RLC Series Circuit

In this exercise you will investigate the effects of changing inductance, capacitance, resistance, and frequency on an RLC series AC circuit.

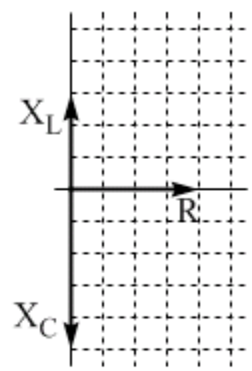
We can define effective resistances for capacitors and inductors:

Inductive reactance: $X_L = \omega L$ Capacitive reactance: $X_C = \dfrac{1}{\omega C}$

Note that the angular frequency, ω, is connected to the frequency, *f*, using $\omega = 2\pi f$

When we have series circuits with resistors connected to inductors and/or capacitors we can find the total resistance of the circuit, which we call the impedance, using:

Impedance: $Z = \sqrt{\left(X_L - X_C\right)^2 + R^2}$



The peak voltage is related to the peak current as follows:

$$\Delta V_{\max} = I_{\max} Z$$

$$\Delta V_{L,\max} = I_{\max} X_L$$

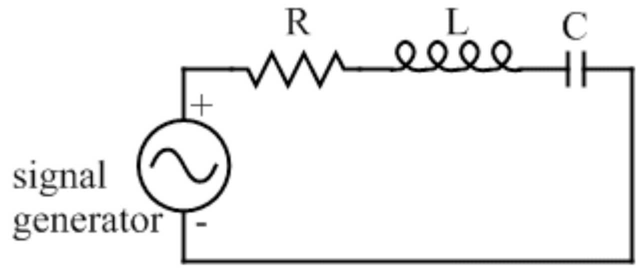
$$\Delta V_{C,\max} = I_{\max} X_C$$

Much of this is tied together with the impedance triangle, in which the inductive reactance is shown on the +y axis, the capacitive reactance is shown on the –y axis, and the resistance is shown on the +x axis. The impedance is then the vector sum of those three things, and the angle between the impedance and the x-axis corresponds to the phase angle between the voltage and current in the circuit.

The phase angle is also the angle between the voltage and the current in the circuit. When the phase angle is positive, the voltage leads the current (the voltage peaks first). When the phase angle is negative, the voltage lags the current (the current peaks first).

The phase angle is given by: $\tan\phi = \dfrac{X_L - X_C}{R}$

PART I – Finding the resonance frequency



1. Start with the following values for resistance, inductance, and capacitance:

   - R = 2 Ω; L = 4 mH = 0.004 H; C = 1 mF = 0.001 F

Question 1: With the frequency set to 100 Hz, what is the amplitude of the resistor voltage? Make a prediction – to maximize the amplitude of the resistor voltage by changing only the frequency from the signal generator, will you need to increase or decrease the frequency? What are you basing your prediction on?

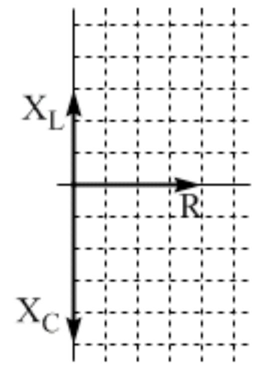
2. Adjust the frequency to determine the frequency that maximizes the resistor voltage.

Question 2: The frequency at which the resistor voltage is maximized has a name – the *resonance frequency*. There is also an equation that predicts what the resonance frequency should be. What is the equation, and does the frequency given by the equation match the frequency you found in step 2? Hint: resonance is when the impedance is minimized.

Question 3: Does anything else special happen at the resonance frequency? Do you notice anything about the voltage across the inductor compared to the voltage across the capacitor at this frequency? If so, what? What about the phase relationship between the input voltage and the three output voltages?

PART II – Understanding the impedance triangle

When we wanted to understand a DC circuit with three resistors in series we would generally find the equivalent resistance of the circuit by simply adding the three resistance values together. The AC circuit we're using here, with three circuit elements in series, is somewhat similar. Again, it is helpful to find the equivalent resistance of the circuit – we call this the **impedance, Z**. Because of the phase relationship between the voltage and the current, however, we add the effective resistances of the different circuit elements *as vectors*.

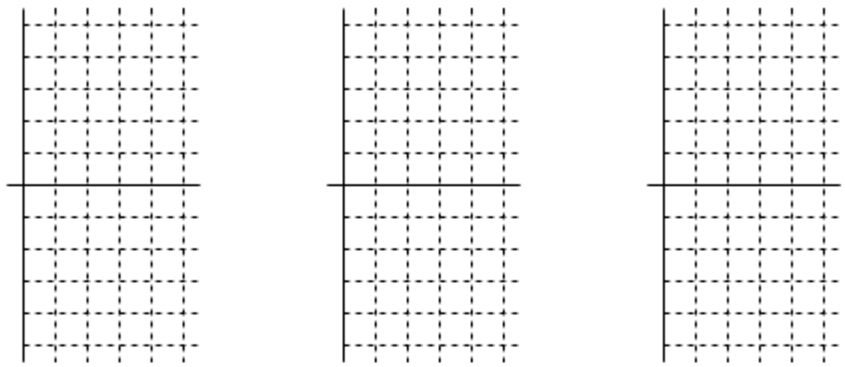


- The resistance goes on the x-axis: voltage and current are in phase for the resistor.
- The effective resistance of the inductor (the **inductive reactance** X_L) is drawn on the y-axis: voltage leads the current by 90˚ for the inductor.
- The effective resistance of the capacitor (the **capacitive reactance** X_C) is drawn on the negative y-axis: voltage lags the current by 90˚ for the inductor.

Question 5: Add the three vectors shown above to find the impedance. The angle between the impedance and the resistance equals the phase angle between the voltage and the current in the circuit. Calculate what that angle is in the situation shown above. Does the voltage lead the current in this case or does the current lead the voltage?

Question 6: If you know the frequency and the inductance, how do you find the inductive reactance? If you know the frequency and the capacitance, how do you find the capacitive reactance?

3. Adjust the frequency until you find the resonance frequency of the circuit (use the R, L, and C values from part I). Using the curves shown on the screen to help you, use the **middle graph below** to sketch, to scale, the impedance triangle corresponding to this situation.



$f = 0.5f_o$ $f = f_o$ $f = 2f_o$

Question 7: As you adjust the frequency, what do you look for on the screen to tell you that you have found the resonance frequency?

Question 8: What does the impedance triangle tell you about how the phase of the input voltage (from the signal generator) compares to the phase of the current at resonance? Does this match what you observe on the screen?

4. Now set the frequency to half of the resonance frequency. Use the **left graph above** to sketch, to scale, the impedance triangle corresponding to this situation.

Question 9: What does the impedance triangle tell you about how the phase of the input voltage (from the signal generator) compares to the phase of the current at this frequency? Does this match what you observe on the screen?

Question 10: The maximum current in the circuit is given by $I_{max} = \frac{V_{max}}{Z}$. Compare your first two impedance triangles to see how the impedance (Z) at this frequency compares to the impedance at the resonance frequency. How should the maximum currents at the two frequencies compare? Is this what you observe?

5. Now set the frequency to twice the resonance frequency. Use the **right graph on the previous page** to sketch, to scale, the impedance triangle corresponding to this situation.

Question 11: What does the impedance triangle tell you about how the phase of the input voltage (from the signal generator) compares to the phase of the current at this frequency? Does this match what you observe on the screen?

Question 12: Now compare your three impedance triangles. At which frequency is the impedance least? At which frequency is the impedance greatest? How should the maximum currents at the three frequencies compare? Is this what you observe?

6. Sketch the impedance triangle that corresponds to a very low frequency.

Question 13: As the frequency decreases what should happen to the impedance? The maximum current? The phase between the voltage and current? Reduce the frequency in the circuit – do your observations correspond with what you stated should happen?

7. Sketch the impedance triangle that corresponds to a very high frequency.

Question 14: As the frequency increases what should happen to the impedance? The maximum current? The phase between the voltage and current? Increase the frequency – do your observations correspond with what you stated should happen?