21-5 The Doppler Effect for Sound

We have probably all had the experience of listening to the siren on an emergency vehicle as it approaches us, and hearing a shift in the frequency of the sound when the vehicle passes us. This shift in frequency is known as the Doppler effect, and it occurs whenever the wave source or the detector of the wave (your ear, for instance) is moving relative to the medium the wave is traveling in. Applications of the Doppler effect for sound include Doppler ultrasound, a diagnostic tool used to study blood flow in the heart. There is a related but slightly different Doppler effect for electromagnetic waves, which we will investigate in the next chapter, that has applications in astronomy as well as in police radar systems to measure the speed of a car.

EXPLORATION 21.5 – Understanding the Doppler effect

Let's explore the principles behind the Doppler effect. We will begin by looking at the situation of a stationary source of sound, and a moving observer.

Step 1 – ***Construct a diagram showing waves expanding spherically from a stationary source that is broadcasting sound waves of a single frequency. If you, the observer, remain stationary, you hear sound of the same frequency as that emitted by the source. Use your diagram to help you explain whether the frequency you hear when you move toward the source, or away from the source, is higher or lower than the frequency emitted by the source.*** We can represent the expanding waves as a set of concentric circles centered on the stationary source, as in Figure 21.6. This picture shows a snapshot of the waves at one instant in time, but remember that the waves are expanding outward from the source at the speed of sound. If you are stationary at position A, the waves wash over you at the same frequency as they were emitted. If you are at position A but moving toward the source, however, the frequency you observe increases, because you are moving toward the oncoming waves. Conversely, if you move away from the source (and you are traveling at a speed less than the speed of sound), you observe a lower frequency as you try to out-run the waves.

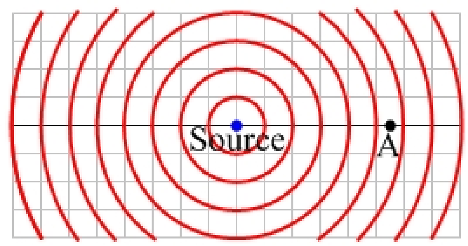

Figure 21.6: Waves emitted by a stationary source (the blue dot) expand out away from the source, giving a pattern of concentric circles centered on the source. You, the observer, are at point A. The frequency of the waves you receive depends on both your speed and the direction of your motion, if you are moving

Step 2 – ***Starting with the usual relationship connecting frequency, speed, and wavelength, $f = v / \lambda$, think about whether the observer moving toward or away from a stationary source effectively changes the wave speed or the wavelength. If the speed of sound is v and the observer's speed is v_o, write an equation for the frequency heard by the observer.*** As we can see from the pattern in Figure 21.6 above, the wavelength has not changed. What changes, when you move through the pattern of waves, is the speed of the waves with respect to you. When you move toward the source, the effective speed of the waves (the relative speed of the waves with respect to you) is $v + v_o$, while when you move away from the source the wave speed is effectively $v - v_o$. The frequency you observe, f', is thus the effective speed over the wavelength:

$$f' = \frac{v \pm v_o}{\lambda} = \frac{v}{\lambda}\left(\frac{v \pm v_o}{v}\right) = f\left(\frac{v \pm v_o}{v}\right),$$ (Eq. 21.9: **Frequency for a moving observer**)

where f is the frequency emitted by the source, and where we use the + sign when the observer moves toward the source, and the – sign when the observer moves away from the source.

Step 3 – ***Construct a diagram showing waves expanding from a source that is moving to the right at half the speed of sound while broadcasting sound waves of a single frequency. Use your diagram to help you explain whether the frequency you hear when you are stationary is higher or lower than that emitted by the source, when the source is moving toward you and when the source is moving away from you.*** In this situation, the result is quite different from that in Figure 21.21, because each wave is centered on the position of the source at the instant the wave was emitted. Because the waves are emitted at different times, and the source is moving, we get the picture shown in Figure 21.22. To the left of the source, such as at point B, the waves are more spread out. Thus, when the source is moving away from the observer, the observed frequency is less than the emitted frequency. The reverse is true for a point to the right of the source: the waves are closer together than usual, so an observer in this region (such as at point C) observes a greater frequency than the emitted frequency.

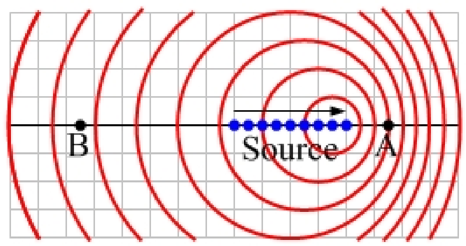

Figure 21.7: When a source of waves is moving relative to the medium, the wave pattern is asymmetric. An observer for which the source is moved away observes a lower-frequency wave, while when the source is moving toward the observer, a higher-frequency wave is observed. In the case shown, the source is moving to the right at half the wave speed.

Step 4 – ***Starting with the usual relationship connecting frequency, speed, and wavelength, $f = v / \lambda$, think about whether the source moving toward or away from a stationary observer effectively changes the wave speed or the wavelength. If the speed of sound is v and the source's speed is v_s, write an equation for the frequency heard by the observer.*** As we can see from the pattern in Figure 21.7, the movement of the source changes the wavelength. The waves still travel at the speed of sound, however. What changes, when you move through the pattern of waves, is the speed of the waves with respect to you. When the source moves toward the observer, the effective wavelength is $(v - v_s)/f$, while when the source moves away the wavelength is effectively $(v + v_s)/f$. The frequency you observe, f', is thus the speed over the effective wavelength:

$$f' = \frac{v}{\lambda'} = \frac{v}{v \mp v_s} f \ ,$$ (Eq. 21.10: **Frequency for a moving source**)

where f is the frequency emitted by the source. Use the – sign when the source moves toward the observer, and the + sign when the source moves away from the observer.

Key idea for the Doppler effect: Motion of a source of sound, or motion of an observer, can cause a shift in the observed frequency of a wave. **Related End-of-Chapter Exercises: 11, 12.**

Essential Question 21.5: Is the Doppler effect simply a relative velocity phenomenon? For instance, is the situation of an observer moving at speed v_1 toward a stationary source the same as a source moving at speed v_1 toward a stationary observer?

Answer to Essential Question 21.5: The Doppler effect for sound (and for all mechanical waves) is not a relative velocity phenomenon. The relative velocity of the source and observer is the same in these two situations, but the observed frequency is different in the two situations. One interesting example is when $v_I = v$, the wave speed. When the observer moves at speed v toward a stationary source, the observed frequency is twice the emitted frequency. When the source moves at a speed v toward a stationary observer, however, the observed frequency is infinite. We will investigate that situation further in the next section.

21-6 Sonic Booms, and the Doppler Effect in General

Essential Question 21.5 raises the question of what happens when a source of waves travels at the wave speed. We should also consider what happens when the source travels faster than the wave source.

Let's begin by drawing a diagram like that in Figure 21.7, but with the source traveling to the right at the wave speed. In this special case, in Figure 21.8, because the source keeps up with the waves, the waves pile up at the source, leading to a large amplitude wave that moves with the source. This is known as a **sonic boom**, because a large amplitude corresponds to a loud sound. The observer at position A would hear the sonic boom when the source passed by.

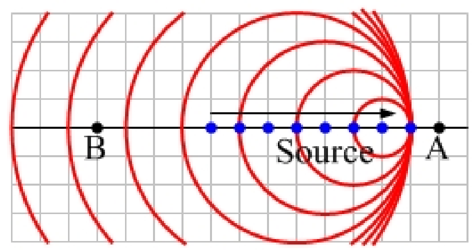

Figure 21.8: When the source moves at the wave speed, the waves pile up on one another at the source, creating a sonic boom.

Let's go further, and see what the picture looks like when the source travels faster than the waves. Figure 21.9 shows what happens when the source travels to the right at twice the wave speed. In this case, the waves pile up along lines that make an angle with the line of travel of the source. This pattern should look familiar to you, given that it looks like the waves left behind by a boat as it travels through water, as in the photograph in Figure 21.10. This tells us that the boat's speed is faster than the speed of the water waves.

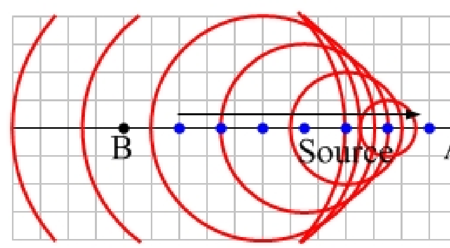

Figure 21.9: When the source moves faster than the waves, the waves create a wake pattern.

Figure 21.10: A common example of the situation of a source of waves traveling faster through the medium than the waves themselves is in the shape of the wake created when a boat passes through water.

In section 21-5, we considered what happens when either the source moves or the observer moves, but not both. Let's now consider what happens in general, when both the source of a wave and the observer are moving with respect to the medium the waves are moving through. The general equation is simply a combination of the equations we derived in section 21-5 for the situations of a moving observer and a moving source

The Doppler effect: The Doppler effect describes the shift in frequency of a wave that occurs when the source of the waves, and/or the observer of the waves, moves with respect to the medium the waves are traveling through. The general equation for the observed frequency is:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right), \qquad \text{(Equation 21.11: \textbf{The general Doppler equation})}$$

where f' is the frequency observed by the observer, f is the frequency of the waves emitted by the source, v is the speed of the wave through the medium, v_o is the speed of the observer, and v_s is the speed of the source. In the numerator, use the top (+) sign if the observer moves toward the source, and the bottom (–) sign if the observer moves away from the source. In the denominator, use the top (–) sign if the source moves toward the observer, and the bottom (+) sign if the source moves away from the observer.

EXAMPLE 21.6 – Catching a moth

A particular bat emits ultrasonic waves with a frequency of 56.0 kHz. The bat is flying at 16.00 m/s toward a moth, which is moving at 2.00 m/s away from the bat. The speed of sound is 340.00 m/s. (a) Assuming the moth could detect the waves, what frequency waves would it observe? (b) The waves reflect off the moth and are detected by the bat. What frequency are the waves detected by the bat?

SOLUTION

(a) Here, we use the general Doppler equation, where f = 56.0 kHz and v = 340 m/s. The observer is the moth, so v_o is 2.00 m/s, and we use the bottom sign (the minus sign) in the numerator because the moth is traveling away from the bat. The bat is the source, so v_s = 16.00 m/s, and we use the top sign (the minus sign) in the denominator because the bat is traveling toward the moth. This gives:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right) = (56.0\text{ kHz})\left(\frac{340.00\text{ m/s} - 2.00\text{ m/s}}{340.00\text{ m/s} - 16.00\text{ m/s}}\right) = (56.0\text{ kHz}) \times 1.0432 = 58.42\text{ kHz}$$

(b) Again, we use the general Doppler equation, but this time the moth acts as the source (because the moth reflects the waves back to the bat) and the bat is the observer. The frequency emitted by the moth is the frequency we calculated in part (a). Let's use f'' to denote the frequency of the waves detected by the bat, so f' = 56.0 kHz and v = 340 m/s. The observer is the bat, so v_o' is 16.00 m/s, and we use the top sign (the plus sign) in the numerator because the bat is traveling toward the moth. The moth is the source, so v_s' = 2.00 m/s, and we use the bottom sign (the plus sign) in the denominator because the moth is traveling away from the bat. This gives:

$$f'' = f'\left(\frac{v \pm v_o'}{v \mp v_s'}\right) = (58.42\text{ kHz})\left(\frac{340.00\text{ m/s} + 16.00\text{ m/s}}{340.00\text{ m/s} + 2.00\text{ m/s}}\right) = (58.42\text{ kHz}) \times 1.0409 = 60.8\text{ kHz}$$

The bat can use the frequency of the detected wave to determine how fast, and in what direction, the moth is flying.

Related End-of-Chapter Exercises: 13 – 15, 38 – 40.

Essential Question 21.6: What happens when a source and observer have identical velocities? Is the observed frequency larger, smaller, or the same as the frequency emitted by the source?