Flowing fluids, such as the water flowing in the photograph at Coors Falls in Colorado, can make interesting patterns. In this chapter, we will investigate the basic physics behind such flow. Photo credit: Digital Vision.

It is interesting to think about how much physics is involved in the situation shown in the photograph. First of all, there is the influence of gravity, which makes the water flow down. Second, you can see the parabolic trajectories followed by the water as it is in free-fall at a number of different locations in the photograph. Third, there is the somewhat ghostly appearance of the water. This is caused by the photographer keeping the camera shutter open for an extended period as the picture was taken, with the motion of the water during this period causing a blurring.

Chapter 9 – Fluids

CHAPTER CONTENTS

In this chapter on fluids, we will introduce some new concepts, but the main focus will be on how to incorporate fluids into the framework of forces and energy that we have examined in the earlier chapters.

Although we will address basic issues related to flowing fluids, in the first half of this chapter our focus will be on static fluids. What determines whether something floats or sinks in a fluid? How can an object sink in one liquid, and yet float in a different liquid? How can a huge ocean liner float, when its mass is so huge, and being made from raw materials that would sink in water? Our study of fluids begins with us addressing questions such as these.

9-1 The Buoyant Force

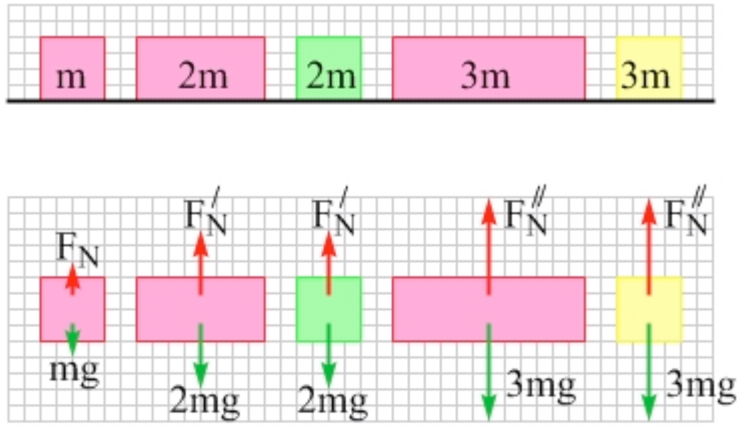


Figure 9.1: A diagram of the blocks we will place in a beaker of water, and the free-body diagram for each block as it sits on a table.

We should begin by defining what a fluid is. Many people think of a fluid as a liquid, but **a fluid is anything that can flow**. By this definition, a fluid can be a liquid or a gas. Flowing fluids can be rather complicated, so let's start with static fluids – fluids that are at rest.

Let's consider some experiments involving various blocks that float in a container of water. The blocks are represented in Figure 9.1, which shows how the masses of the blocks compare, and also shows the free-body diagrams of the blocks as they sit in equilibrium on a table. Starting from the left, the first, second, and fourth blocks are all made from the same material. The other two blocks are both made from different material.

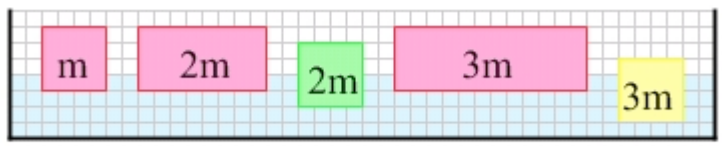
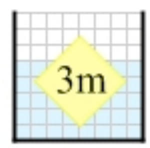


Figure 9.2: A diagram of the blocks floating in the beaker of water.

Our first goal is to look at the similarities between the normal force (a force arising from contact between solid objects) and the force arising from the interaction between an object and a fluid that the object is completely or partly submerged in. Figure 9.2 illustrates how the blocks float when they are placed in the container of water. We have taken some liberties here, because in reality some of the blocks would tilt 45˚ and float as shown in Figure 9.3. Neglecting this rotation simplifies the analysis without affecting the conclusions.

Figure 9.3: We will ignore the fact that blocks that are submerged more than 50% tend to float rotated by 45˚ from the way they are drawn in Figure 9.2. Neglecting this fact will simplify the analysis without affecting the conclusions.



EXPLORATION 9.1 – Free-body diagrams for floating objects

Sketch the free-body diagram of the blocks in Figure 9.2 as they float in the container of water. Note that each block is in equilibrium – what does that imply about the net force acting on each block? Because each block is in equilibrium, the net force acting on each block must be zero.

What forces act on each block? As usual there is a downward force of gravity. Because each block is in equilibrium, however, the net force acting on each block is zero. For now, let's keep things simple and show, on each block, one upward force that balances the force of gravity. The free-body diagrams are shown in Figure 9.4. Note that there is no normal force, because the blocks are not in contact with a solid object. Instead, they are supported by the fluid. We call the upward force applied by a fluid to an object in that fluid **the buoyant force**, which we symbolize as $\vec{F}_B$.

Because the objects are only in contact with the fluid, the fluid must be applying the upward buoyant force to each block. Compare the free-body diagrams in Figure 9.1, when the blocks are in equilibrium on the table, with the free-body diagrams in Figure 9.4, when the blocka are in equilibrium while floating in the fluid. For a floating object, at least, there are a lot of similarities between the buoyant force exerted by a fluid and the normal force exerted by a solid surface.

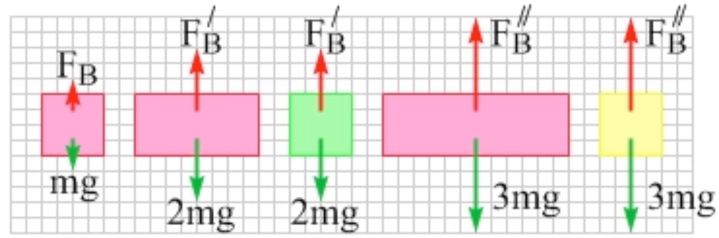


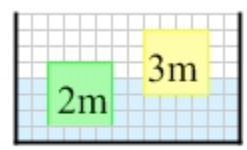
Figure 9.4: Free-body diagrams for the blocks floating in equilibrium in the beaker of water. F_B represents the buoyant force, an upward force applied on each block by the fluid.

Examine Figures 9.2 and 9.4 closely. Even though the two blocks of mass 2*m* are immersed to different levels in the fluid, they displace the same volume of fluid, so they experience equal buoyant forces. The 3*m* blocks displace 50% more volume than do the blocks of mass 2*m*, and they experience a buoyant force that is 50% larger. The block of mass *m*, on the other hand, displaces half the volume of fluid that the blocks of mass 2*m* do, and experiences a buoyant force that is half as large. We can conclude that ***the buoyant force exerted on an object by a fluid is proportional to V_{disp}, the volume of fluid displaced by that object***. We can express this as an equation (where ∝ means "is proportional to"),

$$F_B \propto V_{disp} .$$ (Eq. 9.1: **Buoyant force is proportional to volume of fluid displaced**)

Key idea about the buoyant force: An object in a fluid experiences a net upward force we call the buoyant force, $\vec{F}_B$. The magnitude of the buoyant force is proportional to the volume of fluid displaced by the object. **Related End-of-Chapter Exercise: 2.**

The conclusion above is supported by the fact that if we push a block farther down into the water and let go, the block bobs up. The buoyant force increases when we push the block down because the volume of fluid displaced increases, so, when we let go, the block experiences a net upward force. Conversely, when a block is raised, it displaces less fluid, reducing the buoyant force and giving rise to a net downward force when we let go. Figure 9.5 shows these situations and the corresponding free-body diagrams.



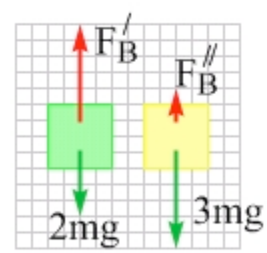


Figure 9.5: In this case, the blocks are not at equilibrium. The block on the left has been pushed down into the water and released. Because it displaces more water than it does at equilibrium, the buoyant force applied to it by the water is larger than the force of gravity applied to it by the Earth and it experiences a net upward force. The reverse is true for the block on the right, which has been lifted up and released. Displacing less water causes the buoyant force to decrease, giving rise to a net downward force.

Figure 9.6: To be able to float, this large ship needs to displace a very large volume of fluid. This large volume of fluid is displaced by the part of the ship that is below the water surface, and which, therefore, is not visible to us in this photograph. Photo credit: Corbis Images.

Essential Question 9.1: Two objects float in equilibrium in the same fluid. Object A displaces more fluid than object B. Which object has a larger mass?

Answer to Essential Question 9.1: Object A. When an object floats in equilibrium, the buoyant force exactly balances the force of gravity. Object A displaces more fluid, so it experiences a larger buoyant force. This must be because object A weighs more than object B.

9-2 Using Force Methods with Fluids

EXAMPLE 9.2 – A block on a string

A block of weight mg = 45 N has part of its volume submerged in a beaker of water. The block is partially supported by a string of fixed length that is tied to a support above the beaker. When 80% of the block's volume is submerged, the tension in the string is 5.0 N.

(a) What is the magnitude of the buoyant force acting on the block?

(b) Water is steadily removed from the beaker, causing the block to become less submerged. The string breaks when its tension exceeds 35 N. What percent of the block's volume is submerged at the moment the string breaks?

(c) After the string breaks, the block comes to a new equilibrium position in the beaker. At equilibrium, what percent of the block's volume is submerged?

SOLUTION

As usual, we should begin with a diagram of the situation. A free-body diagram is also very helpful. These are shown in Figure 9.7.

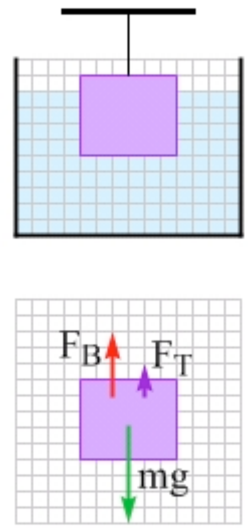


Figure 9.7: A diagram and a free-body diagram for the 45 N block floating in the beaker of water while partly supported by a string.

(a) On the block's free-body diagram, we draw a downward force of gravity, applied by the Earth. We also draw an upward force of tension (applied by the string), and, because the block displaces some fluid, an upward buoyant force (applied by the fluid). The block is in equilibrium, so there must be no net force acting on the block.

Taking up to be positive, applying Newton's Second Law gives: $\sum \vec{F} = 0$.

Evaluating the left-hand side with the aid of the free-body diagram gives: $+F_T + F_B - mg = 0$.

Solving for the buoyant force gives: $F_B = mg - F_T = +45 \text{ N} - 5.0 \text{ N} = +40 \text{ N}$.

(b) As shown in Figure 9.8, removing water from the beaker causes the block to displace less fluid, so the magnitude of the buoyant force decreases. The magnitude of the tension increases to compensate for this. Applying Newton's Second Law again gives us essentially the same equation as in part (a). We can use this to find the new buoyant force, F_B'. Just before the string breaks we have:

$$F_B' = mg - F_T' = +45 \text{ N} - 35 \text{ N} = +10 \text{ N}.$$

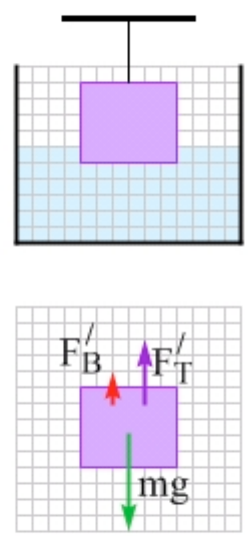


Figure 9.8: A diagram and free-body diagram of the situation just before the string breaks.

Now, we can apply the idea that the buoyant force is proportional to the volume of fluid displaced. If a buoyant force of 40 N corresponds to a displaced volume equal to 80% of the block's volume, a buoyant force of 10 N (1/4 of the original force) must correspond to a displaced volume equal to 20% of the block's volume (1/4 of the original displaced volume).

(c) After the string breaks and the block comes to a new equilibrium position, we have a simpler free-body diagram, as shown in Figure 9.9. The buoyant force now, F_B'', applied to the block by the fluid, must balance the force of gravity applied to the block by the Earth. This comes from applying Newton's Second Law:

$$\sum \vec{F} = 0 .$$

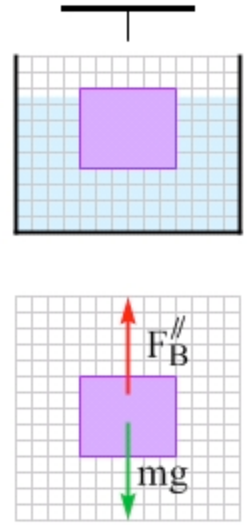


Figure 9.9: A diagram and free-body diagram for the situation after the string breaks, when the block has come to a new equilibrium position in the beaker.

Taking up to be positive, evaluating the left-hand side with the aid of the free-body diagram gives:

$$F_B'' - mg = 0 \text{, so } F_B'' = mg = 45 \text{ N} .$$

Using the same logic as in (b), if a buoyant force of 40 N corresponds to a displaced volume equal to 80% of the block's volume, a buoyant force of 45 N must correspond to a displaced volume equal to 90% of the block's volume.

Related End-of-Chapter Exercises: 21, 36.

Let's now extend our analysis to objects that sink. First, hang a block from a spring scale (a device that measures force) to measure the force of gravity acting on the block. With the block hanging from the spring scale, the scale reads 10 N, so there is a 10 N force of gravity acting on the block. A diagram and two free-body diagrams (one for the spring scale and one for the block) are shown in Figure 9.10.

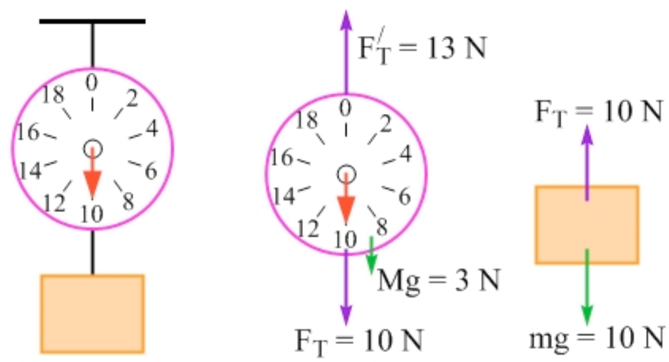


Figure 9.10: A diagram showing a block hanging from a spring scale, as well as free-body diagrams for the spring scale (which itself has a force of gravity of 3 N acting on it) and the block.

Question: With the block still suspended from the spring scale, let's dip the block into a beaker of water until it is exactly half submerged. Make a prediction. As we lower the block into the water, will the reading on the spring scale increase, decrease, or stay the same? Briefly justify your prediction.

Answer: The reading on the spring scale should decrease. This is because the spring scale no longer has to support the entire weight of the block. The more the block is submerged, the larger the buoyant force, and the smaller the spring-scale reading.

Essential Question 9.2: The spring scale reads 10 N when the block is out of the water. Let's say it reads 6.0 N when exactly 50% of the block's volume is below the water surface. What will the scale read when the entire block is below the water surface? Why?

Answer to Essential Question 9.2: We can apply the idea that the buoyant force acting on an object is proportional to the volume of fluid displaced by that object. When the block is half submerged, the buoyant force is 4.0 N up because the buoyant force and the spring scale, which exerts a force of 6.0 N up, must balance the downward 10 N force of gravity acting on the block. When the block is completely submerged, it displaces twice as much fluid, doubling the buoyant force to 8.0 N up. The spring scale only has to apply 2.0 N of force up on the block to make the forces balance. Diagrams and free-body diagrams for these situations are shown in Figure 9.11.

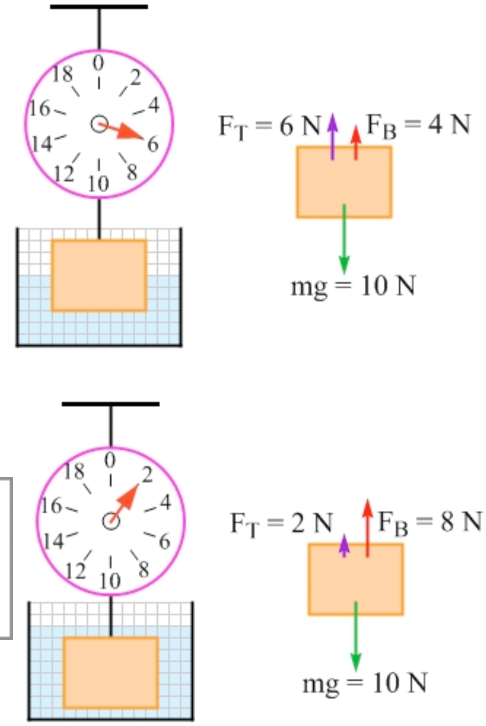


Figure 9.11: The top diagrams show the situation and free-body diagram for a block suspended from a spring scale when the block is half submerged in water. The bottom diagrams are similar, except that the block is completely submerged.

9-3 Archimedes' Principle

EXPLORATION 9.3 – What does the buoyant force depend on?

We know that the buoyant force acting on an object is proportional to the volume of fluid displaced by the object. What else does it depend on? Let's experiment to figure this out. We'll use a special beaker with a spout, as shown in Figure 9.12. In each case, we will fill the beaker to a level just below the spout, so that when we add a block to the beaker any fluid displaced by the block will flow down the spout into a second catch beaker. The catch beaker sits on a scale, so we can measure the weight of the displaced fluid. The fluid in the beaker will be either water or a second liquid, so we can figure out whether the fluid in the beaker makes any difference.

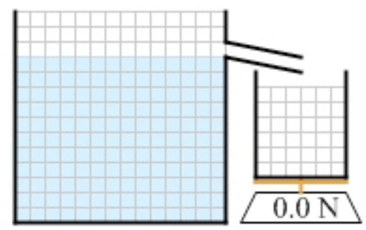


Figure 9.12: The beaker with the spout, and the catch beaker sitting on the scale. The scale is tared so it will read directly the weight of fluid in the catch beaker.

The blocks we will work have equal volumes but different masses. The weights of the blocks are 8 N, 16 N, and 24 N. Before we add a block to the beaker, we will make sure the beaker is filled to just below the level of the spout, and that the catch beaker is empty. If a block sinks in the fluid, we will hang it from a spring scale before completely submerging the block, so we can find the buoyant force from the difference between the force of gravity acting on the block and the reading on the spring scale. Also, the scale under the catch beaker is tared, which means that with the empty catch beaker sitting on it the scale reads zero and will read directly the weight of any fluid in the catch beaker.

The results of the experiments with water are shown in Figure 9.13, along with the corresponding free-body diagrams. In every case, ***the magnitude of the buoyant force acting on the block is equal to the weight of the fluid displaced by the block.***

Does this only work with water? Let's try it with the second fluid. The results are shown in Figure 9.13. Here we notice some differences - the 8 N block still floats buts displaces twice the volume of fluid it did in the water; the 16 N block now sinks; and the 24 N block still sinks but has half the buoyant force it had when it was in the water. Once again, however, the magnitude of the buoyant force on the block is equal to the weight of fluid displaced by the block.

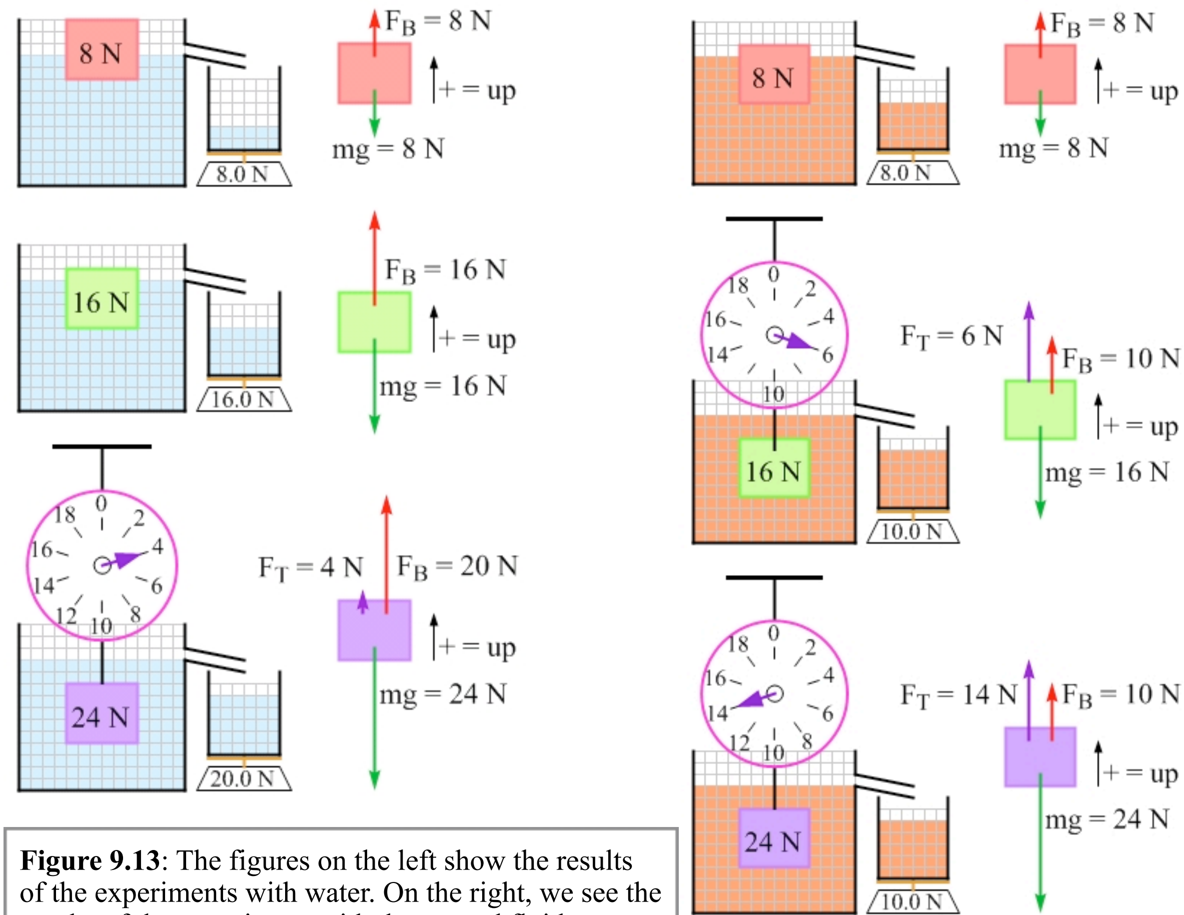


Figure 9.13: The figures on the left show the results of the experiments with water. On the right, we see the results of the experiment with the second fluid.

With the second fluid, we see that the buoyant force the fluid exerts on an object is still proportional to the volume of fluid displaced. However, we can also conclude that displacing a particular volume of water gives a different buoyant force than displacing exactly the same amount of the other fluid. Some property of the fluid is involved here.

To determine which property of the fluid is associated with the buoyant force, let's focus on the fact that the buoyant force is equal to the weight of the fluid displaced by the object:

$$F_B = m_{disp} g \,.$$

If we bring in mass density, for which we use the symbol ρ, we can write this equation in terms of the volume of fluid displaced. The relationship between mass, density, and volume is:

$$m = \rho V \,.$$ (Equation 9.2: **Mass density**)

Using this relationship in the equation for buoyant force gives:

$$F_B = m_{disp} g = \rho_{fluid} V_{disp} \, g \,.$$ (Equation 9.3: **Archimedes' Principle**)

Key Idea regarding Archimedes' Principle: The magnitude of the buoyant force exerted on an object by a fluid is equal to the weight of the fluid displaced by the object. This is known as Archimedes' principle. **Related End-of-Chapter Exercises: 4, 7.**

Essential Question 9.3: How does the mass density of the second fluid in Exploration 9.3 compare to the mass density of water?

Answer to Essential Question 9.3: The second fluid has a density that is half that of water. We can see that because a particular volume of water has a mass that is twice as much as the mass of an equal volume of the second fluid.

9-4 Solving Buoyancy Problems

Archimedes was a Greek scientist who, legend has it, discovered the concept while taking a bath, whereupon he leapt out and ran naked through the streets shouting “Eureka!” Archimedes was thinking about this because the king at the time wanted Archimedes to come up with some way to make sure that the king’s crown was made out of solid gold, and was not gold mixed with silver. Archimedes’ realized that he could use his principle to determine the density of the crown, and he could then compare it to the known density of gold.

Using Equation 9.3, we can now explain the results of the block-and-two-fluid experiment above. The differences we observe between when we place the blocks in water and when we place them in the second fluid can all be explained in terms of the difference between the density of water and the density of the second fluid. In fact, to explain the results of Exploration 9.2 the second fluid must have half the density of water. The 10-N block, for instance, floats in both fluids and therefore the buoyant force is the same in both cases, exactly equal-and-opposite to the 10 N force of gravity acting on the block. Because the density of the second fluid is half the density of water, the block needs to displace twice the volume of fluid in the second fluid to achieve the same buoyant force. The 30-N block, on the other hand, displaces the same amount of fluid in each case. However, it experiences twice the buoyant force from the water as it does from the second fluid because of the factor of two difference in the densities.

What happens with the 20-N block is particularly interesting, because it floats in water and yet sinks in the second fluid. This raises the question, what determines whether an object floats or sinks when it is placed in a fluid?

EXPLORATION 9.4 – Float or sink?

How can we tell whether an object will float or sink in a particular fluid? As we have considered before, when an object floats in a fluid the upward buoyant force exactly balances the downward force of gravity. This gives: $F_B = mg$.

Using Archimedes’ principle, we can write the left-hand side as: $\rho_{fluid} V_{disp} g = mg$.

The factors of g cancel (this tells us that it doesn’t matter which planet we’re on, or where on the planet we are), giving: $\rho_{fluid} V_{disp} = m$.

If we write the right-hand side in terms of the density of the object, we get, for a floating object:

$$\rho_{fluid} V_{disp} = \rho_{object} V_{object} .$$

Re-arranging this equation leads to the interesting result (that applies for floating objects only):

$$\frac{\rho_{object}}{\rho_{fluid}} = \frac{V_{disp}}{V_{object}} . \qquad \text{(Equation 9.4: \textbf{For floating objects})}$$

Equation 9.4 answers the question of what determines whether an object floats or sinks in a fluid – the density. ***If an object is less dense than the fluid it is in then it floats***. An object that is less dense than the fluid it is in floats because the object displaces a volume of fluid smaller than its own volume – in other words, the object floats with part of it sticking out above the surface of the fluid. On the other hand, an object more dense than the fluid it is in must displace a volume of fluid larger than its own volume in order to float. This is certainly not possible for the solid blocks we have considered above. Thus, we can conclude that ***an object with a density larger than the density of the fluid it is in will sink in that fluid***.

Key Ideas: Whether an object floats or sinks in a fluid depends on its density. An object with a density less than that of a fluid floats in that fluid, while an object with a larger density than that of a fluid will tend to sink in that fluid. **Related End-of-Chapter Exercises: 1, 13.**

Equation 9.4 tells us that we can determine the density of a floating object by observing what fraction of its volume is submerged. For instance, if an object is 30% submerged in a fluid its density is 30% of the density of the fluid. Table 9.1 shows the density of various materials.

Material	Density (kg/m^3)	Material	Density (kg/m^3)
Interstellar space	10^{-20}	Planet Earth (average)	5500
Air (at 1 atmosphere)	1.2	Iron	7900
Water (at 4˚C)	1000	Mercury (the metal)	13600
Sun (average)	1400	Black hole	10^{-19}

Table 9.1 The density of various materials.

What about an object that has the same density as the fluid it is in? This is known as **neutral buoyancy**, because the upward buoyant force on the object balances the downward force of gravity on the object when the object is 100% submerged. Because the net force acting on the object is zero it is in equilibrium at any of the positions shown in Figure 9.14. This is true as long as the fluid density does not change with depth, which is something of an idealization. Again we are using a model, with an assumption of the model being that a fluid is incompressible – its density is constant.

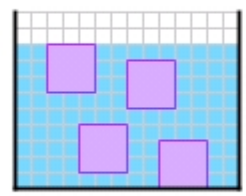


Figure 9.14: A neutrally buoyant object (an object with the same density as the surrounding fluid) will be at equilibrium at any of the positions shown. All other objects will either float at the surface, or sink to the bottom.

The general method for solving a typical buoyancy problem is based on the method we used in chapter 3 for solving a problem involving Newton's Laws. Now, we include Archimedes' principle. In general buoyancy problems are 1-dimensional, involving vertical forces, so that simplifies the method a little.

A General Method for Solving a Buoyancy Problem

1. Draw a diagram of the situation.
2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object as well as an appropriate coordinate system.
3. Apply Newton's Second Law to each free-body diagram.
4. If necessary, bring in Archimedes' principle, $F_B = \rho_{fluid} V_{disp}\, g$.
5. Put the resulting equations together and solve.

Essential Question 9.4: Let's say the four objects shown in Figure 9.14 have densities larger than that of the fluid. Can any of the objects be at equilibrium at the positions shown? Explain.

Answer to Essential Question 9.4: Objects that are denser than the fluid they are in tend to sink to the bottom of the container. One object in Figure 9.14 already rests at the bottom, so it is in equilibrium. For the three higher objects, the force of gravity, acting down, is larger than the buoyant force that acts up. These three objects are not at equilibrium, and will sink to the bottom.

9-5 An Example Buoyancy Problem

EXAMPLE 9.5 – Applying the general method

Let's now consider an object that sinks to the bottom of a beaker of liquid. The object is a block with a weight of 20 N, when weighed in air. The beaker it is to be placed in contains some water, as well as a waterproof scale that rests on the bottom of the beaker. This scale is tared to read zero, and let's assume the scale is unaffected by any changes in the level of the water above it. The beaker itself rests on a second scale that reads 50 N, the combined weight of the beaker, the water, and the scale inside the beaker. When the 20-N block is placed in the beaker, it sinks to the bottom and comes to rest on the scale in the beaker, which now reads 5.0 N. This is known as the **apparent weight** of the block. Let's assume g = 10 m/s^2 to simplify the calculations.

(a) What is the magnitude and direction of the buoyant force applied on the block by the water?
(b) With the block now completely immersed in the water, what is the reading on the scale under the beaker?
(c) What is the block's density and volume?

SOLUTION

Let's begin with the first two steps in the general method, by drawing a diagram of the situation and a free-body diagram of the block. These are shown in Figure 9.15, where up is taken to be the positive direction. Note that three forces act on the block, one of which is the downward force of gravity. The 5.0 N reading on the scale is the magnitude of the downward normal force applied by the block on the scale. By Newton's Third Law, the scale applies an upward 5.0 N normal force on the block. The third force acting on the block is the upward buoyant force applied on it by the water.

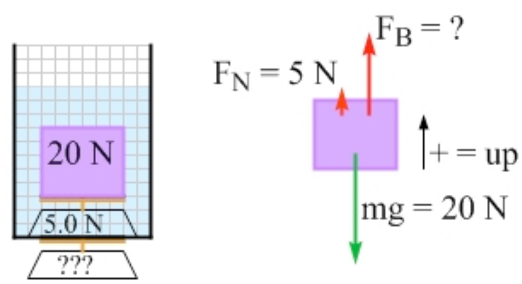


Figure 9.15: A diagram and free-body diagram for the block resting on the scale inside the beaker of fluid.

(a) The block is in equilibrium (at rest with no acceleration), so we can apply Newton's Second Law to determine the buoyant force acting on the block.

$$\Sigma\vec{F} = m\vec{a} = 0 .$$

Looking at the free-body diagram to evaluate the left-hand side gives:

$$+F_B + F_N - mg = 0 .$$

Solving for the buoyant force gives:

$F_B = mg - F_N = 20 \text{ N} - 5.0 \text{ N} = 15 \text{ N}$, directed up.

(b) What is the reading on the scale under the beaker? The scale under the beaker supports everything on top of it, so with the block inside the beaker the scale under the beaker reads 70 N. This comes from adding the full 20-N weight of the block to the original 50 N, from the beaker, water, and scale inside the beaker.

Doesn't the water support 15 N of the block's weight, via the buoyant force? Yes, it does. However, if the water exerts a force of 15 N up on the block, then by Newton's third law the block exerts a 15 N force down on the water. The water passes this force along to the beaker, which passes it along to the scale under the beaker. Similarly, the block exerts a 5.0-N normal force down on the scale inside the beaker, and the scale passes this force along to the beaker, which passes it along to the scale under the beaker. Now matter how you look at it, adding a 20-N block to the beaker ends up increasing the reading on the scale under the beaker by 20 N.

(c) Let's derive a general equation that tells us how the density of a submerged object is related to its weight mg and apparent weight W_{app}. The apparent weight is numerically equal to the normal force experienced by the submerged object.

In part (a), we used Newton's Second Law to arrive at an expression for the buoyant force acting on our submerged object, obtaining: $F_B = mg - F_N$.

Writing this in terms of the apparent weight gives: $F_B = mg - W_{app}$.

For a submerged object, the apparent weight is less than the actual weight. If we call f the ratio of the apparent weight to the actual weight, $f = W_{app} / mg$, we can write the previous equation, using $W_{app} = f\, mg$, as:

$$F_B = mg - f\, mg = (1 - f)mg \, .$$

Now, use Archimedes' principle to transform the left-hand side of the equation:

$$\rho_{fluid} V_{disp}\, g = (1 - f)mg \, .$$

Finally, write the object's mass in terms of its density: $\rho_{fluid} V_{disp}\, g = (1 - f)\rho_{object} V_{object}\, g$.

The volume of fluid displaced by an object that is completely submerged is equal to its own volume, so we can cancel the factors of volume as well as the factors of g, leaving:

$$\rho_{fluid} = (1 - f)\rho_{object} \, .$$

Solving for the density of the object, we can write the equation in various ways:

$$\rho_{object} = \frac{\rho_{fluid}}{1 - f} = \frac{\rho_{fluid}}{1 - \frac{W_{app}}{mg}} = \frac{mg\, \rho_{fluid}}{mg - W_{app}} \, .$$

That applies generally to a completely submerged object. In our case, where we have $f = 1/4$, we find that the density of the block is:

$$\rho_{block} = \frac{4}{3} \rho_{water} = 1330 \text{ kg/m}^3 \, .$$

Related End-of-Chapter Exercises: 9, 18.

Essential Question 9.5: It is possible for small metal objects, such as sewing needles or Japanese yen, to float on the surface of water, if carefully placed there. Can we explain this in terms of the buoyant force?

Answer to Essential Question 9.5: No. These metal objects are denser than water, so we expect them to sink in the water (which they will if they are not placed carefully at the surface). They are held up by the surface tension of the water. Surface tension is beyond the scope of this book but it is similar to how a gymnast is supported by a trampoline – the water surface can act like a stretchy membrane that can support an object that is not too massive.

9-6 Pressure

Where does the buoyant force come from? What is responsible, for instance, for the small upward buoyant force exerted on us by the air when we are surrounded by air? Let's use a model in which the fluid is considered to be made up of a large number of fast-moving particles that collide elastically with one another and with anything immersed in it. For simplicity, let's examine the effect of these collisions on a block of height h and area A that is suspended from a light string, as shown in Figure 9.16.

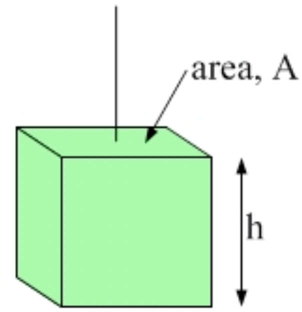


Figure 9.16: A block of height h and area A supported by a light string.

Consider a collision involving an air molecule bouncing off the left side of the block, as in Figure 9.17. Assuming the block remains at rest during the collision (the block's mass is much larger than that of the air molecule), then, because the collision is elastic, the magnitude of the molecule's momentum remains the same and only the direction changes: the component of the molecule's momentum that is directed right before the collision is directed left after the collision. All other momentum components remain the same. The block exerts a force to the left on the molecule during the collision, so the block experiences an equal-and-opposite force to the right.

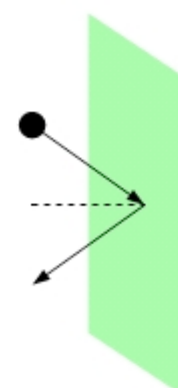


Figure 9.17: A magnified view of a molecule bouncing off the left side of the block.

There are a many molecules bouncing off the left side of the block, producing a sizable force to the right on the block. The block does not accelerate to the right, however, because there is also a large number of molecules bouncing off the right side of the block, producing a force to the left on the block. Averaged over time, the rightward and leftward forces balance. Similarly, the forces on the front and back surfaces cancel one another.

If all the forces cancel out, how do these collisions give rise to the buoyant force? Consider the top and bottom surfaces of the cube. Because the buoyant force exerted on the cube by the air is directed vertically up, the upward force on the block associated with air molecules bouncing off the block's bottom surface must be larger than the downward force on the block from air molecules bouncing off the block's top surface. Expressing this as an equation, and taking up to be positive, we get:

$$+F_{bottom} - F_{top} = +F_B = +\rho_{fluid} V_{disp} g \, .$$

The volume of air displaced by the block is the block's entire volume, which is its area multiplied by its height: $V_{disp} = Ah$. Substituting $V_{disp} = Ah$ into the expression above gives:

$$+F_{bottom} - F_{top} = +F_B = +\rho_{fluid} A h g \, .$$

This is the origin of the buoyant force – the net upward force applied to the block by molecules bouncing off the block's bottom surface is larger in magnitude than the net downward force applied by molecules bouncing off the block's upper surface. This is a gravitational effect – the buoyant force is proportional to g. One way to think about this is that if the molecules at the block's top surface have a particular average kinetic energy, to conserve energy those at the block's bottom surface should have a larger kinetic energy because their gravitational potential energy is less. Thus, molecules bouncing off the bottom surface are more energetic, and they impart a larger average force to the block than the molecules at the top surface.

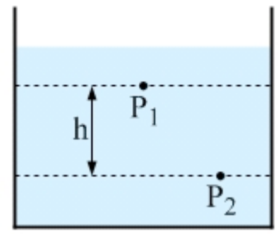
Dividing both sides of the previous equation by the area A gives:

$$+\frac{F_{bottom}}{A} - \frac{F_{top}}{A} = +\rho_{fluid}\, h\, g \,. \qquad \text{(Equation 9.5)}$$

The name for the quantity of force per unit area is **pressure**.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \quad \text{or} \quad P = \frac{F}{A} \,. \qquad \text{(Equation 9.6: Pressure)}$$

The MKS unit for pressure is the pascal (Pa). $1\ \text{Pa} = 1\ \text{N/m}^2$.



Using the symbol P for pressure, we can write Equation 9.5 as:

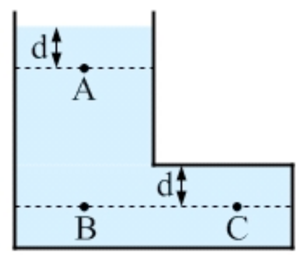
$$P_{bottom} - P_{top} = \rho_{fluid}\, h\, g \,.$$

We can write this equation in a general way, so that it relates the pressures of any two points, points 1 and 2, in a static fluid, where point 2 is a vertical distance h below the level of point 1. This gives:

$$P_2 = P_1 + \rho g h \,. \qquad \text{(Equation 9.7: Pressure in a static fluid)}$$

Figure 9.18: The pressure difference between two points is proportional to the vertical distance between them. Pressure increases with depth in a static fluid.

As represented by Figure 9.18, only the vertical level of the point matters. Any horizontal displacement in moving from point 1 to point 2 is irrelevant.



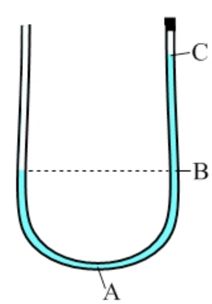
EXPLORATION 9.6 – Pressure in the L

Consider the L-shaped container in Figure 9.19. Rank points A, B, and C in terms of their pressure, from largest to smallest.

Because pressure in a static fluid depends only on vertical position, points B and C have equal pressures, and the pressure at that level in the fluid is higher than that at point A. The fact that there is a column of water of height d immediately above both points A and C is irrelevant. The fact that C is farthest from the opening is also irrelevant. Only the vertical position of the points matters.

Figure 9.19: A container shaped like an L that is filled with fluid and open at the top.

Key ideas: In a static fluid the pressure at any point is determined by that point's vertical position. All points at the same level have the same pressure, and points lower down have higher pressure than points higher up.
Related End-of-Chapter Exercises: 10, 51.



Essential Question 9.6: Water is placed in a U-shaped tube, as shown in Figure 9.20. The tube's left arm is open to the atmosphere, but the tube's right arm is sealed with a rubber stopper. Rank points A, B, and C based on their pressure, from largest to smallest.

Figure 9.20: A U-shaped water-filled tube that is sealed at the top right by a rubber stopper.

Answer to Essential Question 9.6: A>B>C. Point A, being the lowest of the three points, has the highest pressure. Point C, being the highest of the three points, has the lowest pressure.

9-7 Atmospheric Pressure

At sea level on Earth, standard atmospheric pressure is 101.3 kPa, or about 1.0×10^5 Pa, a substantial value. Atmospheric pressure is associated with the air molecules above sea level. Air is not very dense, but the atmosphere extends upward a long way so the cumulative effect is large. The reason we, and most things, don't collapse under atmospheric pressure is that in almost all situations there is pressure on both sides of an interface, so the forces balance. If you can create a pressure difference, however, you can get some interesting things to happen. This is how suction cups work, for instance – by removing air from one side the air pressure on the outside of the suction cup gives rise to a force that keeps the suction cup attached to a surface. It's also fairly easy to use atmospheric pressure to crush a soda can (see end-of-chapter Exercise 6).

In many situations what matters is the **gauge pressure**, which is the difference between the total pressure and atmospheric pressure. The total pressure is generally referred to as the **absolute pressure**. For instance, the absolute pressure at the surface of a lake near sea level is 1 atmosphere (1 atm), so the gauge pressure there would be 0. The gauge pressure 10 meters below the surface of a lake is about 1 atmosphere (1 atm), taking the density of water to be 1000 kg/m^3, because:

$$\rho\, g\, h \approx 1000\,\frac{\text{kg}}{\text{m}^3} \times 10\,\frac{\text{m}}{\text{s}^2} \times 10\text{ m} = 1\times10^5\,\frac{\text{kg m}}{\text{s}^2}\,\frac{1}{\text{m}^2} = 1\times10^5\,\frac{\text{N}}{\text{m}^2} = 1\times10^5\text{ Pa}\,.$$

The absolute pressure 10 m below the surface is about 2 atm. This is particularly relevant for divers, who must keep in mind that every 10 m of depth in water is associated with an additional 1 atmosphere worth of pressure.

EXAMPLE 9.7 – Under pressure

A plastic box is in the shape of a cube measuring 20 cm on each side. The box is completely filled with water and remains at rest on a flat surface. The box is open to the atmosphere at the top. Assume atmospheric pressure is 1.0×10^5 Pa and use g = 10 m/s^2.

(a) What is the gauge pressure at the bottom of the box?
(b) What is the absolute pressure at the bottom of the box?
(c) What is the force associated with this absolute pressure?
(d) What is the force associated with the absolute pressure acting on the inside surface of one side of the box?
(e) What is the net force associated with pressure acting on one side of the box?
(f) What is the net force acting on one side of the box?

SOLUTION

As usual let's begin with a diagram of the situation, shown in Figure 9.21.

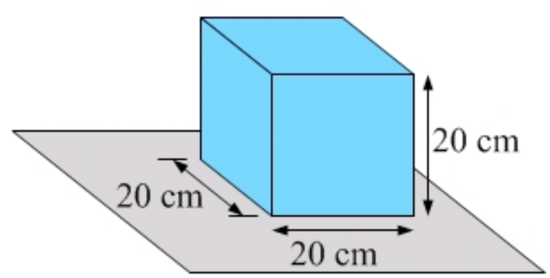


Figure 9.21: A box in the shape of a cube that is open at the top and filled with water.

(a) Because the pressure at the top surface is atmospheric pressure, the gauge pressure at the bottom is simply the pressure difference between the top of the box and the bottom. Applying Equation 9.7, regarding the pressure difference between two points in a static fluid, we get the gauge pressure at the bottom:

$$\Delta P = \rho\, g\, h = 1000\frac{\text{kg}}{\text{m}^3}\times 10\frac{\text{m}}{\text{s}^2}\times 0.20\text{ m} = 2000\text{ Pa}\,.$$

(b) The absolute pressure at the bottom is the gauge pressure plus atmospheric pressure. This gives: $P_{bottom} = P_{atm} + P_{gauge} = 1.0\times 10^5\text{ Pa} + 2000\text{ Pa} = 1.02\times 10^5\text{ Pa}$. Stating this to three significant figures would violate the rules about significant figures when adding, so we should really round this off to $1.0\times 10^5\text{ Pa}$.

(c) To find the force from the pressure we use Equation 9.6, re-arranged to read $\text{Force} = \text{Pressure}\times\text{Area}$. This gives a force of $F_{bottom} = \left(1.0\times 10^5\text{ Pa}\right)\left(0.2\text{ m}\right)^2 = 4000\text{ N}$, directed down at the bottom of the box.

(d) Finding the force associated with the side of the box is a little harder than finding it at the bottom, because the pressure increases with depth in the fluid. In other words, the pressure is different at points on the side that are at different depths. Because the pressure increases linearly with depth, however, we can take the average pressure to be the pressure halfway down the side of the box. The gauge pressure at a point inside the box that is halfway down the side is:

$$P_{gauge} = \rho g h = 1000\frac{\text{kg}}{\text{m}^3}\times 10\frac{\text{m}}{\text{s}^2}\times 0.1\text{ m} = 1000\text{ Pa}.$$

To find the force associated with the pressure we use absolute pressure, so we get: $F_{side} = \left(P_{atm} + P_{gauge}\right)\times\text{Area} = \left(1.0\times 10^5\text{ Pa} + 1000\text{ Pa}\right)\times\left(0.2\text{ m}\right)^2 = 4040\text{ N}$, which we should round off to 4000 N directed out from the center of the box.

(e) In part (d) we were concerned with the fluid pressure applying an outward force on one side of the box. Now we need to account for the air outside the box exerting an inward force on the same side of the box. This force is simply atmospheric pressure multiplied by the area, and is thus 4000 N directed inward. The net force associated with pressure is thus the combination of the 4040 N force directed out and the 4000 N force directed in, and is thus 40 N directed out. The same result can be obtained from $F_{pressure} = P_{gauge}\times\text{Area}$.

(f) Because the box and all its sides remain at rest, the net force on any one side must be zero, so this 40 N outward force associated with the gauge pressure of the water must be balanced by forces applied to one side by the rest of the box.

Related End-of-Chapter Exercises: 27, 28.

Essential Question 9.7: In Example 9.7, we accounted for the change in water pressure with depth, but we did not account for the increase in air pressure with depth, which could affect our calculation of the inward force exerted by the air on a side of the box. Explain why we can neglect this change in air pressure.