18-3 Circuit Analogies, and Kirchoff's Rules

Analogies can help us to understand circuits, because an analogous system helps us build a model of the system we are interested in. For instance, there are many parallels between fluid being pumped through a set of pipes and charge flowing around a circuit. There are also useful parallels we can draw between a circuit and a ski hill, in which skiers are taken to the top of the hill by a chair lift and then ski down via various trails. Let's investigate these in turn.

A particular fluid system is shown in Figure 18.5. The fluid is enclosed in a set of pipes, and a water wheel spins in response to the flow. The fluid circulates through the system by means of a pump, which creates a pressure difference (analogous to potential difference in the circuit) between different sections of the system.

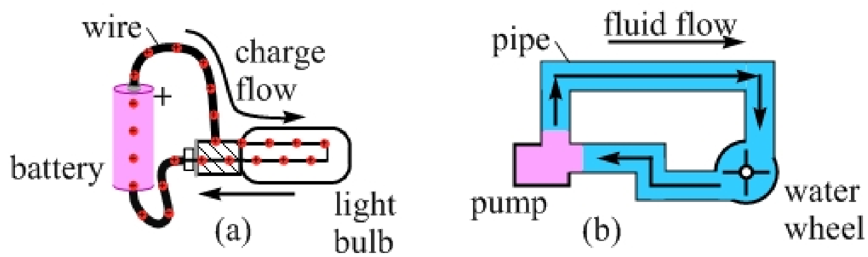

Figure 18.5: Fluid flows through a set of pipes like charges flow through a circuit.

The pump in the fluid system is like the battery in the circuit; the water is like the charge; and the water wheel is like the light bulb in the circuit. The large pipes act like the wires, one pipe carrying water from the pump to the water wheel, and another carrying the water back to the pump, much as charge flows through one wire from the battery to the light bulb, and through a second wire back to the battery. The pressure difference in the fluid system is analogous to the potential difference across the resistor in the circuit.

EXPLORATION 18.3 – Analogies between a circuit and a ski hill

A basic ski hill consists of a chair lift, like that shown in Figure 18.6, that takes skiers up to the top of the hill, and a trail that skiers ski down to the bottom. A short and wide downward slope takes the skiers from the top of the lift to the top of the trail, and another takes the skiers from the bottom of the trail to the bottom of the lift.

Figure 18.6: A chair lift taking skiers up a hill. Photo credit: PhotoDisc/Getty Images.

Step 1 – ***Identify the aspects of the circuit that are analogous to the various aspects of the ski hill. In particular, identify what for the ski hill plays the role of the battery, the flowing charge, and the resistor.*** The chair lift is like the battery in the circuit, while the skiers are like the charges. The chair lift raises the gravitational potential energy of the skiers, and the skiers dissipate all that energy as they ski down the trail (which is like the resistor in the circuit) to the bottom. Similarly, the battery raises the electric potential energy of the charges, and that energy is dissipated as the charges flow through the resistor.

Step 2 – ***Does the analogy have limitations? Identify at least one difference between the ski hill and the electric circuit.*** What happens when the chair lift / battery is turned off? On the ski hill the skiers keep skiing down to the bottom, but in the circuit if a switch is opened the net flow of charge stops. This difference stems from the fact that with the ski hill the potential difference is imposed by something external to the system, the Earth's gravity, while in the circuit the battery provides the potential difference. Another difference is that in the circuit the charges are identical and obey basic laws of physics, while on a ski hill the skiers are not identical, and make choices regarding when to stop for lunch or to enjoy the view, and which route to take down the hill.

Step 3 - Let's use our ski hill analogy to understand two basic rules about circuits, which we will make use of throughout the rest of the chapter. Figure 18.7 shows one ski trail dividing into two, trails A and B. The same thing happens in a circuit, with one path dividing into paths A and B. ***What is the relationship between N, the total number of skiers, and N_A and N_B, the number of skiers choosing trails A and B, respectively? What is the analogous relationship between the current I in the top path in the circuit, and the currents I_A and I_B in paths A and B?***

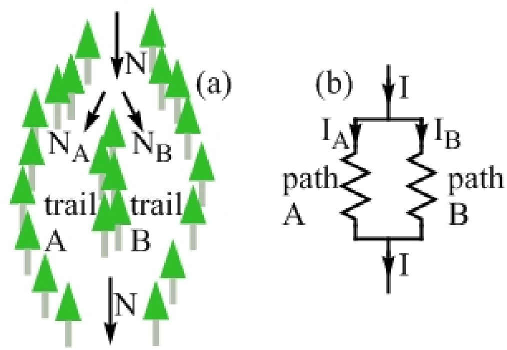

Figure 18.7: (a) One trail splits temporarily into two on the ski hill; (b) one path splits into two in the circuit.

We do not lose or gain skiers, so some skiers choose trail A and the rest choose B, giving $N = N_A + N_B$. The skiers come back together when the trails re-join, and *N* skiers continue down the trail. Similarly, a certain number of charges flow through the top path in the circuit, and the charges take either path A or path B through the circuit before re-combining. The rate of flow of charge is the current, so we can say that $I = I_A + I_B$. This is the **junction rule**.

Step 4 - ***Figure 18.8 shows various points on a ski hill and in a circuit. For a complete loop, say from point 3 back to point 3, if we add up the changes in gravitational potential as we move around the loop, what will we get? If we do the analogous process for the circuit, adding up the electric potential differences as we move around a complete loop, what will we get?***

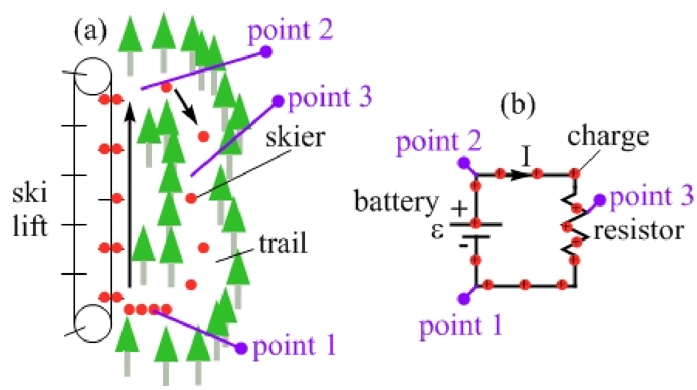

Figure 18.8: Equivalent points marked on a ski hill and on a circuit.

In both cases we get zero. There is as much up as down on the ski hill, so when we return to the starting point the net change in potential is zero. The same applies to the circuit. This is the **loop rule**.

Key Ideas for Analogies: Analogies, particularly the gravitation-based ski-hill analogy, can give us considerable insight into circuits. In this case we used the analogy to come up with what are known as Kirchoff's Rules. The **Junction Rule** – ***the total current entering a junction is equal to the total current leaving a junction***. The **Loop Rule** – ***the sum of all the potential differences for a closed loop is zero***. **Related End-of-Chapter Exercises: 38 and 39.**

Essential Question 18.3: In Figure 18.7, let's say the resistance of path A is larger than that of path B. Which current is larger? What is the blanket statement summarizing this idea?

Answer to Essential Question 18.3: If path A has a larger resistance then path B we would expect the current in path A to be smaller than that in path B, much as we might expect more skiers to choose trail B if it is an easier trail than trail A. The blanket statement about this is – **current prefers the path of least resistance**.

18-7 Series-Parallel Combination Circuits

In many circuits some resistors are in series while others are in parallel. In such series-parallel combination circuits we often want to know the current through, and/or the potential difference across, each resistor. Let's explore one method for doing this. This method can be used if the circuit has one battery, or when multiple batteries can be replaced by a single battery.

EXPLORATION 18.7 – The contraction/expansion method of circuit analysis

Four resistors are connected in a circuit with a battery with an emf of 18 V, as shown in Figure 18.14. The resistors have resistances $R_1 = 4.0\,\Omega, R_2 = 5.0\,\Omega, R_3 = 7.0\,\Omega,$ and $R_4 = 6.0\,\Omega$. Our goal is to find the current through each resistor.

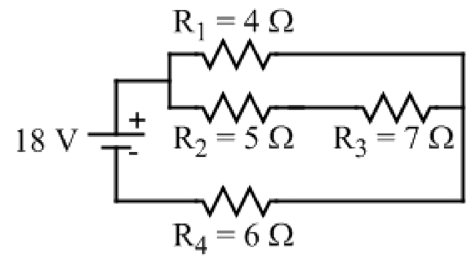

Figure 18.14: A series-parallel combination circuit with one battery and four resistors.

Step 1 – ***Label the currents at various points in the circuit. This can help determine which resistors are in series and which are in parallel.*** This is done in Figure 18.15(a). The current passing through the battery is labeled *I*. This current splits, with a current I_1 through resistor R_1, and a current I_2 through resistor R_2. The current I_2 goes on to pass through R_3, so R_2 and R_3 are in series with one another. The two currents re-combine at the top right of the circuit, giving a net current of *I* directed from right to left through resistor R_4 and back to the battery.

Step 2 – ***Identify two resistors that are either in series or in parallel with one another, and replace them by a resistor of equivalent resistance.*** Resistors R_2 and R_3 are in series, so they can be replaced by their equivalent resistance of 12.0 Ω (resistances add in series), as shown in Figure 18.15(b). Is this the only place we could start in this circuit? For instance, are resistors R_1 and R_2 in parallel? To be in parallel, both ends of the resistors must be directly connected by a wire, with nothing in between. The left ends of resistors R_1 and R_2 are directly connected, but the right ends are not, with resistor R_3 in between. In fact, the only place to start in this circuit is with R_2 and R_3.

Step 3 – ***Continue the process of replacing two resistors by an equivalent resistor until the circuit is reduced to one equivalent resistor.*** In the next step, shown in Figure 18.15(c), the two resistors in parallel, R_1 and R_{23}, are replaced by their equivalent resistance of 3.0 Ω. Finally, in Figure 18.15(d), the two resistors in series are replaced by their equivalent resistance, 9.0 Ω.

Step 4 – ***Apply Ohm's Law to find the total current in the circuit.*** With only one resistor we know both its resistance and the potential difference across it, so we can apply Ohm's Law:

$$I = \frac{\varepsilon}{R_{eq}} = \frac{18\text{ V}}{9.0\,\Omega} = 2.0\text{ A}.$$

Step 5 – ***Label the potential at various points in the single-resistor circuit.*** Choose a point as a reference. Here we choose the negative terminal of the battery to be $V = 0$. The other side of the battery is therefore +18 volts. Wires have negligible resistance, so $\Delta V = IR = 0$ across each wire. Thus, all points along the wire leading from the negative terminal of the battery have $V = 0$, while all points along the wire leading from the positive terminal have $V = +18$ V. See Figure 18.15(e).

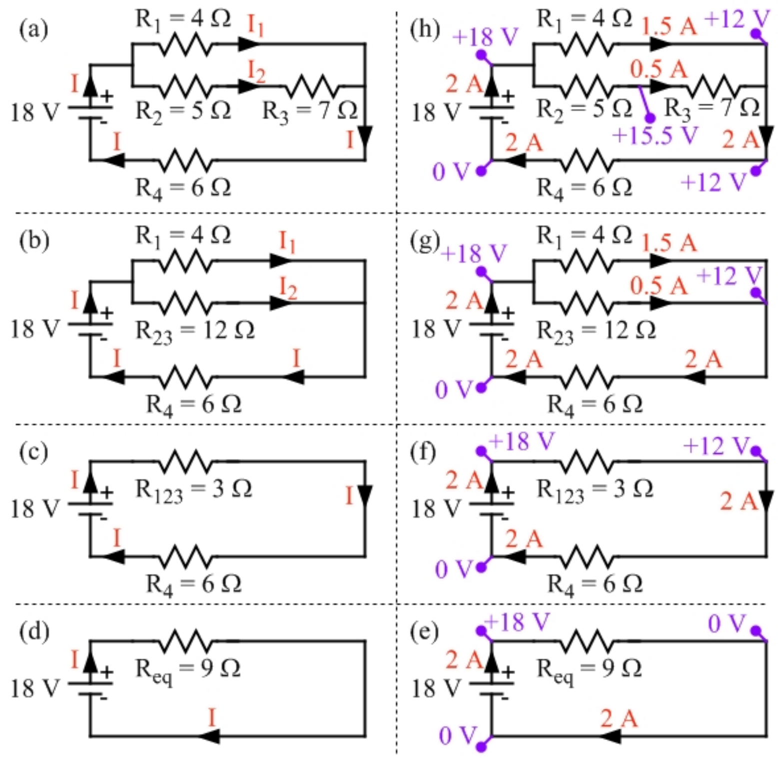

Figure 18.15: The various steps in the contraction/expansion method.

(a) Labeling the current at various points can help identify which resistors are in series and which are in parallel.

(b) R_2 and R_3 are in series, and can be replaced by one equivalent resistor, R_{23}.

(c) – (d) Resistors are replaced a pair at a time to find the circuit's equivalent resistance.

(e) Apply Ohm's Law to find the total current.

(f - h) Expansion reverses the steps. At each expansion step we find the current and potential difference for each resistor.

Step 6 – ***Expand the circuit back from one resistor to two. Find the current through, and potential difference across, both resistors.*** Expansion reverses the steps of the contraction. In Figure 18.15(f) we replace the 9.0 Ω resistor by the 3.0 Ω and 6.0 Ω resistors, in series, it came from. When a resistor is split into two in series, the current (2.0 A in this case) through all three resistors is the same. We can now Ohm's Law to find the potential difference. For the two resistors we get $\Delta V_{3\Omega} = IR = 6.0\text{ V}$ and $\Delta V_{6\Omega} = IR = 12\text{ V}$. Thus the wire connecting the 3.0 Ω and 6.0 Ω resistors is at a potential of V = +12 V. This is consistent with the loop rule, and the fact that the direction of the current through a resistor is the direction of decreasing potential.

Step 7 – ***Expand the circuit back from two resistors to three.*** The 3.0 Ω resistor is replaced by the 4.0 Ω and 12.0 Ω resistors, in parallel, that it came from, as shown in Figure 18.15(g). When one resistor is split into two in parallel, the potential difference across all three resistors is the same. That is 6.0 V in this case. We can then apply Ohm's Law to find the current in each resistor, giving $I_1 = \Delta V / R = 6\text{ V}/4\,\Omega = 1.5\text{ A}$ and $I_2 = \Delta V / R = 6\text{ V}/12\,\Omega = 0.5\text{ A}$. These add to the 2.0 A through their equivalent resistor, as we expect from the junction rule.

Step 8 – ***Continue the expansion process, at each step finding the current through, and potential difference across, each resistor.*** In this circuit there is one more step. This is shown in Figure 18.15(h), where the 12.0 Ω resistor is split into the original 5.0 Ω and 7.0 Ω resistors. These resistors have the same current, 0.5 A, as the 12.0 Ω resistor, and their potential differences can be found from Ohm's Law and sum to the 6.0 V across the 12.0 Ω resistor.

Key ideas for the contraction/expansion method: One way to analyze a circuit is to contract the circuit to one equivalent resistor and then expand it back. In each step in the contraction two resistors that are either in series or in parallel are replaced by one resistor of equivalent resistance. In the expansion when one resistor is expanded to two in series all three resistors have the same current, while when one resistor is expanded to two in parallel all three resistors have the same potential difference across them. **Related End-of-Chapter Exercises: 15, 20, 21, 24.**

Essential Question 18.7: Check the answer above by comparing the power associated with the battery to the total power dissipated in the resistors. Why should these values be the same?

Answer to Essential Question 18.7: The power provided to the circuit by the battery can be found from $P = \varepsilon I = (18\text{ V}) \times (2.0\text{ A}) = 36\text{ W}$. The equation $P = I^2 R$ gives the power dissipated in each resistor: $P_1 = I_1^2 R_1 = (1.5\text{ A})^2 (4.0\Omega) = 9.0\text{ W}$; $P_2 = I_2^2 R_2 = (0.5\text{ A})^2 (5.0\Omega) = 1.25\text{ W}$; $P_3 = I_2^2 R_3 = (0.5\text{ A})^2 (7.0\Omega) = 1.75\text{ W}$; and $P_4 = I^2 R_4 = (2.0\text{ A})^2 (6.0\Omega) = 24\text{ W}$, for a total of 36 W. Thus, the power input to the circuit by the battery equals the power dissipated in the resistors, which we expect because of conservation of energy.

18-8 An Example Problem; and Meters

Let's now explore a situation that involves many of the concepts from the last few sections, and allows us to discuss the role of a switch in a circuit.

EXPLORATION 18.8 – Three bulbs and two switches

Three identical light bulbs, A, B, and C, are placed in the circuit shown in Figure 18.16 along with two switches, 1 and 2, and a battery with an emf of 120 V (like a standard electrical outlet).

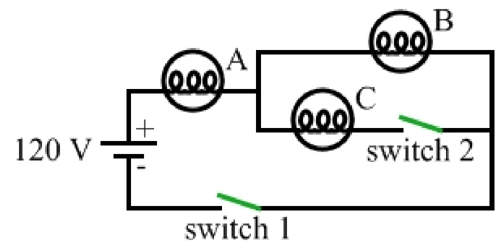

Figure 18.16: A circuit with one battery, two switches, and three identical light bulbs. The switches are initially open.

Step 1 – ***Are any bulbs on when the switches are both open? If so, which bulbs are on? If not, explain why not.*** For a bulb to glow a current must pass through it. For there to be a current there must be a complete circuit, a conducting path for charges to flow through from one terminal of the battery to the other. With switch 1 open there is not a complete circuit, so all the bulbs are off.

Step 2 – ***Kirchoff's loop rule is true even when the switches are open. How is this possible?*** This is possible because the potential difference across switch 1 is equal to the battery emf. If we define the wire connecting the negative terminal of the battery to the left side of switch 1 to be at $V = 0$, all other parts of the circuit, including the right side of the switch, are at a potential of $V = +120\text{ V}$. There are no potential differences across the bulbs because there is no current.

Step 3 – ***Complete these sentences. An open switch has a resistance of _________. A closed switch has a resistance of ________.*** We generally treat an open switch as having a resistance of infinity. A closed switch acts like a wire, so we assume it has a resistance of zero.

Step 4 – ***Rank the bulbs based on their brightness when switch 1 is closed and switch 2 is open. What is the potential difference across each bulb?*** Bulb C is off – because switch 2 is open there is no current in that part of the circuit. Thus, the circuit has bulbs A and B in series with one another and the battery. Because bulbs A and B are identical, and have the same current through them, they are equally bright. The ranking is A=B>C. Bulbs A and B share the emf of the battery, with a potential difference of 60 V across each bulb. Bulb C has no potential difference across it.

Step 5 – ***What happens to the brightness of each bulb when switch 2 is closed (so both switches are closed)? What is the potential difference across each bulb?*** With switch 2 closed, charge flows through bulb C, so C comes on and is brighter than before. Bulbs B and C are in parallel, and have equal resistance, so half the current passes through B and half through C. Bulb B got all the current before switch 2 was closed, so you might think that bulb B is now obviously dimmer. However, closing switch 2 decreases the overall resistance of the circuit, increasing the current. So, bulb B only gets half the current, but the total current increases – which effect dominates?

Because all the current passes through bulb A, increasing the current in the circuit increases both the brightness of, and the potential difference across, bulb A. By the loop rule, increasing A's potential difference means B's potential difference decreases, so B's current and brightness must also be less. To summarize, A and C get brighter, while B gets dimmer. B and C are now equally bright, and A is the brightest of all. Assuming the bulbs have the same resistance, A has 80 V across it, while B and C each have 40 V across them, as shown in Figure 18.17.

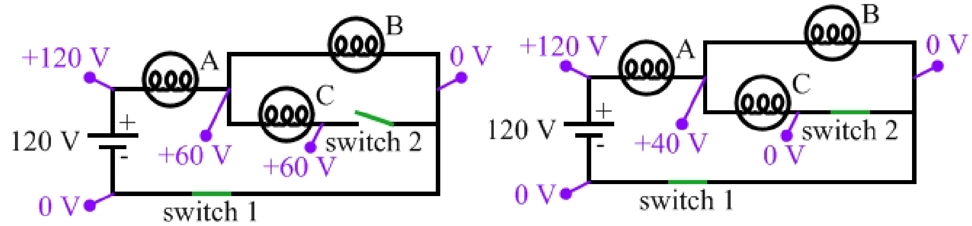

Figure 18.17: Labeling the potential at various points helps us understand what happens to the bulbs when switch 2 is closed.

Key Ideas for Switches: We can treat an open switch as having infinite resistance, and a closed switch as having no resistance. **Related End-of-Chapter Exercises**: **47 – 49.**

Ammeters Measure Current

A meter that measures current is known as an **ammeter**. Should an ammeter be wired in series or parallel? Should the ammeter have a small resistance or a large resistance? Does adding an ammeter to the circuit increase or decrease the current through the resistor of interest?

Circuit elements that are in series have the same current passing through them. Thus, to measure the current through a resistor an ammeter should be placed in series with that resistor, as in Figure 18.18. Adding the ammeter, which has some resistance, increases the equivalent resistance of the circuit and thus reduces the current in the circuit. The resistance of the ammeter should be as small as possible to minimize the effect of adding the ammeter to the circuit.

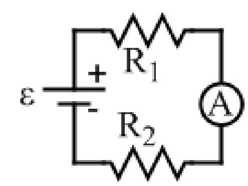

Figure 18.18: An ammeter, represented by an A inside a circle, is used to measure the current through whatever is in series with it. In this case that's everything in the circuit.

Voltmeters Measure Potential Difference

A meter that measures potential difference is known as a **voltmeter**. Should a voltmeter be wired in series or parallel? Should it have a small or a large resistance? How does adding a voltmeter to a circuit affect the circuit?

Circuit elements in parallel have the same potential difference across them. Thus, to measure the potential difference across a resistor a voltmeter should be placed in parallel with that resistor, as in Figure 18.19. Connecting the voltmeter, which has some resistance, in parallel decreases the resistance of the circuit, increasing the current. The resistance of the voltmeter should be as large as possible to minimize the effect of adding the voltmeter.

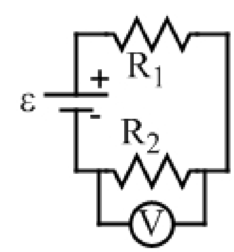

Figure 18.19: A voltmeter, represented by a V inside a circle, is used to measure the potential difference of whatever is in parallel with it. In this case that's resistor R_2.

Related End-of-Chapter Exercises: 35, 58.

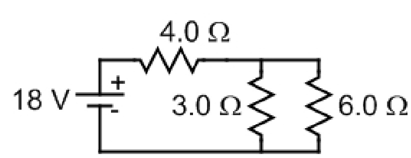

Figure 18.20: The circuit for Essential Question 18.8.

Essential Question 18.8: Can you add a 5.0 Ω resistor to the circuit in Figure 18.20 so that some current passes through it while the current through original resistors is unchanged? Explain.