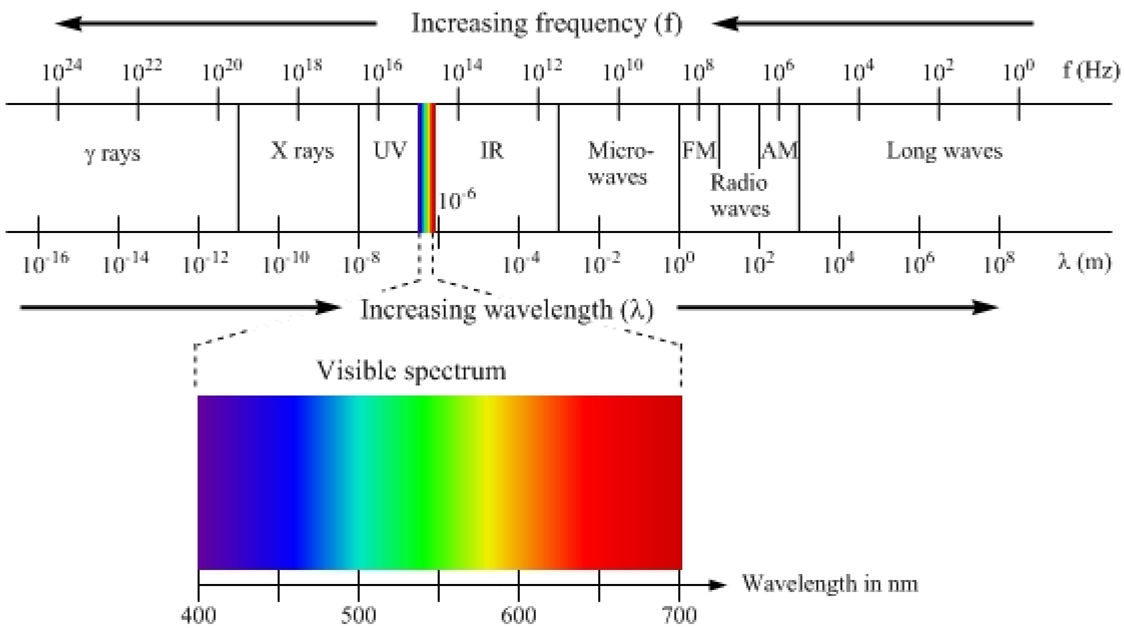


Figure 22.2: An overview of the electromagnetic spectrum. Note that the diagram covers 24 orders of magnitude in both frequency and wavelength.

Our eyes are sensitive to electromagnetic (EM) waves that have wavelengths in the visible spectrum, between 400 and 700 nm, but our bodies can be affected by EM waves in other ways, too. We have some sensors on the backs of our hands, in particular, that are sensitive to infrared radiation, which we can use to tell, without touching it, whether an object is hot. Ultraviolet radiation, in small doses, can produce tanning of the skin, but it is also associated with premature aging of the skin and cataracts. As you move to the left on the diagram in Figure 22.2, the energy associated with packets of EM radiation increases, and thus x-rays and gamma rays have enough energy to pass into and through our bodies. This makes them useful, because they can be used for diagnostic imaging (x-rays) our in cancer treatment (gamma rays), but they also have enough energy to be able to change DNA molecules, which is generally not a good thing. At the other end of the spectrum, we are generally insensitive to EM waves that have a longer wavelength than microwave radiation, such as radio waves.

Electromagnetic waves are produced by accelerating charged particles. Visible light, for instance, can be produced by electrons changing energy levels inside atoms, or by vibrating atoms. X-rays are produced by firing electrons at a metal target, with the x-rays being given off when the electrons are slowed abruptly by the metal atoms. Radio waves are produced by connecting a source of oscillating voltage to one or more metal rods (antennas), causing electrons to oscillate back and forth along the rod. The charge separation is associated with the production of electric fields, while the current associated with the moving electrons generates the magnetic fields.

Properties of electromagnetic waves

Figure 22.3 shows a snapshot of a particular kind of electromagnetic wave, known as a plane (or linearly) polarized wave. The red vectors represent the electric field vectors in the wave at the particular instant shown, while the blue vectors represent the magnetic field vectors at the same instant. Note that all the electric field vectors are in one plane, while all the magnetic field vectors are aligned in a plane that is perpendicular to the plane defined by the electric field vectors.

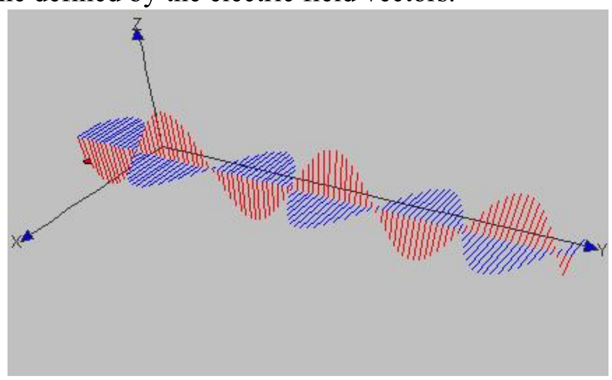


Figure 22.3: A linearly-polarized electromagnetic wave. The red lines represent the electric field vectors, while the blue lines represent the magnetic field vectors. The wave is shown at a particular instant in time. As time goes by, the wave propagates to the right at the speed of light.

Some general features of electromagnetic waves include:

- the energy carried by an electromagnetic wave is divided equally between the electric fields and the magnetic fields.
- the electric and magnetic fields are in phase with one another.
- both the electric field vectors and the magnetic field vectors are perpendicular to the direction of propagation of the wave. Thus, an EM wave is classified as a transverse wave.
- the direction of propagation can be determined by applying a right-hand rule. Start with the fingers on your right hand pointing in the direction of the electric field at a particular point on the wave. If you align your hand so that you can curl your fingers from the electric field direction to the magnetic field direction, your thumb, when it is stuck out, will point in the propagation direction of the wave.
- at all points on the wave, the ratio of the electric field to the magnetic field is given by

$$c = \frac{E}{B}.$$ (Eq. 22.2: **Ratio of electric and magnetic fields in an EM wave**)

Related End-of-Chapter Exercises: 15 – 17.

Essential Question 22.2: (a) What is the wavelength of an x-ray beam if its frequency is 1×10^{18} Hz? How does it compare to the wavelength of the radio wave emitted by the FM radio station WBUR, which broadcasts at a frequency of 90.9 MHz? Assume both waves travel through the air. (b) If the electric field vectors in a linearly-polarized laser beam oscillate with an amplitude of 75 millivolts / meter, what is the amplitude at which the magnetic field vectors in the beam oscillate?

Answer to Essential Question 22.2: (a) To find the wavelength, we can combine the equation $\lambda = v / f$ with the fact that the speed of light in air is 3.00×10^8 m/s. Thus, a frequency of 1×10^{18} Hz corresponds to a wavelength of 3×10^{-10} m, while a frequency of 90.9 MHz corresponds to a wavelength of 3.30 m. (b) Using Equation 22.2, with $c = 3.00 \times 10^8$ m/s, gives an amplitude of $B_{\max} = E_{\max} / c = (0.075 \text{ V/m}) / (3.00 \times 10^8 \text{ m/s}) = 2.5 \times 10^{-10} \text{ T}$.

22-3 Energy, Momentum and Radiation Pressure

All waves carry energy, and electromagnetic waves are no exception. We often characterize the energy carried by a wave in terms of its intensity, which is the power per unit area. At a particular point in space that the wave is moving past, the intensity varies as the electric and magnetic fields at the point oscillate. It is generally most useful to focus on the average intensity, which is given by:

$$I_{average} = \frac{\text{average power}}{\text{area}} = \frac{E_{\max} B_{\max}}{2\mu_0} .$$ (Eq. 22.3: **The average intensity in an EM wave**)

Note that Equations 22.2 and 22.3 can be combined, so the average intensity can be calculated using only the amplitude of the electric field or only the amplitude of the magnetic field.

Momentum and radiation pressure

As we will discuss later in the book, there is no mass associated with light, or with any EM wave. Despite this, an electromagnetic wave carries momentum. The momentum of an EM wave is the energy carried by the wave divided by the speed of light. If an EM wave is absorbed by an object, or it reflects from an object, the wave will transfer momentum to the object. The longer the wave is incident on the object, the more momentum is transferred. This time dependence complicates matters, though, so let's define something about this situation that does not depend on time, which is called radiation pressure, P.

When we looked at an analogous situation for a rubber ball bouncing off an object, in Chapter 7, the ball transfers twice as much momentum to the object when the collision causes the ball's velocity to be equal-and-opposite to what it was before the collision than it does when the ball is stopped completely by the collision. For electromagnetic waves, the pressure is twice as large when the wave reflects from a perfect reflector than when it is 100% absorbed.

$$P = \frac{2I}{c} .$$ (Equation 22.4: **Radiation pressure when a wave reflects 100%**)

$$P = \frac{I}{c} .$$ (Equation 22.5: **Radiation pressure when a wave is 100% absorbed**)

Simply shining a flashlight onto an object causes a pressure to be exerted on the object. For an ordinary flashlight, however, the pressure is so small that it is negligible. For comparison, atmospheric pressure is approximately 10^5 Pa. To exert that pressure with an electromagnetic wave that reflects 100% requires an electromagnetic wave with an intensity of 1.5×10^{13} W/m^2, which is about 10 orders of magnitude more intense than bright sunlight!

Related End-of-Chapter Exercises: 18, 20, 35, 45 – 49.

It has been proposed that spacecraft use radiation pressure for propulsion. The idea is that the craft would unfurl a low-mass large-area reflective sail, and sunlight reflecting from the sail would provide a force to accelerate the spacecraft. Such a spacecraft is known as a **solar sailboat**. Radiation pressure associated with sunlight striking solar panels is exploited on some satellites to make minor adjustments in their motions without needing to use the on-board power source.

EXPLORATION 22.3 – Designing a solar sailboat

Let's design a solar sailboat that we can use to explore the solar system, making use of the following data. Mass of the Sun: $M = 2 \times 10^{30}$ kg; mass of the solar sailboat: $m = 1000$ kg; power emitted by the Sun in the form of electromagnetic waves: $power = 4 \times 10^{26}$ W.

Step 1 – ***Find an expression for the gravitational force exerted on the satellite by the Sun, if the satellite is a distance r from the Sun.*** Applying Newton's Law of universal gravitation, which we covered in Chapter 8, we find

$$F_g = \frac{GmM}{r^2}, \text{ where the constant } G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2.$$

Step 2 – ***Find an expression for the intensity of sunlight reaching the spacecraft.*** For a source like the Sun, which emits waves uniformly in all directions, the intensity at a particular distance is the radiated power divided by the surface area of a sphere with a radius equal to that distance. So,

$$I = \frac{\text{power}}{4\pi r^2}.$$

Step 3 – ***Assuming the sail deployed by the spacecraft is perfectly reflective, find an expression for the force exerted on the sail by the reflecting sunlight. Assume also that the sails are oriented to reflect the sunlight straight back toward the Sun.*** The force associated with the radiation pressure is $F_{rad} = P_{rad} A$, where P_{rad} is the radiation pressure and A is the sail area. Using Equation 22.4, along with the result from Step 2, to determine the radiation pressure, we get

$$F_{rad} = \frac{2I}{c} A = \frac{2 \times power}{c \times 4\pi r^2} A = \frac{A \times power}{2\pi r^2 c}.$$

Step 4 – ***Determine the sail area required to balance the gravitational force exerted on the spacecraft by the Sun.*** Setting the gravitational force equal to the force associated with the radiation pressure gives:

$$\frac{GmM}{r^2} = \frac{A \times power}{2\pi r^2 c} \qquad \Rightarrow \qquad A = \frac{2\pi c GmM}{power}.$$

Interestingly, the area required to balance the forces does not depend on the distance the spacecraft is from the Sun, because both forces are inversely proportional to r^2. Plugging in the values for the various constants, and the values stated above, in this situation the sail area works out to 630000 m^2, which, if the sail was square, would require a sail almost 800 m × 800 m.

Key ideas for solar sailboats: Radiation pressure from sunlight reflecting from a very light metal sail can be a propulsion mechanism for a spacecraft. Solar sailboats need no fuel, and thus can be much lighter than a conventional spacecraft. **Related End-of-Chapter Exercises: 21, 22, 44.**

Essential Question 22.3: Return to Exploration 22.3. Using a sail area 10% larger than that calculated in Step 4, determine the acceleration of the spacecraft if it is the same distance from the Sun that the Earth is ($r = 1.5 \times 10^{11}$ m). Use this acceleration to approximate the spacecraft's speed one week after it starts from rest.

Answer to Essential Question 22.3: With a sail area 10% larger than that needed to balance the forces, there is a net force on the spacecraft of 10% of F_{rad} from step 3. The acceleration, by Newton's Second Law, is thus

$$a = \frac{0.1 \times F_{rad}}{m} = \frac{0.1 \times A \times power}{c \times 4\pi r^2 m} = 3 \times 10^{-4} \text{ m/s}^2.$$

Using $v = v_i + at$, with t = 604800 s in one week, gives a speed of v = 180 m/s. Even though the acceleration is very small, after many weeks the spacecraft builds up a large speed. The acceleration decreases as the distance from the Sun increases, but the spacecraft continually picks up speed as it moves away from the Sun.

22-4 The Doppler Effect for EM Waves

In Chapter 21, we spent considerable effort in coming to understand the Doppler effect for sound. We looked at how, for instance, it is not simply a relative-velocity phenomenon. The sound waves travel through a medium, so what matters is how the source of the waves as well as the observer of the waves moves with respect to the medium.

Because electromagnetic waves do not need a medium, the Doppler effect for EM waves is simply a relative-velocity phenomenon. The shift in frequency observed by an observer depends only on the relative velocity, $\vec{v}$, between the source and the observer. If the source emits EM waves that have a frequency f, the observed frequency $f^{/}$ is given by

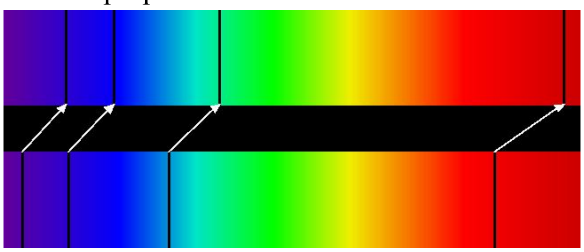
$$f' = f\left(1 \pm \frac{v}{c}\right),$$ (Equation 22.6: **The Doppler effect for electromagnetic waves**)

where v is the magnitude of the relative velocity between the source and the observer. As with the Doppler effect for sound, we use the top (+) sign when the source and observer are moving toward one another, and the bottom (–) sign when the source and observer are moving farther apart.

Applications of the Doppler effect for EM waves

Figure 22.4 illustrates a common application in which the Doppler effect is exploited by astrophysicists to determine how fast, and in what direction, distant stars or galaxies are moving with respect to us here on Earth. At the bottom is the spectrum received on Earth from the Sun. The dark lines at specific wavelengths correspond to light that is absorbed by hydrogen atoms in the Sun. These same lines are seen in the spectrum received from a distant source, at the top, except all the lines are Doppler-shifted toward the red end of the spectrum, indicating that the source is moving away from the Earth. It was data such as this that was used by Edwin Hubble (1889 – 1953) to show that the universe is expanding.

Figure 22.4: The spectrum on the bottom represents light coming to us from the Sun, which is essentially at rest with respect to the Earth, at least for Doppler effect purposes. The four dark lines in the spectrum are caused by the absorption of light of particular wavelengths by hydrogen atoms in the Sun. The spectrum at the top represents light coming to us from a distant star, which is moving away from the Earth at 5% of the speed of light. The characteristic hydrogen absorption lines in the top spectrum have been shifted toward the red end of the spectrum. This is known as redshift.



A second application of the Doppler effect is Doppler radar, which is used in weather forecasting, as it can pick up rotating storm systems. Doppler radar has many sports applications, too, such as detecting the speed of a serve in tennis, or of a pitch in baseball. Doppler radar is also used by police to catch speeding motorists. Let's explore this last application in more detail.

EXPLORATION 22.4 – To catch a speeder
A police officer in a stationary police car aims a radar gun at a truck traveling directly toward the police car. The frequency of the radar gun is 10.525 GHz (10.525×10^9 Hz), and the frequency of the waves reflecting from the truck and returning to the radar gun is shifted from the emitted by frequency by 1600 Hz. If the speed limit on the road is 60 km/h, should the officer pull the truck over to give the driver a ticket? Let's work through the problem to decide.

Step 1 – ***Is the frequency of the waves coming back to the radar gun higher or lower than the frequency of the emitted waves?*** The relative velocity of the two vehicles brings them closer, which effectively lowers the wavelength of the waves, corresponding to a higher frequency.

Step 2 – ***If the truck is traveling at a speed v, write an expression for f', the frequency of the waves received by the truck.*** To do this, we can simply apply Equation 22.6.

$f' = f\left(1+\frac{v}{c}\right)$, where f is the emitted frequency. We use the plus sign in the equation because the truck is moving toward the police car.

Step 3 – ***Write an expression for f'', the frequency of the waves that are picked up by the radar gun after reflecting from the truck. Your expression should be in terms of f, rather than f'.***
We apply Equation 22.6 again, and use the result from step 2.

$f'' = f'\left(1+\frac{v}{c}\right) = f\left(1+\frac{v}{c}\right)^2$, where f is the emitted frequency. Again, we use the plus sign in the equation because the truck is moving toward the police car.

Step 4 – ***Solve for the speed of the truck, and decide whether the truck driver should get a speeding ticket .***

Expanding the bracket in the expression from step 3 gives $f'' = f\left(1+\frac{2v}{c}+\frac{v^2}{c^2}\right) \approx f\left(1+\frac{2v}{c}\right)$.

The speed of light is orders of magnitude larger than the speed of the truck, so v^2/c^2 will be so much smaller than v/c that the v^2/c^2 term can be neglected. Writing the expression in terms of the known frequency shift, 1600 Hz, allows us to solve for the speed of the truck.

$$1600\text{ Hz} = f'' - f = \frac{2fv}{c} \qquad \Rightarrow \qquad v = \frac{c \times 1600\text{ Hz}}{2f} = \frac{(3\times10^8\text{ m/s})\times1600\text{ Hz}}{2\times(10.525\times10^9\text{ Hz})} = 22.8\text{ m/s}.$$

Converting the speed to km/h gives a speed of 22.8 m/s × 0.001 km/m × 3600 s/h = 82 km/h. This is well over the speed limit, so the driver should certainly get a speeding ticket.

Key idea for Doppler radar: In typical Doppler radar applications, the waves are emitted and detected at the same place, and thus the Doppler effect is applied twice to calculate the frequency shift between the emitted and detected waves. **Related End-of-Chapter Exercises: 3, 4, 50 – 53.**

Essential Question 22.4: Return to Exploration 22.4. If the truck is traveling away from the police car at 82 km/h instead of traveling toward it, would the frequency shift of the waves received by the radar gun still be 1600 Hz?

Answer to Essential Question 22.4: If the truck travels away, the frequency shifts lower instead of higher. The difference when we apply Equation 22.6 is that we use a minus sign instead of a plus sign. None of the numbers change, so the frequency shift still has a magnitude of 1600 Hz.

22-5 Polarized Light

Polarizing film consists of linear molecules aligned with one another. When an electromagnetic wave is incident on the film, electric field components that are parallel to the molecules cause electrons to oscillate back and forth along the molecules. This transfers energy from the wave to the molecules, so that part of the wave is absorbed by the film. Waves with electric field vectors in a direction perpendicular to the molecules do not transfer energy to the molecules, however, so they pass through the polarizing film without being absorbed. An electromagnetic wave emerges from the polarizing film **linearly polarized** – all its electric field vectors are aligned with the transmission axis of the polarizing film (which we call a polarizer), the transmission axis being perpendicular to the long molecules in the film.

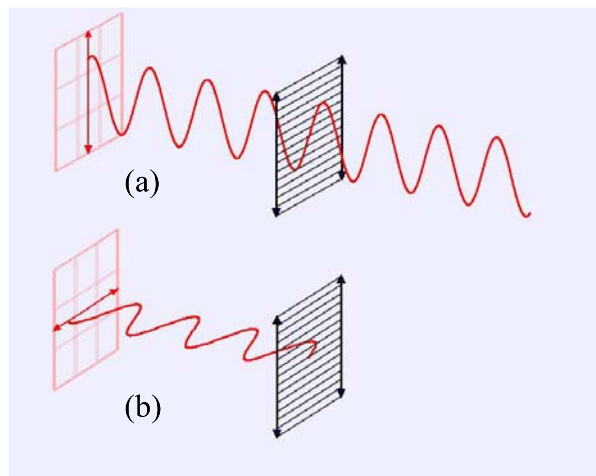


Figure 22.5: A schematic view of what happens to a linearly polarized electromagnetic wave (in red) that is incident on a polarizer (in black). The red arrow at left shows the polarization direction of the wave, while the black arrows on the polarizer indicate the transmission axis for the polarizer. The black lines on the polarizer show the orientation of the long molecules in the polarizer. (a) The wave passes through the polarizer, because the polarization direction matches the transmission axis. (b) The wave is completely blocked by the polarizer, because the polarization direction of the polarizer is perpendicular to the polarizer's transmission axis.

Figure 22.5 shows two special cases, in which the wave is linearly polarized with its electric field vectors either parallel to, or perpendicular to, the polarizer's transmission axis. Figure 22.6 shows the more general case in which the electric field vectors make an angle $\Delta\theta$ with the transmission axis. Splitting the electric field vectors into components parallel to and perpendicular to the transmission axis, the parallel component is transmitted while the perpendicular component is entirely absorbed by the polarizer.

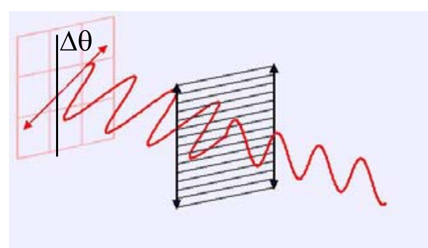


Figure 22.6: The general case of a linearly polarized wave that is incident on a polarizer.

The light emerging from the polarizer has two important features:

- it is linearly polarized, with the polarization direction of the wave matching the transmission axis of the polarizer it just passed through, and
- the intensity of the wave is reduced, because energy is absorbed by the polarizer.

If the magnitude of the electric field in the incident wave is E_0, the magnitude of the electric field in the wave emerging from the polarizer is $E_1 = E_0 \cos(\Delta\theta)$, because it is the cosine component in Figure 22.6 that is transmitted by the polarizer. In general, a wave is characterized by its intensity, which is proportional to the square of the amplitude of the electric fields. Thus,

$I_1 = I_0 \cos^2(\Delta\theta)$. (Eq. 22.7: **Malus' Law for the intensity of light emerging from a polarizer**)

Malus' Law applies when the incident light is linearly polarized. Figure 22.7 shows unpolarized light incident on a polarizer. On average, half the energy is associated with waves in which the electric field vectors are parallel to the polarizer's transmission axis, which are 100% transmitted by the polarizer, while the other half of the energy is associated with waves in which the electric field vectors are perpendicular to the transmission axis, which are 100% absorbed. Thus, the beam emerging from the polarizer is half as intense as that of the incident light. As with a linearly polarized incident wave, the wave emerging from the polarizer is linearly polarized in the direction of the polarizer's transmission axis.

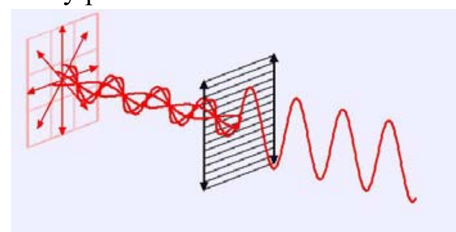


Figure 22.7: When unpolarized light is incident on a polarizer, the emerging beam is (i) half as intense as the incident beam, and (ii) linearly polarized in a direction parallel to the polarizer's transmission axis.

EXAMPLE 22.5 – A sequence of polarizers
As shown in Figure 22.8, light with its polarization direction at 30° to the vertical passes through a sequence of three polarizers. The light has an intensity of 800 W/m^2. Measured from the vertical, the transmission axes of the polarizers are at angles of 0°, 30°, and 75°, respectively. What is the intensity of the light when it emerges from (a) the first polarizer, (b) the second polarizer, and (c) the third polarizer?

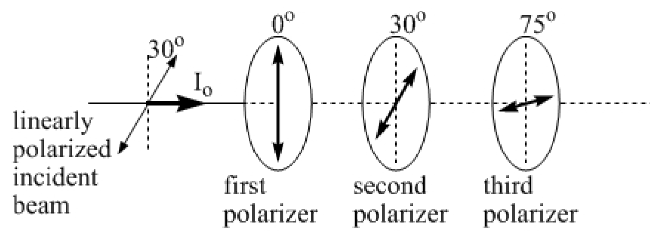


Figure 22.8: Linearly polarized light passes through a sequence of three polarizers, for Example 22.5.

SOLUTION

(a) Because the light is linearly polarized to begin with, we apply Malus' Law. In this case, the angle between the polarization direction of the light and the transmission axis of the polarizer the light is incident on is $\Delta\theta_1 = 30° - 0° = 30°$. Applying Malus' Law gives:

$$I_1 = I_0 \cos^2(\Delta\theta_1) = (800 \text{ W/m}^2)\cos^2(30°) = (800 \text{ W/m}^2) \times \frac{3}{4} = 600 \text{ W/m}^2 .$$

(b) When the light emerges from the first polarizer, it is polarized at an angle of 0° to the vertical, matching the orientation of the transmission axis of the polarizer it just passed through. With the second polarizer at 30°, the angle between the light and the transmission axis is $\Delta\theta_2 = 30° - 0° = 30°$. Applying Malus' Law gives:

$$I_2 = I_1 \cos^2(\Delta\theta_2) = (600 \text{ W/m}^2)\cos^2(30°) = (600 \text{ W/m}^2) \times \frac{3}{4} = 450 \text{ W/m}^2 .$$

(c) When the light emerges from the second polarizer, it is once again polarized at an angle of 30° to the vertical, matching the orientation of the transmission axis of the polarizer it just passed through. With the third polarizer at 75°, the angle between the light and the transmission axis is $\Delta\theta_3 = 75° - 30° = 45°$. Applying Malus' Law gives:

$$I_3 = I_2 \cos^2(\Delta\theta_3) = (450 \text{ W/m}^2)\cos^2(45°) = (450 \text{ W/m}^2) \times \frac{1}{2} = 225 \text{ W/m}^2 .$$

Related End-of-Chapter Exercises: 6 – 12.

Essential Question 22.5: Crossed polarizers are two polarizers with transmission axes that are perpendicular to one another. What is the final intensity (a) if unpolarized light with an intensity of 600 W/m^2 is incident on crossed polarizers, and (b) if a third polarizer, with a transmission axis at 45° to the first polarizer, is placed between the original two polarizers?

Answer to Essential Question 22.5: (a) For crossed polarizers, no light emerges from the second polarizer. For the second polarizer, applying Malus' Law with an angle of $\Delta\theta = 90°$ leads to zero final intensity. (b) Surprisingly, placing a third polarizer between crossed polarizers leads to some light emerging. With unpolarized light, the first polarizer reduces the intensity by a factor of 2, to 300 W/m^2. Applying Malus' Law twice, with $\Delta\theta = 45°$ in each case, gives two factors of ½, with an intensity of 150 W/m^2 after the middle polarizer and 75 W/m^2 after the last polarizer.

22-6 Applications of Polarized Light

Let's summarize the basic method we applied to the polarizer situation in Example 22.5.

A General Method for Solving a Problem Involving Light Passing Through Polarizers

1. For the first polarizer, what we do depends on whether the incident light is unpolarized or polarized. If the light is unpolarized, the intensity is reduced by a factor of 2. If the light is polarized, we apply Malus' Law, where $\Delta\theta$ is the angle between the polarization direction of the incident light and the transmission axis of the polarizer.

2. The light emerging from the first polarizer is always polarized in the direction of the transmission axis of the first polarizer. For the second polarizer, then, we apply Malus' Law, where $\Delta\theta$ is the angle between the polarization direction of the light that emerges from the first polarizer and the transmission axis of the second polarizer. This angle is the same as the angle between the transmission axes of the first and second polarizers.

3. Repeat step 2 for each of the remaining polarizers in the sequence. Remember that $\Delta\theta$, the angle in Malus' Law, is the angle between the polarization direction of the light (this is the same as the angle of the transmission axis of the polarizer the light just emerged from) and the transmission axis of the next polarizer in the sequence.

Related End-of-Chapter Exercises: 28 – 32.

Applications of polarized light

Light emitted by the Sun is unpolarized, but sunlight can become at least partly polarized when it reflects from a flat surface. In general, the direction of polarization is parallel to the plane of the reflecting surface. Light commonly reflects from horizontal surfaces, such as bodies of water. The lenses in polarized sunglasses have vertical transmission axes, so they block light polarized horizontally. This greatly reduces glare from light reflecting off horizontal surfaces.

You can test whether your sunglasses are polarized by looking at the sky on a sunny day while you are wearing your sunglasses. Sunlight scattered through an angle of 90° in the atmosphere is polarized, so if you look in a direction that is at 90° to the direction of the Sun you will have linearly polarized light incident on your sunglasses. If you tilt your head to one side and then the other while you are looking at this part of the sky, you should see the sky brighten and darken if your sunglasses are polarized. By tilting your head, you change the angle between the light and the transmission axes of your sunglass lenses. By Malus' Law, this changes the intensity of the light passing through the sunglasses into your eyes. If you do not see such a change in intensity, then your sunglasses are not polarized.

Rotating the direction of polarization

As we discussed previously, crossed polarizers (polarizers that have their transmission axes at 90° to one another) generally block all light from passing through them. However, if you add something in between the crossed polarizers that changes the polarization direction of the light, some light can get through the system. In Essential Question 22.5, we looked at how adding a third polarizer between the crossed polarizers can result in light being transmitted. However, other transparent materials, such as Karo syrup, bits of mica, some cellophane tape, or clear plastic objects such as plastic forks, when placed between the crossed polarizers also result in light being transmitted. Interestingly, such materials generally affect different wavelengths of light differently, so by varying the thickness of the transparent material the light passes through, colorful patterns can be created. Such patterns have been used to make art installations, such as the one by Austine Wood Comarow at the Museum of Science in Boston, which has ever-changing pictures when viewed through a rotating polarizer.

Stress in a material can also affect the extent to which the polarization direction of a light wave passing through the material is rotated. Engineers exploit this material property by placing models between crossed polarizers to study the stress patterns.

Liquid-crystal displays

Another common application of polarized light is in liquid-crystal displays (LCDs), such as those on digital watches. You may have noticed that an LCD readout can be unreadable if you look at it through polarized sunglasses. This is because the light coming off the LCD is polarized, and thus can all be absorbed by polarizing sunglasses when the display is at a particular angle. The basis structure of an LCD display is shown in Figure 22.9(b). Key components are the crossed polarizers, separated by liquid crystals, and the mirror surface at the back. Light incident on the display from the right first passes through one polarizer, then through layers of liquid crystals. Successive layers of liquid crystals are rotated with respect to one another, and the net effect is that the polarization direction of the light is rotated by 90°. This aligns the light so that it passes through a second polarizer, with its transmission axis perpendicular to the first. The light then reflects off the mirror and reverses the steps, emerging from the sandwich.

By applying a potential difference to the transparent electrodes, however, the liquid crystal layers un-twist, so the light's polarization axis is not rotated. In that case, all the light is blocked by the second polarizer, and that part of the readout looks dark. In a typical LCD readout, numbers are formed using seven-segment displays, in which the appropriate potential difference is applied across a given segment to turn it black, if desired, or no potential difference is applied to make the segment bright, like the background of the display.

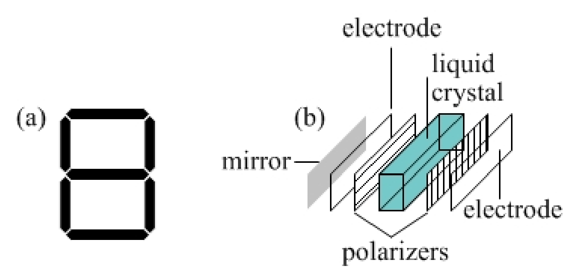


Figure 22.9: (a) Liquid-crystal displays are generally made of seven-segment displays, with different segments being turned on or off to make different numbers. (b) Each segment is formed from a sandwich of crossed polarizers and liquid crystals. Light entering from the right reflects, and the segment appears bright, when no potential difference is applied. Applying a potential difference causes light to be blocked, and the segment to go dark.

Related End-of-Chapter Exercises: 2, 5, 57, 58.

Essential Question 22.6: Light passes through two polarizers. The intensity of the light emerging from the second polarizer is 65% of the intensity of the incident light. The incident light is either unpolarized or linearly polarized. Which is it?