23-1 The Ray Model of Light

We will start our investigation of geometrical optics (optics based on the geometry of similar triangles) by learning the basics of the ray model of light. We will then apply this model to understand reflection and mirrors, in this chapter, and refraction and lenses, in chapter 24. Using the triangles that result from applying the ray model, we will derive equations we can apply to predict where the image created by a mirror or lens will be formed.

A **ray** is as a narrow beam of light that tends to travel in a straight line. An example of a ray is the beam of light from a laser or laser pointer. In the ray model of light, a ray travels in a straight line until it hits something, like a mirror, or an interface between two different materials. The interaction between the light ray and the mirror or interface generally causes the ray to change direction, at which point the ray again travels in a straight line until it encounters something else that causes a change in direction. An example in which the ray model of light applies is shown in Figure 23.1, in which the laser beam travels in straight lines between the mirrors it is interacting with. Even after striking the bright object at the center, the many beams follow straight paths.

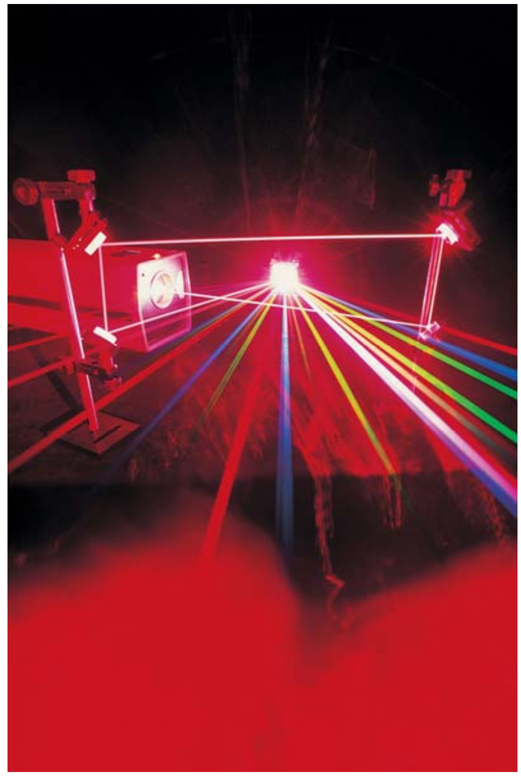


Figure 23.1: The photograph shows a situation in which the ray model of light applies. The laser beam follows straight lines as it travels between each mirror that it interacts with. (Photo credit: Digital Vision)

A laser emits a single ray of light, but we can also apply the ray model in situations in which a light source sends out many rays, in many directions. Examples of such sources include the filaments of light bulbs, and the Sun. If we are far away from such a source, in relation to the size of the source itself, we often treat the source as a **point source**, and assume that the source emits light, usually in all directions, from a single point. Light bulbs, and the Sun, are often treated as point sources. In other situations, such as when we are close to a light bulb that has a long filament, we treat the source as a **distributed source**. Each point on the source can be treated as a point source, so a distributed source is like a collection of point sources, as shown in Figure 23.2.

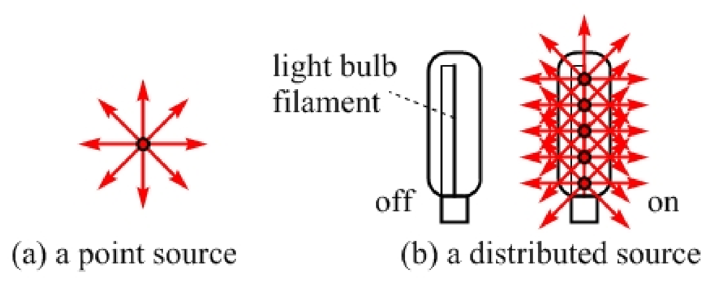
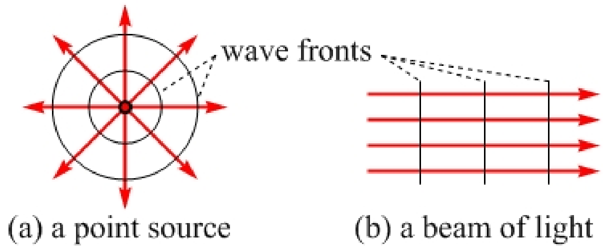


Figure 23.2: (a) A point source of light emits light uniformly in all directions. (b) A distributed source of light, such as a light-bulb filament in the shape of a line, can generally be treated as a collection of point sources.

Wave fronts

In addition to rays of light, we will also mention wave fronts. A wave front is a surface connecting light that was emitted by the light source at the same time. As shown in Figure 23.3, the wave fronts for a point source are spherical shells centered on the source, which propagate away from the source at the speed of light. For a beam of light, like that from a flashlight, in which the rays are parallel, the wave fronts are parallel lines that are perpendicular to the beam.

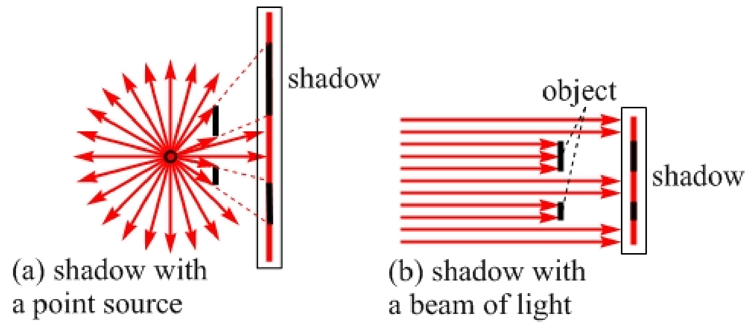
Figure 23.3: (a) For a point source, the wave fronts are spherical shells that are centered on the source. The larger the radius of the wave front, the more time has passes since the light was emitted. (b) When the rays are part of a beam of light that is traveling in a particular direction, the wave fronts are parallel lines that are perpendicular to the beam.



Shadows

The ray model of light can also be used to understand shadows. Figure 23.4 shows how the shadow cast by a point source can be larger than the object creating the shadow, while that from parallel rays is the same size as the object, as long as the surface on which the shadow is cast is perpendicular to the direction of the rays. Distributed sources create more complicated shadows, but they can be understood as the superposition of the shadows from multiple point sources.

Figure 23.4: We can use the ray model of light, in which travels in straight lines, to explain shadows. When the light source is a point source (a), the shadow is generally larger than the object casting the shadow. When the light rays are all going in the same direction, however, the shadow is the same size of the object when the shadow is cast on a surface that is perpendicular to the light rays.



Treating sources that do not themselves emit light as light sources

In some cases, we will use objects that actually emit light, such as a light bulb or the Sun, as the objects that send light toward a mirror or lens. In other cases we will use objects, such as you, that do not emit light themselves. How can we do this? In general, objects that do not emit light themselves are illuminated by other light sources. As shown in Figure 23.5, such sources can be treated as if they emit light, because they scatter much of the light incident on them in many different directions.

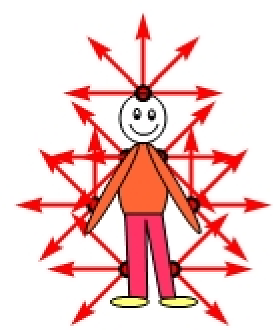
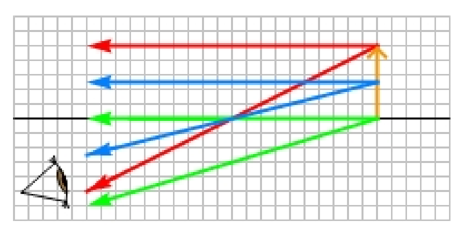


Figure 23.5: An illuminated object can itself be treated as a source of light, because much of the light shining on it is scattered off the object in all directions.

How we see objects

To see an object, rays of light need either to be emitted by, or reflected from, the object, and then pass into our eyes. Our brains assume that the rays of light travel in straight lines, so we trace the rays of light back until they meet at the location of the object, as shown in Figure 23.6.

Figure 23.6: Objects send out light in many directions. We see the object if enough of this light enters our eye. Only a small number of the rays are shown for this object, color-coded red for rays from the top, blues for rays from the middle, and green for rays from the bottom of the object.



Related End-of-Chapter Exercises: 26 – 28.

Essential Question 23.1: The Sun is a very large object, much larger than the Earth. Give an example in which we can treat the Sun as a point source when applying the ray model. Give an example in which the Sun must be treated as a distributed source of light.

Answer to Essential Question 23.1: To explain the formation of your own shadow on a sunny day, we can treat the Sun as a point source located 150 million km away. On the other hand, the shadow that the Earth casts on the Moon during a lunar eclipse has a very dark region (the umbra) and a semi-dark region (the penumbra). This more complex shadow pattern can be partly explained by treating the Sun as a distributed source.

23-2 The Law of Reflection; Plane Mirrors

A ray of light that reflects from a surface obeys a very simple rule, known as the law of reflection. See, also, the illustrations in Figure 23.7.

The Law of Reflection: for a ray of light reflecting from a surface, the angle of incidence is equal to the angle of reflection. These angles are generally measured from the normal (perpendicular) to the surface.

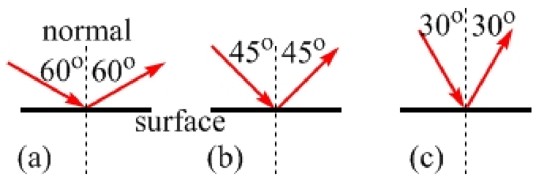


Figure 23.7: In each case, the ray obeys the Law of Reflection, in that the angle of incidence, measured from the normal, is equal to the angle of reflection. In the specific examples shown, both the incident ray and the reflected ray are at an angle, measured from the normal, of (a) 60°, (b) 45°, and (c) 30°.

A surface acts as a mirror when the Law of Reflection is followed on a large scale, as shown in Figure 23.8 (a). In that case, the whole beam of light, with many parallel rays, reflects as expected according to the law. This is known as **specular reflection**: mirror-like reflection that preserves the wave-front structure. In Figure 23.8 (b), however, the surface does not seem to obey the law of reflection. If we look at the magnified view, in (c), however, we see that the surface is irregular. The law of reflection is obeyed for each ray individually, but the irregularities in the surface cause the rays to move off in many different directions after being reflected. This is known as **diffuse reflection**: reflection in which the wave fronts are not preserved. Diffuse reflection explains why some surfaces that appear to be flat, such as a table or a road, do not act as mirrors. As far as light is concerned, these surfaces are far from flat.

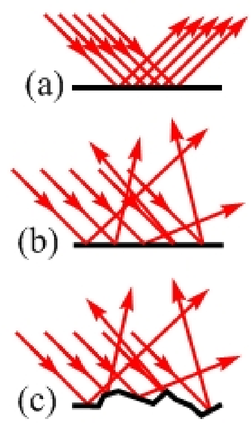
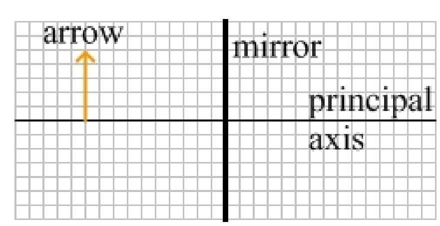


Figure 23.8: (a) Specular reflection from a flat mirror, in which all rays reflect at the same angle. (b) Many flat surfaces exhibit diffuse reflection, in which rays reflect at different angles. (c) A magnified view of the situation in (b). Even though the surface may appear flat to us, the surface is actually quite irregular as far as light is concerned.

The surface in Figure 23.8(a) is known as a plane mirror. Common examples are the mirrors in every bathroom. When we look at ourselves in such a mirror, where do we see our image? How large is the image? To answer such questions, we can use a ray diagram to determine where an image is formed and what its characteristics are.

EXPLORATION 23.2 – Using a ray diagram to find the location of an image

Figure 23.9: An arrow located some distance in front of a plane mirror.



Step 1 – ***An arrow is placed in front of a vertical plane mirror, as shown in Figure 23.9. Sketch two rays of light, which travel in different directions, that leave the tip of the arrow and reflect off the mirror. Show the direction of these rays after they reflect from the mirror.***

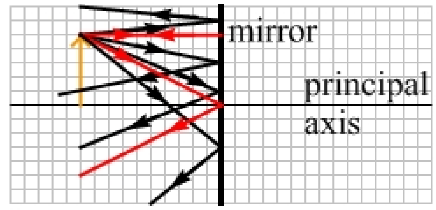


Figure 23.10: A selection of reflected rays. In each case, the reflected rays are drawn in the direction that is consistent with the law of reflection.

Figure 23.10 shows a number of rays leaving the arrow and reflecting from the mirror, obeying the law of reflection. One of these, the horizontal ray, in red, that strikes the mirror at a 0° angle of incidence (measured from the normal to the mirror) is special, in that it reflects back along the path the ray came in on, and is thus easy to draw. However, you do not need to use this particular ray in the ray diagram – any two rays of light that reflect from the mirror can be used.

Step 2 – ***The point where the reflected rays meet is where the tip of the image is located. Sketch the image of the arrow.*** The reflected rays diverge as they travel away from the mirror to the left, but if we extend the reflected rays back through the mirror to the right, we find a point where they intersect. This is where the image of the tip of the arrow is located. Note that all the reflected rays, when they are extended back, pass through this point, which is why we can use any two reflected rays to create the ray diagram. Because the base of the arrow, which we call the object, is located on the principal axis (the horizontal line bisecting the mirror), we know that the base of the image will also be located there, so we draw an image of the arrow between the point where the image of the tip is and the principal axis.

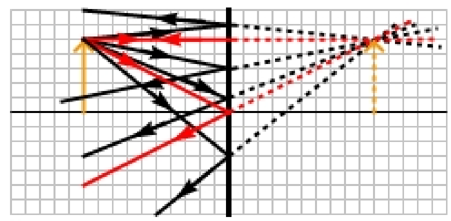


Figure 23.11: For a plane mirror, the reflected rays must be extended back through the mirror to find the location where they meet. Because the rays left the tip of the arrow, the point where the reflected rays meet is the location of the tip of the image of the arrow.

Step 3 – ***Prove that the image is located as shown in Figure 23.11 by drawing two more ray diagrams, one showing the location of the image of the midpoint of the arrow, and one showing the location of the image of the base (bottom) of the arrow.*** Figure 23.12 combines the ray diagram for the tip with those of the arrow's base and midpoint, showing that the image really is at the location shown in Figure 23.11.

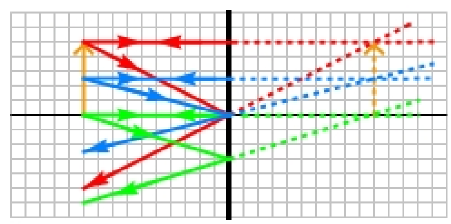


Figure 23.12: We can sketch a ray diagram for any point on an object to find the location of its image. In this case, the red rays are for the tip of the arrow, the blue for the midpoint, and the green for the base.

Step 4 – ***Compare the reflected rays in Figure 23.13 to the rays in Figure 23.6.*** Our brains cannot tell the difference between the two situations, which is why we see an image of the arrow formed at the location shown in Figure 23.13.

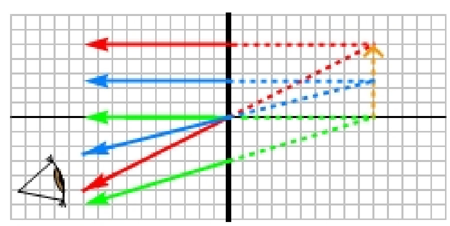


Figure 23.13: Our brains trace the reflected rays back along straight lines until they meet, and we see an image at that location.

Key idea for ray diagrams: The location of the image of any point on an object, when the image is created by a mirror, can be found by drawing rays of light that leave that point on the object and reflect from the mirror. The direction of the reflected rays must be consistent with the law of reflection. The point where the reflected rays meet is where the image of that point is.
Related End-of-Chapter Exercises: 1, 2, 4, 5

Essential Question 23.2: First, make a prediction. When the arrow in Exploration 20.2 is moved closer to the mirror, will its image be larger, smaller, or the same size as the image we found in Step 2 above? Sketch a new ray diagram to check your prediction.

Answer to Essential Question 23.2: Images from plane mirrors are always the same size as the original object. We can see this in the ray diagram in Figure 23.14.

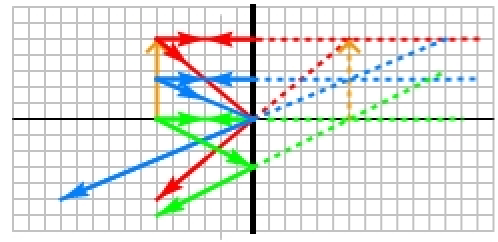


Figure 23.14: Compare this figure to Figure 23.12. Moving the object closer to the mirror moves the image closer to the mirror, but the height of the image is still equal to the height of the object.

23-3 Spherical Mirrors: Ray Diagrams

Let us move now from plane mirrors to spherical mirrors, which curve like the surface of a sphere. Spherical mirrors can be convex, such as the mirrors on the passenger side of cars, or concave, such as shaving or makeup mirrors. Unlike plane mirrors, which always produce an image that is the same size as the object, the image in a convex mirror is always smaller than the object, while the image in a concave mirror can be larger, smaller, or the same size as the object.

The focal point of a spherical mirror is defined by what the mirror does to a set of rays of light that are parallel to one another and to the principal axis of the mirror. As shown in Figure 23.15, a concave mirror reflects the rays so they converge to pass through the focal point, *F*. A convex mirror, in contrast, reflects parallel rays so that they diverge away from the focal point. Note that each ray obeys the law of reflection when it reflects from the mirror. The location of the focal point depends on the curvature of the mirror. The smaller the radius of curvature of the mirror, the closer the focal point is to the mirror's surface.

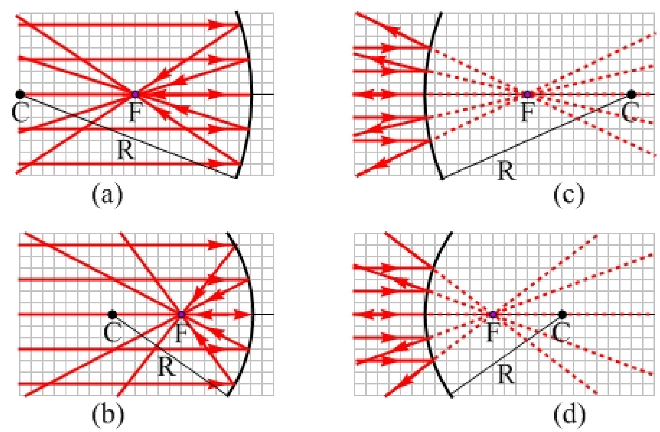


Figure 23.15: Focal points are shown for four different mirrors. In (a) and (b), concave mirrors reflect parallel rays so that they converge to a single point called the focal point. In (c) and (d), convex mirrors reflect parallel rays so that they diverge away from the mirror's focal point. The location of the focal point depends on the mirror's radius of curvature. In each case, *C* is the mirror's center of curvature, *R* is the radius of curvature, and *F* is the focal point.

What we show here for spherical mirrors is an approximation, valid for rays that are not too far from the principal axis, in relation to the magnitude of the mirror's focal length. A mirror actually needs to have a parabolic shape to reflect all parallel rays through one point (or away from one point, for a diverging mirror). The fact that spherical mirrors do not really bring all such rays to a point (or diverge them away from a point) is a defect called **spherical aberration**.

Focal length of a spherical mirror: The focal point of a spherical mirror is located halfway between the surface of the mirror and the mirror's center of curvature. Thus, the focal length of a spherical mirror has a magnitude of $R/2$, where R is the radius of curvature of the mirror. By convention, an object that diverges parallel light rays has a negative focal length (f), while an object that converges parallel light rays has a positive focal length. Thus:

For a concave mirror: $f = +\frac{R}{2}$. (Eq. 23.1) For a convex mirror: $f = -\frac{R}{2}$. (Eq. 23.2)

In the limit that the radius of curvature approaches infinity, the mirror becomes a plane mirror and the focal length is either + infinity or – infinity (it does not matter which sign is used).

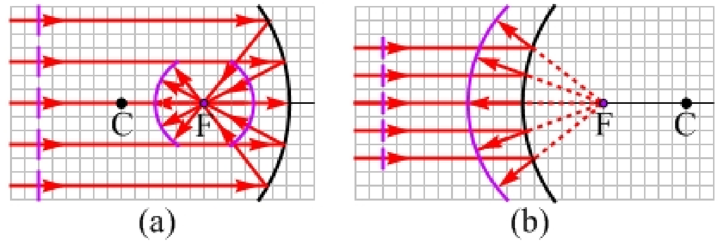


Figure 23.16: (a) For light rays that are parallel to the principal axis, a converging mirror re-unites the wave front (in purple) at the focal point. From there, the wave front diverges almost as if the focal point is a point source. (b) A diverging mirror deflects the parallel rays so the wave fronts appear to diverge from the mirror's focal point.

Following the wave fronts

Figure 23.16 shows what spherical mirrors do to wave fronts. For the converging mirror, the waves take the same time to get from the left to the focal point. For the diverging mirror, once the waves reflect from the mirror, it is as if they left the focal point at the same time.

EXPLORATION 23.3 – Ray diagram for a convex mirror

We will follow a process similar to that of plane mirrors to draw a ray diagram for a convex (diverging) mirror.

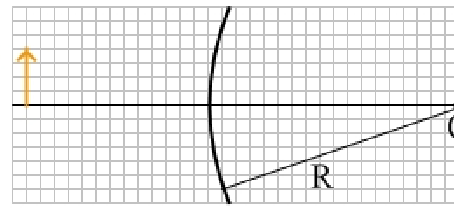


Figure 23.17: An object, represented by an arrow, is placed in front of a convex (diverging) mirror. The mirror's center of curvature is shown.

Step 1 – ***First, locate the mirror's focal point. Then, draw a light ray that leaves the tip of the object (its top) and goes parallel to the principal axis. Show how this parallel ray reflects from the mirror.*** The focal point is halfway between the point where the principal axis intersects the mirror, and the center of curvature. For a convex mirror, all parallel rays appear to diverge from the focal point, so we draw the reflected ray reflecting along a line that takes it directly away from the focal point.

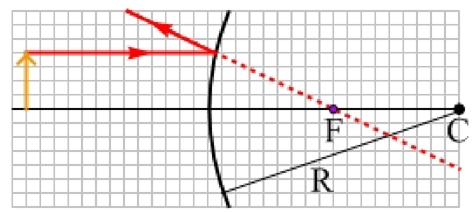


Figure 23.18: The parallel ray reflects from the mirror in such a way that it travels directly away from the focal point.

Step 2 – ***Sketch a second ray that leaves the tip of the object and reflects from the mirror. Using your reflected rays, draw the image.*** One useful ray, in blue in Figure 23.19, reflects from the point on the mirror that the principal axis passes through. At that location, the reflection is like that from a vertical plane mirror. Another useful ray, in green, goes straight toward the mirror's center of curvature. This ray has a 0° angle of incidence, and thus reflects back along the same line. The reflected rays diverge on the left of the mirror, but we can extend them back to meet on the right side of the mirror, showing us where the tip of the image is. The image, smaller than the object, is drawn from the tip down to the principal axis.

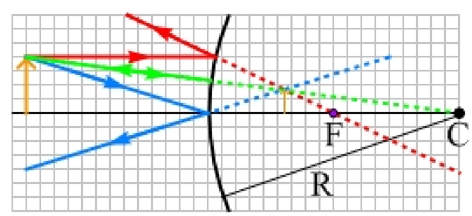


Figure 23.19: In addition to the parallel ray, two other rays are easy to draw the reflection for. The ray (in blue) that strikes the mirror at the principal axis reflects as if the mirror was a vertical plane mirror. The ray in green travels directly toward the center of curvature, striking the mirror at 90° to the surface, and thus reflecting straight back.

Key idea: As with a plane mirror, when a number of rays leave the same point on an object and reflect from a spherical mirror, the corresponding point on the image is located at the intersection of the reflected rays. **Related End-of-Chapter Exercises: 6, 9, 18, 60, 61.**

Essential Question 23.3: (a) Modify the ray diagram in Figure 23.19 to show what happens to the image when the object is moved closer to the mirror. (b) Add several more rays (leaving the tip of the object) to your modified ray diagram, showing what the rays do when they reflect from the mirror. How do you know how to draw the reflected rays?

Answer to Essential Question 23.3: (a) The parallel ray is nice to work with, because its reflection does not change direction when the object is moved left or right. The ray that strikes the mirror at the principal axis, however, comes in at a larger angle of incidence, and thus reflects at a larger angle. As shown in Figure 23.20, when the object is closer to the mirror, the image increases in size and moves closer to the mirror.

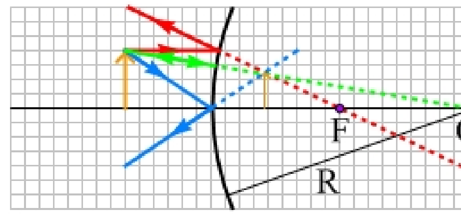


Figure 23.20: The new ray diagram shows that moving the object closer to the mirror results in the image increasing in size (but remaining smaller than the object) and moving closer to the surface of the mirror.

(b) We have two ways of knowing how to draw the reflected rays properly. First, the law of reflection must be obeyed when the rays reflect from the mirror. Second, we know that when we extend the reflected rays back, they will pass through the tip of the image, which we located in Figure 23.20. Three additional rays are shown in Figure 23.21.

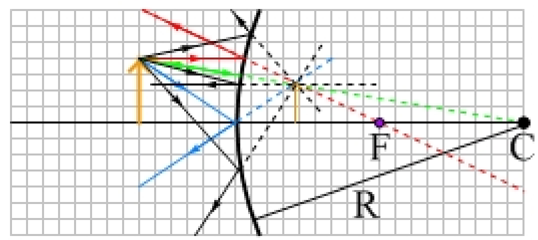


Figure 23.21: For all rays of light that leave the tip of the object and reflect from the mirror, the reflected rays can be extended back to pass through the tip of the image.

23-4 A Qualitative Approach: Image Characteristics

So far, we have looked at ray diagrams in two cases, the case of a plane mirror (section 23-2) and that of a convex mirror (section 23-3). In both cases, we had to extend the reflected rays back through the mirror to get the rays to intersect, giving us an image behind the mirror. We see this all the time with plane mirrors. If you stand 1.0 m in front of a plane mirror, you see an image of yourself 1.0 m behind the mirror. Is it always true that the image created by a mirror is located behind the mirror? We will first investigate the concave mirror, the last case we will deal with, and we will then summarize various image characteristics.

EXAMPLE 23.4 – A ray diagram for a concave mirror
An object, represented by an arrow, is located 15.0 cm in front of a concave mirror that has a focal length of +10.0 cm, as shown in Figure 23.22. Sketch a ray diagram to find the location of the tip of the image of the arrow, and sketch the image on the diagram.

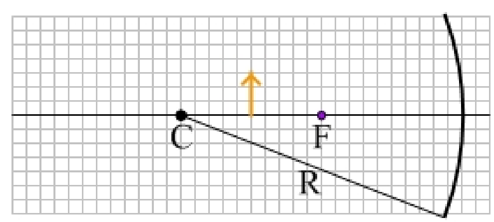


Figure 23.22: An object in front of a concave mirror. The squares in the grid measure 1.0 cm × 1.0 cm.

SOLUTION

Let's use the same procedure we have used previously, starting by drawing rays of light that leave the tip of the object. Again, one useful ray, shown in red, is the ray that travels parallel to the principal axis. The concave mirror reflects this ray so that it passes through the mirror's focal point. A second ray that we know how to draw is the one, in blue, that reflects from the mirror at the principal axis, reflecting at that point as if the mirror were a vertical plane mirror. Note that these two rays meet to the left of the mirror, giving us the location of the tip of the image. As usual, we draw the image of the arrow from that point to the principal axis, because the base of the object is also on the principal axis.

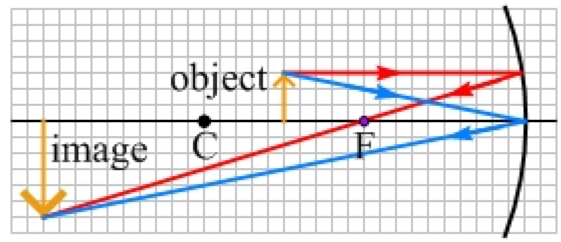


Figure 23.23: A ray diagram showing two of the several possible rays we can draw to locate the tip of the image.

Wave fronts

Figure 23.24 shows how the converging mirror affects the wave fronts (in purple). The light leaving the tip of the object, reflecting from the mirror, and arriving at the tip of the image, takes the same time no matter which path it takes.

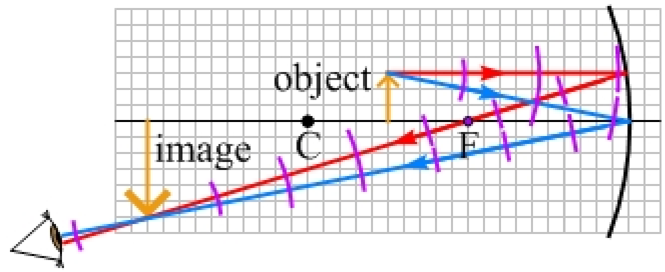


Figure 23.24: The wave fronts that leave the tip of the object converge on the tip of the image – the rays take the same time to reach the image. The wave fronts then diverge away from the image. To your brain, the image looks like an object.

Image characteristics

The image in Figure 23.23 is quite different from images we have seen earlier in the chapter. First, the image is inverted (upside down) compared to the object. Second, the image is larger than the object. Third, the image is formed from light rays that actually pass through the image. Note that concave mirrors do not always form images with these characteristics, as we will investigate in more detail in section 23-6. For now, however, let's discuss some general issues related to image characteristics.

Upright or inverted?

As we have seen, plane mirrors and convex mirrors produce an upright image. This is an image that is in the same orientation as the object. An inverted image, like that in Example 23.4, is one in which the image is upside down in relation to the object.

Real or virtual?

Most of the images we see on a daily basis in mirrors are virtual. A virtual image is one that the light does not actually pass through. Instead, our brains see an image there because, when we look in the mirror at the object, our brains are so used to light traveling in straight lines that we trace all the reflected rays back to their apparent source, the point behind the mirror where the light appears to come from. For a single mirror, when the image is virtual it is also upright.

In Example 23.4, we saw a situation in which the light rays passed through the mirror, creating a real image. Real images, from concave mirrors, have a three-dimensional quality that virtual images do not have, and it is worth going out of your way to see one. For a single mirror, when the image is real it is also inverted.

Larger or smaller?

A plane mirror, as we investigated earlier, produces an image that is always the same size as the object. Convex (diverging) mirrors, on the other hand, always produce an image that is smaller than the object. Concave (converging) mirrors, as we will investigate further in section 23-6, can produce an image that is larger, smaller, or the same size as the object. We will discuss these ideas in a more quantitative way when we define the magnification of a mirror, in section 23-5.

Related End-of-Chapter Exercises: 7, 37, 38.

Essential Question 23.4: Consider the ray diagram in Figure 23.23. If an object of the same size of the image was placed at the image's position, where would its image be located?

Answer to Essential Question 23.4: The image would be located where the object in Figure 23.23 is located. This demonstrates an important fact about light rays – they are reversible. As Figure 23.25 shows, we can simply reverse the direction of the rays from Figure 23.23 to obtain the appropriate ray diagram. Note that we can do this only when the image is a real image.

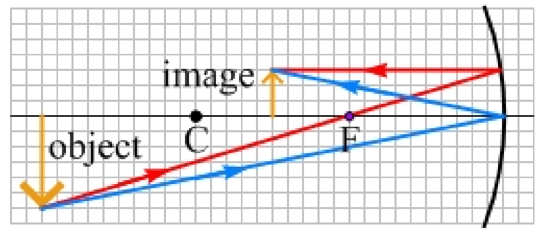


Figure 23.25: When the image is a real image, the ray diagram is reversible.

23-5 A Quantitative Approach: The Mirror Equation

The branch of optics that involves mirrors and lenses is generally called geometrical optics, because it is based on the geometry of similar triangles. Let's investigate this geometry, and use it to derive an important relationship between the image distance, object distance, and the focal length.

Let's look again at the ray diagram we drew in Figure 23.23 of section 23-4, shown again here in Figure 23.26.

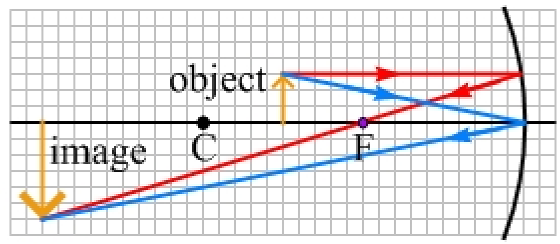


Figure 23.26: The ray diagram we constructed in section 23-4, for an object in front of a concave mirror.

Remove the red rays, and examine the two triangles in Figure 23.26, one shaded green and one shaded yellow, bounded by the blue rays, the principal axis, and the object and image. The two triangles are similar, because the three angles in one triangle are the same as the three angles in the other triangle. We can now define the following variables: d_o is the object distance, the distance of the object from the center of the mirror; d_i is the image distance, the distance of the image from the center of the mirror; h_o is the height of the object; h_i is the height of the image.

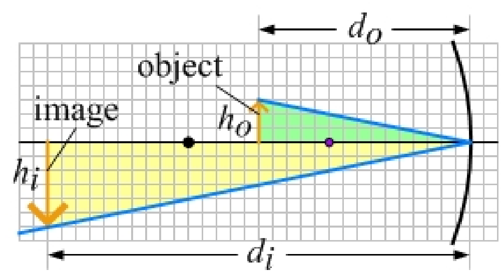


Figure 23.27: Similar triangles, bounded by the principal axis, the object and image, and the blue ray that reflects from the mirror.

Using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that:

$$-\frac{h_i}{h_o} = \frac{d_i}{d_o} \, . \qquad \text{(Equation 23.3)}$$

The image height is negative because the image is inverted, which is why we need the minus sign in the equation. Let's now return to Figure 23.26, and remove the blue rays. This gives us the shaded similar triangles shown in Figure 23.28.

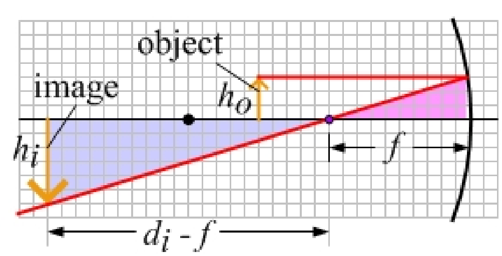


Figure 23.28: Similar triangles, with two sides bounded by the principal axis and the red ray, and a third side that is equal to the object height (pink triangle) or the image height (blue triangle).

We use an approximation, which is valid as long as the object height is relatively small, that the length of the pink triangle is *f*, the focal length. Again, using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that:

$$\frac{d_i - f}{f} = -\frac{h_i}{h_o}.$$

Simplifying the left side, and bringing in equation 23.3, we get: $\frac{d_i}{f} - 1 = \frac{d_i}{d_o}$.

Dividing both sides by d_i gives: $\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$, which is generally written as:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}. \qquad \text{(Equation 23.4: \textbf{The mirror equation})}$$

The mnemonic "If I do I di" can help you to remember the mirror equation.

Often, we know the focal length f and the object distance d_o, so equation 23.4 can be solved for d_i, the image distance:

$$d_i = \frac{d_o \times f}{d_o - f} \qquad \text{(Equation 23.5: \textbf{The mirror equation, solved for the image distance})}$$

Sign conventions

We derived the mirror equation above by using a specific case involving a concave mirror. The equation can also be applied to a plane mirror, a convex mirror, and all situations involving a concave mirror if we use the following sign conventions.

The focal length is positive for a concave mirror, and negative for a convex mirror.

The image distance is positive if the image is on the reflective side of the mirror (a real image), and negative if the image is behind the mirror (a virtual image).

The image height is positive when the image is above the principal axis, and negative when the image is below the principal axis. A similar rule applies to the object height.

Magnification

The magnification, m, is defined as the ratio of the height of the image (h_i) to the height of the object (h_o). Making use of Equation 23.3, we can write the magnification as:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}. \qquad \text{(Equation 23.6: \textbf{Magnification})}$$

The relative sizes of the image and object are as follows:

- The image is larger than the object if $|m| > 1$.
- The image and object have the same size if $|m| = 1$.
- The image is smaller than the object if $|m| < 1$.

The sign of the magnification tells us whether the image is upright (+) or inverted (–) compared to the object.

Related End-of-Chapter Exercises: 11 – 15.

Essential Question 23.4: As you are analyzing a spherical mirror situation, you write an equation that states: $\frac{1}{f} = \frac{1}{+12\text{ cm}} + \frac{1}{+24\text{ cm}}$. What is the value of $1/f$ in this situation? What is f?

Answer to Essential Question 23.4: To add fractions you need to find a common denominator.
$\frac{1}{f} = \frac{1}{+12\text{ cm}} + \frac{1}{+24\text{ cm}} = \frac{2}{+24\text{ cm}} + \frac{1}{+24\text{ cm}} = \frac{3}{+24\text{ cm}}$. This gives $f = \frac{+24\text{ cm}}{3} = 8.0\text{ cm}$.

23-6 Analyzing the Concave Mirror

In section 23-4, we drew one ray diagram for a concave mirror. Let's investigate the range of ray diagrams we can draw for such a mirror.

EXPLORATION 23.6 – Ray diagrams for a concave mirror

Step 1 – *Draw a ray diagram for an object located 40 cm from a concave mirror that has a radius of curvature of 20 cm. Verify the image location on your diagram with the mirror equation.* In drawing a ray diagram, it is helpful to know where the mirror's focal point is. For a spherical mirror, the focal point is halfway between the mirror's center of curvature and the point at which the principal axis intersects the mirror. Thus, the focal length in this case is +10 cm.

In Figure 23.29, two rays are shown. One is the parallel ray, which leaves the tip of the object, travels parallel to the principal axis, and reflects from the mirror so that it passes through the focal point. The second ray reflects off the mirror at the point at which the principal axis meets the mirror, reflecting as if the mirror was a vertical plane mirror.

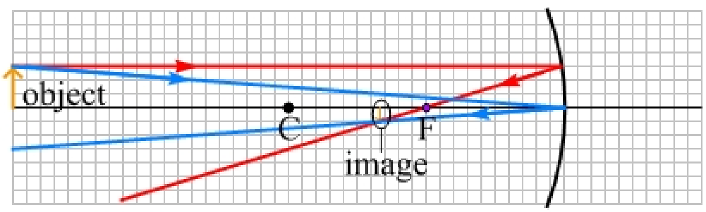


Figure 23.29: A ray diagram for the situation in which the object is far from the mirror. The squares in the grid measure 1.0 cm × 1.0 cm.

Applying the mirror equation, in the form of equation 23.5, to find the image distance:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(40\text{ cm})\times(+10\text{ cm})}{(40\text{ cm})-(+10\text{ cm})} = \frac{+400\text{ cm}^2}{30\text{ cm}} = +13.3\text{ cm}.$$

This image distance is consistent with the ray diagram in Figure 23.29.

Step 2 – *Repeat step 1, with the object now moved to the center of curvature.* The parallel ray follows the same path as it did Figure 23.29. As shown in Figure 23.30, the ray (in blue) that reflects from the center of the mirror follows a different path, because shifting the object changes the angle of incidence for that ray. This situation is a special case. When the object is located at the center of curvature, the image is inverted, also at the center of curvature, and the same size as the object because the object and image are the same distance from the mirror.

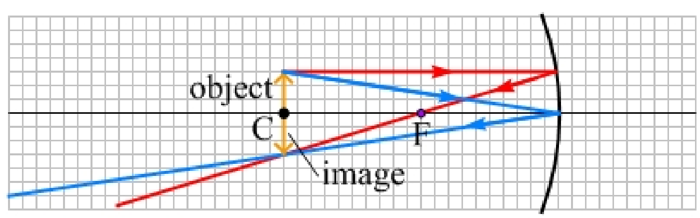


Figure 23.30: When the object is at the mirror's center of curvature, so is the image.

Applying the mirror equation to find the image distance, we get:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(20\text{ cm})\times(+10\text{ cm})}{(20\text{ cm})-(+10\text{ cm})} = \frac{+200\text{ cm}^2}{10\text{ cm}} = +20\text{ cm}$$, matching the ray diagram.

Step 3 – *Repeat step 1, with the object 15 cm from the mirror.* No matter where the object is, the parallel ray follows the same path. The path of the second ray, in blue, depends on the object's position. The ray diagram (Figure 23.31) shows that the image is real, inverted, larger than the object, and about twice as far from the mirror as the object.

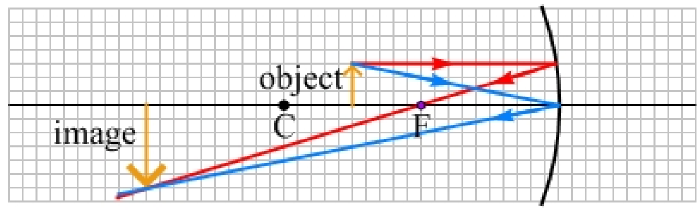


Figure 23.31: A ray diagram for a situation in which the object is between the mirror's center of curvature and its focal point.

Applying the mirror equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(15\text{ cm})\times(+10\text{ cm})}{(15\text{ cm})-(+10\text{ cm})} = \frac{+150\text{ cm}^2}{5.0\text{ cm}} = +30\text{ cm}$$, matching the ray diagram.

Step 4 – *Repeat step 1, with the object at the mirror's focal point.* As shown in Figure 23.32, the two reflected rays are parallel to one another, and never meet. In such a case the image is formed at infinity.

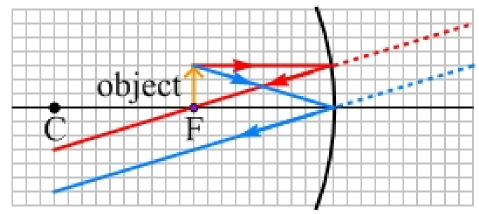


Figure 23.32: A ray diagram for a situation in which the object is at the focal point.

Applying the mirror equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(10\text{ cm})\times(+10\text{ cm})}{(10\text{ cm})-(+10\text{ cm})} = \frac{+100\text{ cm}^2}{0\text{ cm}} = +\infty$$, which agrees with the ray diagram.

Step 5 – *Repeat step 1, with the object 5.0 cm from the mirror.* When the object is closer to the mirror than the focal point, the two reflected rays diverge to the left of the mirror, and they must be extended back to meet on the right of the mirror. The result is a virtual, upright image that is larger than the object, as shown in Figure 23.33.

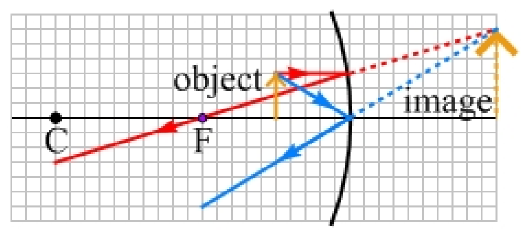


Figure 23.33: A ray diagram for a situation in when the object is between the mirror's surface and its focal point.

Applying the mirror equation to find the image distance gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(5.0\text{ cm})\times(+10\text{ cm})}{(5.0\text{ cm})-(+10\text{ cm})} = \frac{+50\text{ cm}^2}{-5.0\text{ cm}} = -10\text{ cm}.$$

Recalling the sign convention that a negative image distance is consistent with a virtual image, the result from the mirror equation is consistent with the ray diagram.

Key idea for concave mirrors: Depending on where the object is placed relative to a concave mirror's focal point, the mirror can form an image of the object that is real or virtual. If the image is real, it can be larger than, smaller than, or the same size as the object. If the image is virtual, the image is larger than the object. **Related End-of-Chapter Exercises: 23 and 47 – 49.**

Essential Question 23.6: When an object is placed 20 cm from a spherical mirror, the image formed by the mirror is larger than the object. What kind of mirror is it? What, if anything, can you say about the mirror's focal length?

Answer to Essential Question 23.6: The mirror must be concave, because a convex mirror cannot produce an image that is larger than the object. A concave mirror produces an image larger than the object only when the object is between the mirror and twice the focal length. So, twice the focal length must be at least +20 cm, and the focal length must be at least +10 cm. All we can say is that the focal length is greater than or equal to +10 cm.

23-7 An Example Problem

Let's begin by discussing a general approach we can use to solve problems involving mirrors. We will then apply the method to a particular situation.

A general method for solving problems involving mirrors

1. Sketch a ray diagram, showing rays leaving the tip of the object and reflecting from the mirror. Where the reflected rays meet is where the tip of the image is located. The ray diagram gives us qualitative information about the location and size of the image and about the characteristics of the image.
2. Apply the mirror equation and/or the magnification equation. Make sure that the signs you use match those listed in the sign convention in section 23-5. The equations provide quantitative information about the location and size of the image and about the image characteristics.
3. Check the results of applying the equations with your ray diagram, to see if the equations and the ray diagram give consistent results.

Rays that are easy to draw the reflections for

To locate an image on a ray diagram, you need a minimum of two rays. If you draw more than two rays, however, you can check the image location you find with the first two rays. Remember, too, that you can draw any number of rays reflecting from the mirror, and that all the rays should obey the law of reflection. There are at least four rays that are easy to draw the reflections for. These rays are shown on Figure 23.34, and include:

1. The ray, in red, that goes parallel to the principal axis, and reflects so that it passes through the focal point (concave mirror), or away from the focal point (convex mirror).
2. The ray, in blue, that reflects from the point on the principal axis that intersects the surface of the mirror. The principal axis is perpendicular to the surface of the mirror, so the angle between the incident ray and the principal axis is the same as the angle between the reflected ray and the principal axis.
3. The ray, in green, that travels along the straight line connecting the tip of the object and the mirror's center of curvature. This ray is incident on the mirror along the normal to the mirror's surface, and thus reflects straight back along the same line.
4. The ray, in purple, that travels along the straight line connecting the tip of the object and the focal point. This ray reflects to go parallel to the principal axis.

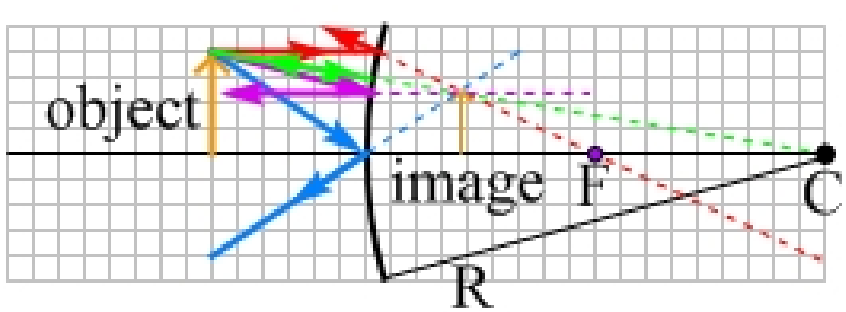


Figure 23.34: An example of the four rays that are easy to draw the reflections for. All the reflected rays meet at the tip of the image.

EXAMPLE 23.7 – Applying the general method

When you stand in front of a mirror that has a radius of curvature of 40 cm, you see an image that is half your size. What kind of mirror is it? How far from the mirror are you? Sketch a ray diagram to check your calculations.

SOLUTION

In this case, let's first apply the equations and then draw the ray diagram. The mirror could be convex, because convex mirrors always produce images that are smaller than the object. A convex mirror produces a virtual, upright image, so the sign of the magnification is positive. Applying the magnification equation, we get:

$$m = +\frac{1}{2} = -\frac{d_i}{d_o}, \text{ which tells us that } \frac{1}{d_i} = -\frac{2}{d_o}.$$

For a convex mirror, the focal length is –*R*/2, which in this case is –20 cm. Applying the mirror equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} - \frac{2}{d_o} = -\frac{1}{d_o}.$$

Thus, we find that $d_o = -f = +20\text{ cm}$, and we can show that $d_i = -10\text{ cm}$.

The ray diagram for this situation is shown in Figure 23.35, confirming the calculations.

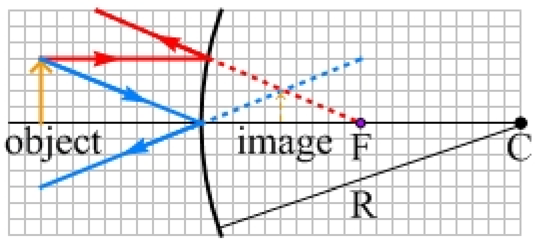


Figure 23.35: A ray diagram for the solution involving a convex mirror. Each box on the grid measures 2 cm × 2 cm.

The solution above is only one of the possible answers. The mirror could also be concave, because a concave mirror can produce a real, inverted image, so the sign of the magnification is negative. Applying the magnification equation, we get:

$$m = -\frac{1}{2} = -\frac{d_i}{d_o}, \text{ which tells us that } \frac{1}{d_i} = \frac{2}{d_o}.$$

For a concave mirror the focal length is +*R*/2, which in this case is +20 cm. Applying the mirror equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{2}{d_o} = \frac{3}{d_o}.$$

Thus, we find that $d_o = 3f = +60\text{ cm}$, and we can show that $d_i = +30\text{ cm}$.

The ray diagram for this situation is shown in Figure 23.36, again confirming the calculations above.

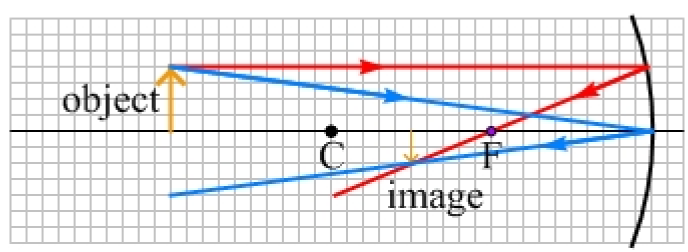


Figure 23.36: A ray diagram for the situation involving a concave mirror. Each box on the grid measures 2 cm × 2 cm.

Related End-of-Chapter Exercises: 16, 17, 19, 20, 43.

Essential Question 23.7: Return to the situation described in Example 23.7. Would there still be two solutions if the image was larger than the object? Explain.

Answer to Essential Question 23.7: Yes, there would still be two solutions, but we would not have a solution associated with a convex mirror, because the convex mirror cannot produce an image that is larger than the object. Instead, both solutions would be associated with a concave mirror. One solution would be for a real image, and the other solution would be for a virtual image.

Chapter Summary

Essential Idea: Reflection and Mirrors.

To understand the image that is created by a mirror, we make use of a simple model of light called the ray model. In the ray model, a ray of light travels in a straight line until it encounters an object. When a ray of light reflects from an object, the light obeys the Law of Reflection.

The Law of Reflection

The angle of incidence is equal to the angle of reflection. These angles are generally measured from the normal to the surface.

Image Formation

For a mirror to form an image of an object, light rays must leave the object and be reflected by the mirror. If the rays leaving a single point on the object are reflected so that they pass through a single point, a real image is formed. If, instead, such reflected rays appear to diverge from a single point behind the mirror (as is the case for a typical bathroom mirror), a virtual image is formed.

Ray Diagrams

When drawing a ray diagram, we generally show rays leaving the tip of the object and reflecting from the mirror. Where the reflected rays meet is where the tip of the image is located. The ray diagram gives us qualitative information about the location and size of the image and about the image characteristics. All rays obey the Law of Reflection when they reflect from the mirror, but some reflected rays are particularly easy to draw. A summary of four such rays is given in section 23-7.

Plane and Spherical Mirrors

Type of mirror	Focal length	Image characteristics
Plane	∞	The image is virtual, upright, the same size as the object, and the same distance behind the mirror that the object is in front of the mirror.
Convex (diverging)	$-R/2$, R being the mirror's radius of curvature	The image is virtual, upright, smaller than the object, and located between the mirror and the mirror's focal point.
Concave (converging)	$+R/2$	The image can be real or virtual, and larger than, smaller than, or the same size as the object. See the table below for details.

Table 23.1: A summary of the mirrors we investigated in this chapter.

Images formed by a Concave (Converging) Mirror

Object position	Image position	Image characteristics
∞	At the focal point.	Real image with height of zero.
Moving from ∞ toward the center of curvature.	Moving from the focal point toward the center of curvature.	The image is real, inverted, and smaller than the object. The image moves closer to the center of curvature, and increases in height, as the object is moved closer to the center of curvature.
At the center of curvature.	At the center of curvature.	The image is real, inverted, and the same size as the object.
Moving from the center of curvature toward the focal point.	Moving from the center of curvature toward infinity.	The image is real, inverted, and larger than the object. The image moves farther from the mirror, and increases in height, as the object is moved closer to the focal point.
At the focal point.	At infinity.	The image is at infinity, and is infinitely tall.
Closer to the mirror than the focal point.	Behind the mirror	The image is virtual, upright, and larger than the object. The image moves closer to the mirror, and decreases in height, as the object is moved closer to the mirror.

Table 23.1: A summary of the image positions and characteristics for various object positions with a concave mirror.

The mirror equation

The mirror equation relates the object distance, d_o, the image distance, d_i, and the mirror's focal length, f. The mnemonic "If I do I di" can help you to remember the mirror equation.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}.$$ (Equation 23.4: **The mirror equation**)

$$d_i = \frac{d_o \times f}{d_o - f}$$ (Equation 23.5: **The mirror equation, solved for the image distance**)

Sign conventions

The focal length is positive for a concave mirror, and negative for a convex mirror.

The image distance is positive if the image is on the reflective side of the mirror (a real image), and negative if the image is behind the mirror (a virtual image).

The image height is positive when the image is above the principal axis, and negative when the image is below the principal axis. A similar rule applies to the object height.

The image height is positive when the image is upright, and negative when the image is inverted. A similar rule applies to the object height.

Magnification

The magnification, m, is the ratio of the image height (h_i) to the object height (h_o).

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}.$$ (Equation 23.6: **Magnification**)

- The image is larger than the object if $|m| > 1$.
- The image and object have the same size if $|m| = 1$.
- The image is smaller than the object if $|m| < 1$.

The magnification is positive if the image is upright, and negative if the image is inverted.