## 20-1 Magnetic Flux

Let's begin by introducing the concept of flux. Flux means something quite different in physics than it does in everyday conversation. In physics, the flux through an area is simply a measure of the number of field lines passing through an area. In chapter 16, for instance, we could have defined an electric flux, a measure of the number of electric field lines passing through an area, in a way analogous to the following definition of magnetic flux. As we will see in section 20-2, magnetic flux turns out to play a crucial role in the generation of electricity.

Magnetic flux is a measure of the number of magnetic field lines passing through an area. The symbol we use for flux is the Greek letter capital phi,  $\Phi$ . The equation for magnetic flux is:  $\Phi = BA \cos \theta,$ (Equation 20.1: Magnetic flux)

where  $\theta$  is the angle between the magnetic field  $\overline{B}$  and the area vector  $\overline{A}$ . The area vector has a magnitude equal to the area of a surface, and a direction perpendicular to the plane of the surface. The SI unit for magnetic flux is the weber (Wb). 1 Wb = 1 T m<sup>2</sup>.

### **EXAMPLE 20.1 – Determining the magnetic flux**

A rectangular piece of stiff paper measures  $20 \text{ cm} \times 25 \text{ cm}$ . You hold the piece of paper in a uniform magnetic field that has a magnitude of  $4.0 \times 10^{-3}$  T. For each situation below, sketch a diagram showing the magnetic field and the paper, and determine the magnitude of the magnetic flux through the paper, when the magnitude of the flux is (a) maximized, (b) minimized, and (c) halfway between its maximum and minimum value.

### SOLUTION

(a) How should we hold the paper so that the largest number of field lines pass through it? As shown in Figure 20.1, we hold it so that the plane of the paper is perpendicular to the direction of the magnetic field. We can also understand this orientation by considering equation 20.1. To maximize the flux with an area vector of constant magnitude and a field of constant magnitude, we need to maximize the factor of  $\cos \theta$ . The factor of  $\cos \theta$  reaches its maximum magnitude of 1 when  $\theta$ , the angle between the area vector and the magnetic field, is either 0° or 180°. In other words, the area vector must be parallel to the magnetic field, which is the case when the plane of the paper is perpendicular to the magnetic field.

**Figure 20.1**: To maximize the magnetic flux through a flat area, orient the area so the plane of the area is perpendicular to the direction of the magnetic field. (a) shows a perspective view, while (b) shows the view looking along the field lines. In this case, the area vector is in the same direction as the field lines.



(b)

Because  $\cos \theta$  has a magnitude of 1, the magnitude of the maximum flux equals the area multiplied by the magnetic field:  $\Phi_{\text{max}} = AB = 0.20 \text{ m} \times 0.25 \text{ m} \times (4.0 \times 10^{-3} \text{ T}) = 2.0 \times 10^{-4} \text{ T} \text{ m}^2$ .

(b) The factor of  $\cos \theta$  in equation 20.1 can be zero. Thus, the minimum magnitude of the magnetic flux is zero ( $\Phi_{min} = 0$ ). How do we hold the paper so that there is no magnetic flux? As shown in Figure 20.2, if the plane of the paper is parallel to the magnetic field, no field lines pass through the paper and the magnetic flux is zero. (a)

**Figure 20.2**: There is no flux when the plane of the area is parallel to the field. (a) shows a perspective view, while (b) shows the view looking along the field lines. In this case, the area vector is perpendicular to the field lines.

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(c) Starting from the situation in Figure 20.1, tilting the loop by 60° (see Figure 20.3) gives a factor of  $\cos\theta$  of  $\frac{1}{2}$ , halfway between its maximum and minimum value. In this case the magnetic flux is  $\Phi = 1.0 \times 10^{-4}$  T m<sup>2</sup>, half its value from part (a). (a

**Figure 20.3**: Tilting the loop from the orientation in Figure 20.1 reduces the flux. (a) shows a perspective view, while (b) shows the view along the field lines.

### EXPLORATION 20.1 – Ranking situations based on flux

The four areas in Figure 20.4 are in a magnetic field. The field has a constant magnitude,

and is directed into the page in the left half of the region and out of the page in the right half. Rank the areas based on the magnitude of the net flux passing through them, from largest to smallest. Note that, when we calculate net flux, field lines passing in one direction through an area cancel an equal number of field lines passing in the opposite direction through an area.

**Figure 20.4**: Four different regions in a magnetic field. The field has the same magnitude everywhere, but it is directed into the page in the left half of the field and out of the page in the right half.

Region 1 is tied with region 4 for the largest area, but all the field lines in region 1 pass through in the same direction, giving region 1 the largest-magnitude flux. In contrast, the net flux through region 4 is zero because the flux through the right half of region 4 cancels the flux through the right side. The field lines in regions 2 and 3 all pass through in the same direction. Region 3 has an area of 3 boxes, while region 2 has an area of  $\pi r^2$ , where the radius is 1 unit, so region 2 has an area of  $\pi$  boxes. Because  $\pi$  is larger than 3, the magnetic flux through region 2 is larger than that through region 3. Thus, ranking by flux magnitude gives 1 > 2 > 3 > 4.

**Key idea for net flux:** In calculating net flux, field lines passing one way through an area cancel an equal number of field lines passing in the opposite direction through that area. **Related End-of-Chapter Exercises: 1, 2, 4, 41.** 

#### An aside: Electric Flux and Gauss' Law

We did not mention electric flux when we talked about electric field, but we can define electric flux in an analogous way to magnetic field. Electric flux is a measure of the number of electric field lines passing through a surface. The equation for electric flux is:

 $\Phi_E = EA\cos\theta,$ 

where  $\theta$  is the angle between the electric field  $\vec{E}$  and the area vector  $\vec{A}$ .

There is a law called Gauss' Law, which says that the net electric flux passing through a closed surface is proportional to the net charge enclosed by that surface. Using the correct proportionality constant, Gauss' Law can be used to calculate electric fields in highly-symmetric situations. It is interesting to note that the analogous law in a magnetic situation, Gauss' Law for magnetism, is not nearly so useful. Because magnetic field lines are always continuous loops, the net magnetic flux passing through a closed surface is always zero – if a magnetic field line emerges from a surface, it must re-enter the surface at some other location, giving a net flux for that field line of zero, to ensure that the field line is a continuous loop.

*Essential Question 20.1*: Return to the situation described in Exploration 20.1, and shown in Figure 20.4. If we define out of the page as the positive direction for magnetic flux, rank the four areas by their net flux, from most positive to most negative.

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$\otimes$	$\otimes$	$\otimes$	© <sup>2</sup>	0	0
$\otimes$	$\otimes$	$\otimes$	0	0	0
$\otimes$	4⊗	$\otimes$	0	0	0



Answer to Essential Question 20.1: In this case, the ranking is 2 > 3 > 4 > 1. Regions 2 and 3 have positive flux, because field lines directed out of the page pass through those regions. Region 4 still has a flux of zero, while region 1 has a negative flux because of the field lines passing through the region into the page.

# 20-2 Faraday's Law of Induction

Table 20.1 summarizes some experiments we do with a magnet and a loop of wire. The loop has a galvanometer in it, which is a sensitive current meter. When the needle on the meter is in the center there is no current in the loop. The direction and size of the needle's deflection reflects the direction and size of the current in the loop.

Experiment	Initial and final states	Meter reading		
1. The north pole of the magnet is brought closer to the loop.	(a) (b) (b) (c) (c) (c) (c) (c) (c) (c) (c) (c) (c	While the magnet is moving closer, the meter needle deflects to the right.		
2. The north pole is held at rest close to the loop.	(a) (b) (b) (c) (c) (c) (c) (c) (c) (c) (c) (c) (c	The needle does not deflect at all.		
3. The north pole of the magnet is moved away from the loop.	(a) (b) (b) (c) (c) (c) (c) (c) (c) (c) (c) (c) (c	While the magnet is moving away, the meter needle deflects to the left.		
4. The north pole of the magnet is brought closer to the loop, but at a faster rate than it was in experiment 1.	(a) (b) (c). (a) (b) (c). (b) (c). Figure 20.8: The same situation as Figure 20.5, but with less time between (a) and (b).	While the magnet is moving closer, the needle deflects farther to the right, but for less time, than in experiment 1.		
5. The magnet is rotated back and forth in front of the loop.	Figure 20.9: The flux oscillates from large to small and back again.	As the magnet oscillates, the needle oscillates back and forth at, in this case, double the frequency of the magnet.		

**Table 20.1**: Various experiments involving a magnet and a wire loop connected to a current meter. The views in the figures are looking through the loop at the magnet.

Among the conclusions we can draw from these experiments are the following:

- A magnet interacting with a conducting loop can produce a current in the loop.
- A current arises only when the magnetic flux through the loop is changing.
- The current is larger when the magnetic flux changes at a faster rate.

Exposing a loop or coil to a *changing* magnetic flux gives rise to a voltage, called an **induced emf.** Following Ohm's law, the induced emf gives rise to an **induced current** in the loop or coil. The emf induced by a changing magnetic flux in each turn of a coil is equal to the time rate of change of that flux. Thus, for a coil with *N* turns, the net induced emf is given by:

$$\varepsilon = -N \frac{\Delta \Phi}{\Delta t} = -N \frac{\Delta (BA \cos \theta)}{\Delta t}.$$
 (Eq. 20.2: **Faraday's Law of Induction**)  
The minus sign in equation 20.2 will be explained in section 20-3

### EXPLORATION 20.2 – Using graphs with Faraday's Law

A flat square conducting coil, consisting of 5 turns, measures  $5.0 \text{ cm} \times 5.0 \text{ cm}$ . The coil has a resistance of  $3.0 \Omega$  and, as shown in Figure 20.10, moves at a constant velocity of 10 cm/s to the right through a region of space in which a uniform magnetic field is confined to the 20 cm long region shown in the figure. The field is directed out of the page, with a magnitude of 3.0 T.

**Figure 20.10**: A flat conducting coil moves at constant velocity to the right through a region of space in which a uniform magnetic field is confined. The small squares on the diagram are 5.0 cm on each side.

Draw the coil's motion diagram, and a graph of the magnetic flux through the coil as a function of time. Define out of the page as the positive direction for flux. Finally, draw a graph of the emf induced in the coil as a function of time. The diagrams are in Figure 20.11. For the first 1.0 s there is no flux, because no field passes through the coil. The flux grows linearly with time during the halfsecond the coil moves into the field. The flux is constant, at

 $B \times A = 7.5 \times 10^{-3}$  T m<sup>2</sup>, for the next 1.5 s, and then drops linearly to zero in the half-second the coil takes to leave the field.

**Figure 20.11**: (a) A motion diagram showing the coil's position at 1-second intervals. Graphs, as a function of time, of (b) the magnetic flux through the coil and (c) the emf induced in the coil.

Faraday's Law tells us that the induced emf is related to  $\Delta \Phi / \Delta t$ , which is the slope of the flux versus time graph. While the flux is increasing, the slope of the flux graph is constant with a value

of  $\Delta \Phi / \Delta t = (7.5 \times 10^{-3} \text{ T m}^2)/(0.50 \text{ s}) = 1.5 \times 10^{-2} \text{ V}$ . Multiplying by the factor of -N from Faraday's Law, where N = 5 turns, gives an induced emf of  $-7.5 \times 10^{-2} \text{ V}$ . The induced emf drops to zero while the flux is constant, and then has a constant value of  $+7.5 \times 10^{-2} \text{ V}$  during the half-second period while the coil is leaving the magnetic field and the flux is decreasing.

**Key idea**: The induced emf is proportional to the negative of the slope of the graph of flux as a function of time. **Related End-of-Chapter Exercises:** 14 - 17, 23 - 26.

*Essential Question 20.2*: What is the magnitude of the maximum current induced in the coil in the system described in Exploration 20.2?

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Answer to Essential Ouestion 20.2: To find the current of the largest magnitude, we apply Ohm's Law, using the induced emf of the largest magnitude. The resistance was given as 3.0  $\Omega$  in Exploration 20.2, so we get  $I_{\text{max}} = \varepsilon_{\text{max}} / R = (0.075 \text{ V}) / (3.0 \Omega) = 0.025 \text{ A}$ .

## 20-3 Lenz's Law and a Pictorial Method for Faraday's Law

Thus far we have discussed the fact that exposing a coil to a changing magnetic flux induces an emf in the coil. If there is a complete circuit, this emf gives rise to an induced current. In what direction is this induced current? The direction of the current also relates to the negative sign in Faraday's law. That negative sign is associated with a whole other law, Lenz's law.

Lenz's law: The emf induced by a changing magnetic flux tends to produce an induced current. The induced current produces a magnetic flux that acts to oppose the original change in flux.

Let's go over a pictorial method for determining the direction of the induced current. As part of this method, recall from chapter 19 that, as shown in Figure 20.12, a current directed clockwise around a loop gives rise to a magnetic field directed into the page inside the loop, while a counterclockwise current produces a field directed out of the page in the loop. This can be confirmed with the right-hand rule. Curl the fingers on your right hand in the direction of the current, and your thumb points in the direction of the field inside the loop.

Figure 20.12: (a) A loop with a clockwise current gives rise to a magnetic field directed into the page within the loop. (b) The magnetic field within the loop is directed out of the page if the current is counterclockwise.

**EXPLORATION 20.3 – A pictorial method for determining current direction** Step 1 - The loop in Figure 20.13 is moved from the Before position to the After position, closer to a long straight wire that carries current to the left. Sketch a diagram showing the magnetic field lines, produced by the current in the straight wire, that pass through the loop when the loop is in the Before position.

Figure 20.13: A conducting loop, above a long straight current-carrying wire, is moved from the Before position to the After position, closer to the wire.

Let's use the right-hand rule to find the direction of the field from the wire. Point the thumb on your right hand in the direction of the current. Then, curl your fingers on your right hand – they show the direction the field lines circle around the wire. This tells us that, above the wire, where the loop is, the field lines from the wire are directed into the page, as in Figure 20.14.

Figure 20.14: When the loop is in the Before position, the field from the long straight wire passes through the loop into the page.

Step 2 – Sketch a diagram showing the magnetic field lines, produced by the current in the straight wire, that pass through the loop when the loop is in the After position. If the flux changes from Before to After, show this on the diagram. The loop is above the wire in the After position, so the field is still directed into the page. However, the loop is now closer to the wire, where the field is larger, so we show more field lines on the diagram in Figure 20.15.

Figure 20.15: When the loop moves to the After position, coming closer to the wire where the field from the wire is stronger, we draw more field lines passing through the loop.





Before

Step 3 – Draw a "To Oppose" picture, with one field line representing the direction of the field the induced current in the loop creates to oppose the change in flux the loop experiences in moving from the Before position to the After position. Based on the direction of the field in the To Oppose picture, determine the direction of the induced current. To oppose the increase in magnetic flux that occurs as the loop moves closer to the wire, the induced current in the loop creates a magnetic field in the opposite direction, out of the page. The pictorial method is qualitative, so we only need to draw one field line directed into the page in Figure 20.16.

**Figure 20.16**: To oppose the change in flux as the loop moves from Before to After, the loop creates a magnetic field out of the page with a counterclockwise induced current.

Using the right-hand rule, when the thumb is directed out of the page, the fingers curl counterclockwise, in the direction of the induced current.

**Key ideas for the pictorial method**: A pictorial method can be used to determine the direction of the induced current in a loop or coil that experiences a change in magnetic flux. The steps are:

- 1. Draw a Before picture, showing the field lines passing through before a change is made.
- 2. Draw an After picture, showing the field lines passing through after a change is made.
- Draw a To Oppose picture, with a single field line to represent the direction of the field needed to oppose the change from the Before picture to the After picture. Then, apply the right-hand rule to find the direction of the induced current needed to produce this field.
  Related End-of-Chapter Exercises: 18 – 20, 22.

### EXAMPLE 20.3 – A quantitative analysis

Rank the four single-turn loops in Figure 20.17 based on the magnitude of their induced current, from largest to smallest. The loops are either moving into or out of a region of uniform magnetic field. The field is zero outside the region.

**Figure 20.17**: Four conducting loops, all of the same resistance, are moving with the velocities indicated either into (loops 1 and 4) or out of (loops 2 and 3) a region of uniform magnetic field.

#### SOLUTION

Combining Ohm's law with Faraday's law, we find that the magnitude of the current is given by: f(x) = f(x) + f(x

$$I_{induced} = \frac{\varepsilon}{R} = \frac{1}{R} \left( \frac{\Delta(BA\cos\theta)}{\Delta t} \right)$$

 $\cos \theta = 1$ , and *B* does not change with time so:  $I_{induced} = \frac{\varepsilon}{R} = \frac{B}{R} \left( \frac{\Delta A}{\Delta t} \right)$ .

Writing the area in terms of the length *L* and width *W*:  $I_{induced} = \frac{B}{R} \left( \frac{\Delta(LW)}{\Delta t} \right).$ 

Defining *L* as the length of the loop perpendicular to the velocity, which is constant, then the magnitude of  $\Delta W / \Delta t$  is the speed. The magnitude of the current is thus:  $I_{induced} = BLv / R$ .

The field and resistance are the same, so the induced current for the loops is proportional to the product of the speed multiplied by the length of the loop that is perpendicular to the velocity. This gives a ranking by current magnitude of 4 > 1 = 2 = 3. Loop 3 has half the length, perpendicular to its velocity, as loops 1 and 2, but makes up for that factor of two in its speed.

Essential Question 20.3: Find the direction of the induced current in the loops in Figure 20.17.

induced

Τо

Oppose

