Resistance of a light bulb

Let's use the power equation to calculate the resistance of a 100 W light bulb.

The bulb's power is 100 W when the potential difference is 120 V, so we can find the resistance from:

\[ P = I^2 R = \left( \frac{\Delta V}{R} \right)^2 R \]

\[ \Rightarrow R = \frac{\Delta V^2}{P} = \frac{120^2}{100} = 144 \, \Omega \]
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We can check this by measuring the resistance with a ohmmeter, \textit{when the bulb is hot}. 
Resistance of a light bulb

Let's use the power equation to calculate the resistance of a 40 W light bulb.

The bulb's power is 40 W when the potential difference is 120 V, so we can find the resistance from:

\[ P = I^2 R = \left( \frac{\Delta V}{R} \right)^2 R \]

\[ \Rightarrow R = \frac{\Delta V^2}{P} = \frac{120^2}{40} = 360 \, \Omega \]
Resistors in series

When resistors are in series they are arranged in a chain, so the current has only one path to take – **the current is the same through each resistor**. The sum of the potential differences across each resistor equals the total potential difference across the whole chain.

\[ V = V_1 + V_2 \]

\[ IR_{eq} = IR_1 + IR_2 \]

The \( I \)'s are the same, and we can generalize to any number of resistors, so **the equivalent resistance of resistors in series is**:

\[ R_{eq} = R_1 + R_2 + R_3 + \cdots \]
Resistors in parallel

When resistors are arranged in parallel, the current has multiple paths to take. The potential difference across each resistor is the same, and the currents add to equal the total current entering (and leaving) the parallel combination.

\[ I = I_1 + I_2 \]

\[ \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} \]

The \( V \)'s are all the same, and we can generalize to any number of resistors, so the equivalent resistance of resistors in parallel is:

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \]
A 100-W light bulb is connected in parallel with a 40-W light bulb, and the parallel combination is connected to a standard electrical outlet. The 40-W light bulb is then unscrewed from its socket. What happens to the 100-W bulb?

1. It turns off
2. It gets brighter
3. It gets dimmer (but stays on)
4. Nothing at all – it stays the same
Light bulbs in series

A 100-W light bulb is connected in series with a 40-W light bulb and a standard electrical outlet. Which bulb is brighter?

1. The 40-watt bulb
2. The 100-watt bulb
3. Neither, they are equally bright
Light bulbs in series

The brightness is related to the power (not the power stamped on the bulb, the power actually being dissipated in the bulb). The current is the same through the bulbs, so consider:

\[ P = I^2 R \]

We already showed that the resistance of the 100 W bulb is 144 \( \Omega \) at 120 volts. A similar calculation showed that the 40 W bulb has a resistance of 360 \( \Omega \) at 120 volts. Neither bulb has 120 volts across it, but the key is that the resistance of the 40 W bulb is larger, so it dissipates more power and is brighter.
Light bulbs in series, II

A 100-W light bulb is connected in series with a 40-W light bulb and a standard electrical outlet. The 100-W light bulb is then unscrewed from its socket. What happens to the 40-W bulb?

1. It turns off
2. It gets brighter
3. It gets dimmer (but stays on)
4. Nothing at all – it stays the same
Bulbs and switches

Four identical light bulbs are arranged in a circuit. What is the minimum number of switches that must be closed for at least one light bulb to come on?
Bulbs and switches

What is the minimum number of switches that must be closed for at least one light bulb to come on?

1. 1
2. 2
3. 3
4. 4
5. 0
Bulbs and switches

Is bulb A on already?
Bulbs and switches

Is bulb A on already?

No. For there to be a current, there must be a complete path through the circuit from one battery terminal to the other.
Bulbs and switches

To complete the circuit, we need to close switch D, and either switch B or switch C.
Bulbs and switches, II

Which switches should be closed to maximize the brightness of bulb D?

1. All four switches.
2. Switch D and either switch B or switch C
3. Switch D and both switches B and C
4. Switch A, either switch B or switch C, and switch D
5. Only switch D.
Bulbs and switches, II

What determines the brightness of a bulb?
Bulbs and switches, II

What determines the brightness of a bulb?

The power.

\[ P = I^2 R \]

For a bulb of fixed resistance, maximizing power dissipated in the bulb means maximizing the current through the bulb.
Bulbs and switches, II

We need to close switch D, and either switch B or switch C, for bulb D to come on. Do the remaining switches matter?
Bulbs and switches, II

We need to close switch D, and either switch B or switch C, for bulb D to come on. Do the remaining switches matter?

Consider this. How much of the current that passes through the battery passes through bulb D?
Bulbs and switches, II

We need to close switch D, and either switch B or switch C, for bulb D to come on. Do the remaining switches matter?

Consider this.
How much of the current that passes through the battery passes through bulb D?

All of it.
Bulbs and switches, II

If we open or close switches, does it change the total current in the circuit?
Bulbs and switches, II

If we open or close switches, does it change the total current in the circuit?

Absolutely, because it changes the total resistance (the equivalent resistance) of the circuit.

\[ I_{total} = \frac{V_{\text{battery}}}{R_{eq}} \]
Bulbs and switches, II

Does it matter whether just one of switches B and C are closed, compared to closing both of these switches?
Bulbs and switches, II

Does it matter whether just one of switches B and C are closed, compared to closing both of these switches?

Yes. Closing both switches B and C decreases the resistance of that part of the circuit, decreasing $R_{eq}$. That increases the current in the circuit, increasing the brightness of bulb D.
Bulbs and switches, II

What about switch A?
Bulbs and switches, II

What about switch A?
An open switch is a path of _______ resistance.
A closed switch is a path of _______ resistance.
Bulbs and switches, II

What about switch A?
An open switch is a path of infinite resistance.
A closed switch is a path of zero resistance.
Bulbs and switches, II

What about switch A?

Closing switch A takes bulb A out of the circuit. That decreases the total resistance, increasing the current, making bulb D brighter.

Close all 4 switches.
How do we analyze a circuit like this, to find the current through, and voltage across, each resistor?

$R_1 = 6 \, \Omega \quad R_2 = 36 \, \Omega \quad R_3 = 12 \, \Omega \quad R_4 = 3 \, \Omega$
A combination circuit

First, replace two resistors that are in series or parallel by one equivalent resistor. Keep going until you have one resistor. Find the current in the circuit. Then, expand the circuit back again, finding the current and voltage at each step.
Combination circuit: rules of thumb

Two resistors are in series when the same current that passes through one resistor goes on to pass through another.

Two resistors are in parallel when they are directly connected together at one end, directly connected at the other, and the current splits, some passing through one resistor and some through the other, and then re-combines.
A combination circuit

Where do we start?

\[ R_1 = 6 \, \Omega \quad R_2 = 36 \, \Omega \quad R_3 = 12 \, \Omega \quad R_4 = 3 \, \Omega \]
A combination circuit

Where do we start?

\[ R_1 = 6 \ \Omega \quad R_2 = 36 \ \Omega \quad R_3 = 12 \ \Omega \quad R_4 = 3 \ \Omega \]

Resistors 2 and 3 are in parallel.
A combination circuit

\[
\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{36 \, \Omega} + \frac{1}{12 \, \Omega} = \frac{1}{36 \, \Omega} + \frac{3}{36 \, \Omega} = \frac{4}{36 \, \Omega}
\]

\[R_{23} = \frac{36 \, \Omega}{4} = 9 \, \Omega\]
A combination circuit

What next?

R_1 = 6 \, \Omega \quad R_{23} = 9 \, \Omega \quad R_4 = 3 \, \Omega
A combination circuit

What next?

R_1 = 6 \, \Omega \quad R_{23} = 9 \, \Omega \quad R_4 = 3 \, \Omega

Resistors 2-3 and 4 are in series.
A combination circuit

\[ R_{234} = R_{23} + R_4 = 9 \, \Omega + 3 \, \Omega = 12 \, \Omega \]

Now what?
\[ R_1 = 6 \, \Omega \quad R_{234} = 12 \, \Omega \]
A combination circuit

\[ R_{234} = R_{23} + R_4 = 9 \, \Omega + 3 \, \Omega = 12 \, \Omega \]

Now what? These resistors are in parallel.

\[ R_1 = 6 \, \Omega \quad R_{234} = 12 \, \Omega \]
A combination circuit

\[
\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_{234}} = \frac{1}{6 \, \Omega} + \frac{1}{12 \, \Omega} = \frac{2}{12 \, \Omega} + \frac{1}{12 \, \Omega} = \frac{3}{12 \, \Omega}
\]

\[
R_{eq} = \frac{12 \, \Omega}{3} = 4 \, \Omega
\]
Now, find the current in the circuit.
Now, find the current in the circuit.

\[ I_{total} = \frac{V_{battery}}{R_{eq}} = \frac{12 \text{ V}}{4 \text{ } \Omega} = 3 \text{ A} \]
A combination circuit

Expand the circuit back, in reverse order.
A combination circuit

When expanding an equivalent resistor back to a parallel pair, the voltage is the same, and the current splits. Apply Ohm’s Law to find the current through each resistor. Make sure the sum of the currents is the current in the equivalent resistor.
When expanding an equivalent resistor back to a series pair, the current is the same, and the voltage divides. Apply Ohm’s Law to find the voltage across each resistor. Make sure the sum of the voltages is the voltage across the equivalent resistor.
A combination circuit

The last step.
Three identical light bulbs are connected in the circuit shown. When the power is turned on, and with the switch beside bulb C left open, how will the brightnesses of the bulbs compare?

1. A = B = C
2. A > B > C
3. A > B = C
4. A = B > C
5. B > A > C
Three identical bulbs, II

When the switch is closed, bulb C will turn on, so it definitely gets brighter.

What about bulbs A and B?
1. Both A and B get brighter
2. Both A and B get dimmer
3. Both A and B stay the same
4. A gets brighter while B gets dimmer
5. A gets brighter while B stays the same
6. A gets dimmer while B gets brighter
7. A gets dimmer while B stays the same
8. A stays the same while B gets brighter
9. A stays the same while B gets dimmer
Three identical bulbs, II

Closing the switch brings C into the circuit - this reduces the overall resistance of the circuit, so the current in the circuit increases.
Three identical bulbs, II

Closing the switch brings C into the circuit - this reduces the overall resistance of the circuit, so the current in the circuit increases.

Increasing the current makes A brighter.
Three identical bulbs, II

Closing the switch brings C into the circuit - this reduces the overall resistance of the circuit, so the current in the circuit increases.

Increasing the current makes A brighter. Because $\Delta V = IR$, the potential difference across bulb A increases.
Three identical bulbs, II

Closing the switch brings C into the circuit - this reduces the overall resistance of the circuit, so the current in the circuit increases.

Increasing the current makes A brighter. Because $\Delta V = IR$, the potential difference across bulb A increases. This decreases the potential difference across B, so its current drops and B gets dimmer.