# Newton's Law of Universal Gravitation

Two objects of mass *m* and *M*, with their centers of gravity separated by a distance *r*, exert attractive forces on one another. The magnitude of this gravitational force is given by:

$$F_g = \frac{GmM}{r^2}$$

where G is the universal gravitational constant:

$$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$$

The direction of the force exerted on one object is toward the center of gravity of the second object – the force is attractive.

# Newton's Law of Universal Gravitation

Newton's form of the equation for the force of gravity must be consistent with the *mg* we have been using up to this point in the course:

$$\frac{GmM}{r^2} = mg$$

For an object of mass *m* at the surface of the Earth, this tells us that:

$$g = \frac{GM_E}{R_E^2} = \frac{(6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2) \times (5.97 \times 10^{24}\ \text{kg})}{(6.38 \times 10^6\ \text{m})^2} = 9.8\ \text{m/s}^2$$

# At the center of the Earth

If we bring an object from far away toward the Earth, the gravitational force increases. The closer it gets, the bigger the force. This is certainly true when the mass is outside the Earth - what happens if we bring it right to the surface and then keep going, tunneling into the Earth?

What is the force of gravity on an object if is right at the center of the Earth?

1. zero
2. infinite

# Inside the Earth

The net gravitational force on an object at the center of the Earth is zero – forces from opposite sides of the Earth cancel out.

This is a consequence of Gauss' Law for Gravity. One implication of Gauss' Law is that inside a uniform spherical shell, the force of gravity due to the shell is zero. Outside the shell, the force is exactly the same as that from a point object of the same mass as the shell, placed at the center of the shell.

Newton's Law of Universal Gravitation applies as long as one object is not overlapping the other.

# Field

A field is something that has a magnitude and a direction at every point in space. An example is a gravitational field, symbolized by *g*. The electric field, *E*, plays a similar role for charged objects that *g* does for objects that have mass.

*g* has a dual role, because it is also the acceleration due to gravity. If only gravity acts on an object:

$$\vec{F}_g = m\vec{g} = m\vec{a} \qquad \Rightarrow \qquad \vec{a} = \vec{g}$$

For a charged object acted on by an electric field only, the acceleration is given by:

$$\vec{F}_E = q\vec{E} = m\vec{a} \qquad \Rightarrow \qquad \vec{a} = \frac{q\vec{E}}{m}$$

Simulation

# Gravitational field

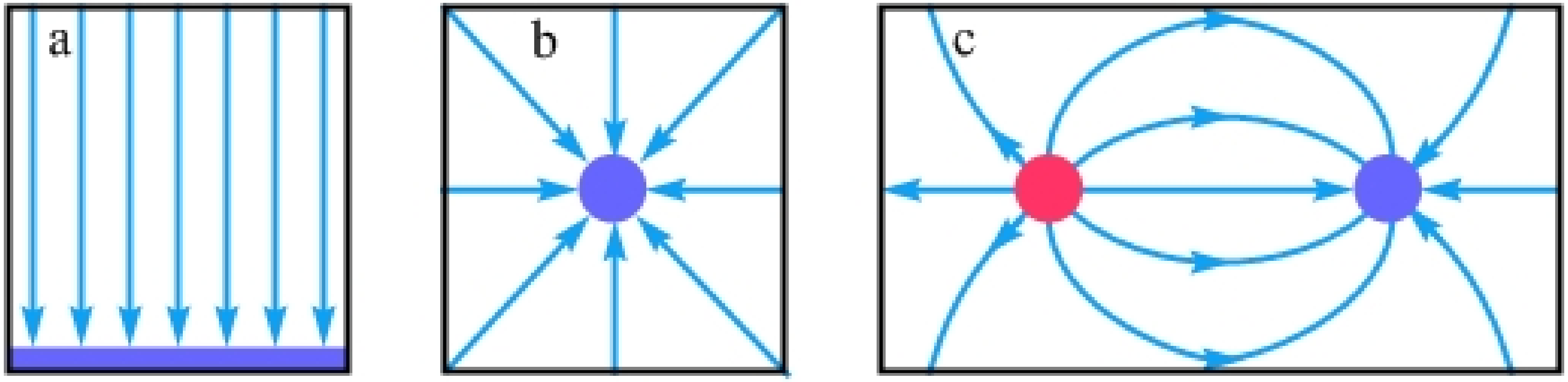
$\vec{g}$, the magnitude of the gravitational field at a point is the gravitational force per unit mass that an object experiences when it is placed at that point.

$$\vec{g} = \frac{\vec{F}_g}{m}$$

The units of gravitational field are N/kg, which is the same as $m/s^2$.

# Field lines

Field line diagrams show the direction of the field, and give a qualitative view of the magnitude of the field at various points. The field is strongest where the lines are closer together.



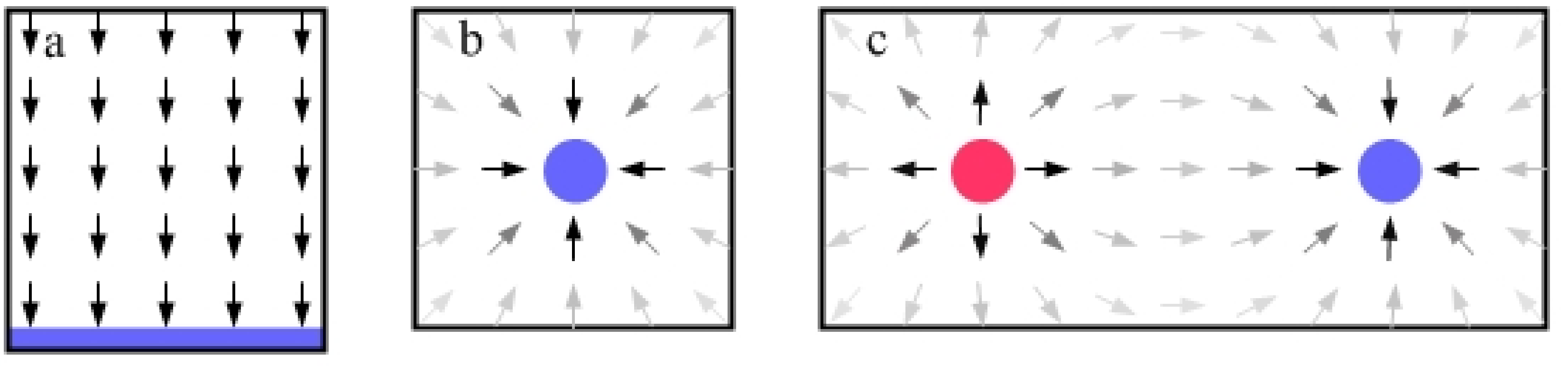

a – a uniform field directed down

b – the field near a point object (like the Earth)

c – field lines start on positive charges and end on negative charges. This is an electric dipole – two charges of opposite sign and equal magnitude separated by some distance.

# Field vectors

Field vectors give an alternate picture, and reinforce the idea that there is a field at all points in space. The field is strongest where the vectors are darker.




a – a uniform field directed down

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c – field lines start on positive charges and end on negative charges. This is an electric dipole – two charges of opposite sign and equal magnitude separated by some distance.

# Getting quantitative about field

The field line and field vector diagrams are nice, but when we want to know about the field at a particular point those diagrams are not terribly useful.

Instead, we use superposition. The net field at a particular point is the vector sum of the individual fields at that point. The individual fields sometimes come from individual objects. We assume these objects to be highly localized, so we call them point objects.

Gravitational field from a point object: $g = \frac{Gm}{r^2}$

The field points toward the object.

# Earth and Moon

Using the fact that the gravitational field at the surface of the Earth is about six times larger than that at the surface of the Moon, and the fact that the Earth's radius is about four times the Moon's radius, determine how the mass of the Earth compares to the mass of the Moon.

$$g = \frac{Gm}{r^2}$$

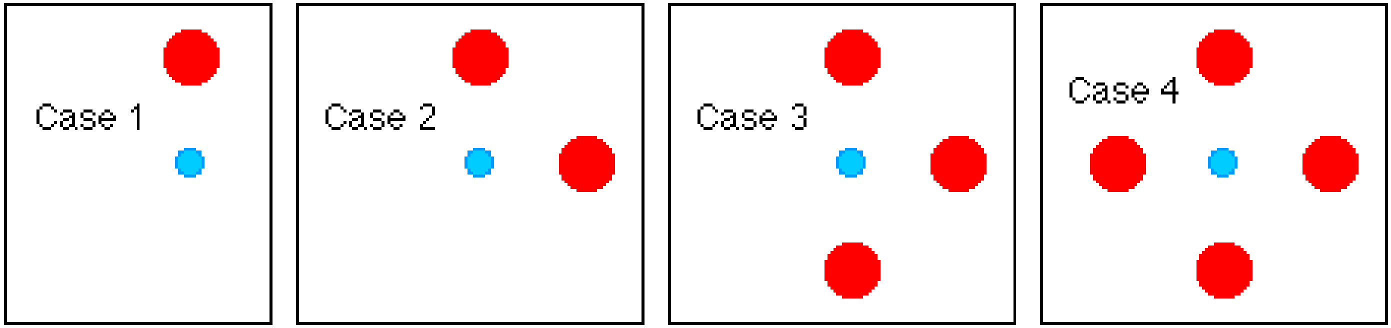
# Superposition

If an object experiences multiple forces, we can use:

The principle of superposition - the net force acting on an object is the vector sum of the individual forces acting on that object.

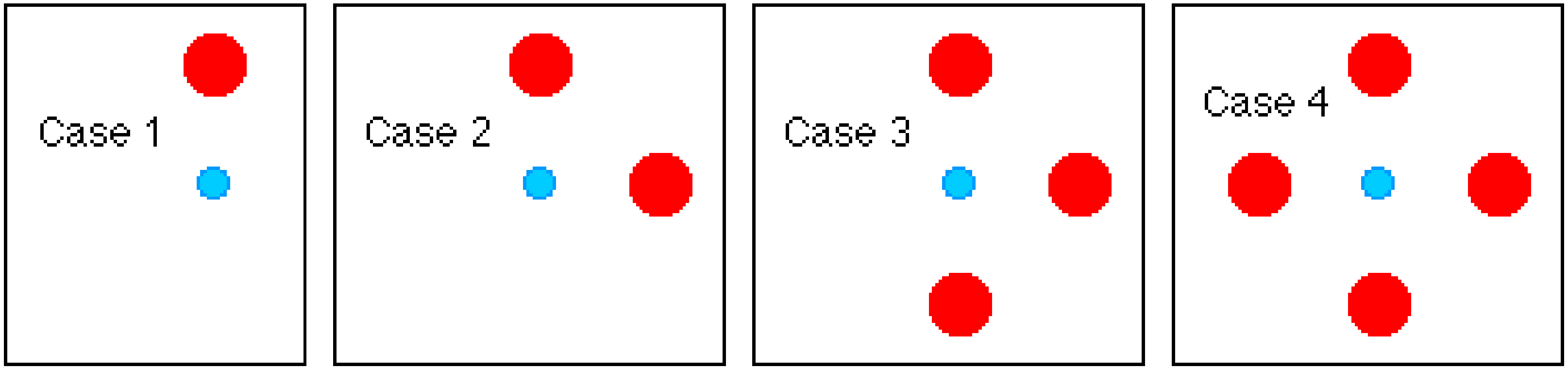
# Rank these situations

A small blue ball has one or more large red balls placed near it. The red balls are all the same mass and the same distance from the blue one. Rank the different cases based on the net gravitational force experienced by the blue ball due to the neighboring red ball(s).




1. 1=3>2>4
2. 3>2>1>4
3. 4>3>2>1
4. 2>1=3>4
5. 4>1=2=3

# Ranking the situations

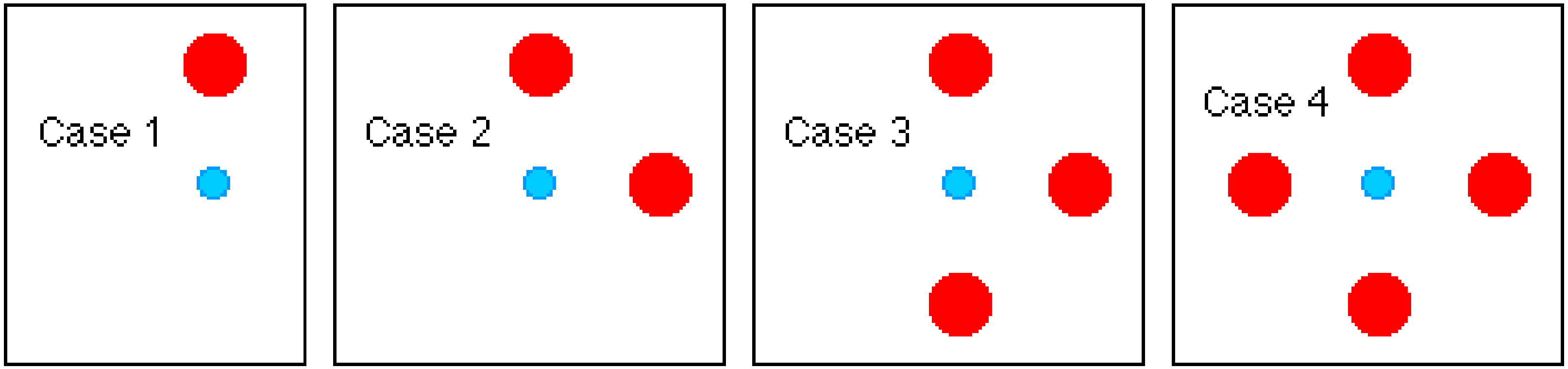



Does case 4 give the largest net force or the smallest?

Is there another case with the same magnitude net force as case 1?

Remember this kind of situation when we look at charged objects later in the program. There are many similarities between gravitational interactions and interactions between charged objects.

# Ranking the situations




Does case 4 give the largest net force or the smallest?

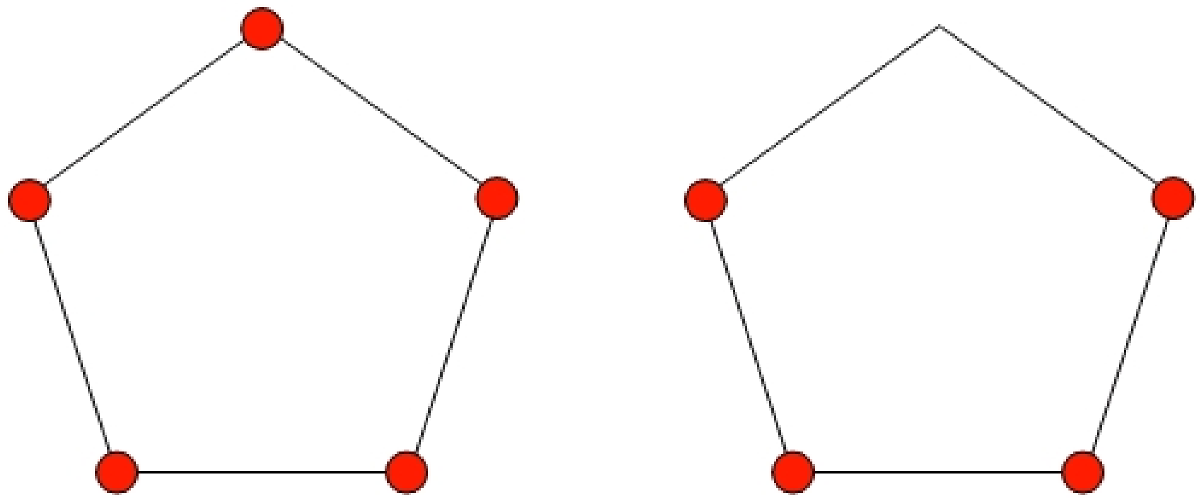
The smallest – there is no net force in case 4.

Is there another case with the same magnitude net force as case 1? Yes, case 3.

Remember this kind of situation when we look at charged objects later in the program. There are many similarities between gravitational interactions and interactions between charged objects.

# Superposition

Five balls of equal mass are placed so that there is one ball at each corner of a pentagon. Each ball is a distance R from the center. What is the net gravitational field at the center of the pentagon?



If the ball at the top corner is removed, what is the net field at the center?

# Superposition

A ball of mass 6*m* is placed on the *x*-axis at *x* = -2*a*. There is a second ball of unknown mass at *x* = +*a*. The net gravitational field at the origin due to the two balls has a magnitude of

$$g_{net} = \frac{Gm}{a^2}$$

What is the mass of the second ball? Find all possible solutions.

# Gravitational potential energy

The energy of interaction (that is, the gravitational potential energy) of two objects of mass *m* and *M* separated by a distance *r* is:

$$U_g = -\frac{GmM}{r}$$

The negative sign just tells us that the interaction is attractive.

Note that with this equation the potential energy is defined to be zero when *r* = infinity.

What matters is the change in gravitational potential energy. For small changes in height at the Earth’s surface, the equation above gives the same change in potential energy as *mgh*.

# Escape speed

How fast would you have to throw an object so it never came back down? Ignore air resistance. Let's find the escape speed - the minimum speed required to escape from a planet's gravitational pull.

How should we try to figure this out?

Attack the problem from a force perspective?

From an energy perspective?

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From an energy perspective?

Forces are hard to work with here, because the size of the force changes as the object gets farther away. Energy is easier to work with in this case.

# Escape speed

Let’s start with the conservation of energy equation.

$$U_i + K_i + W_{nc} = U_f + K_f$$

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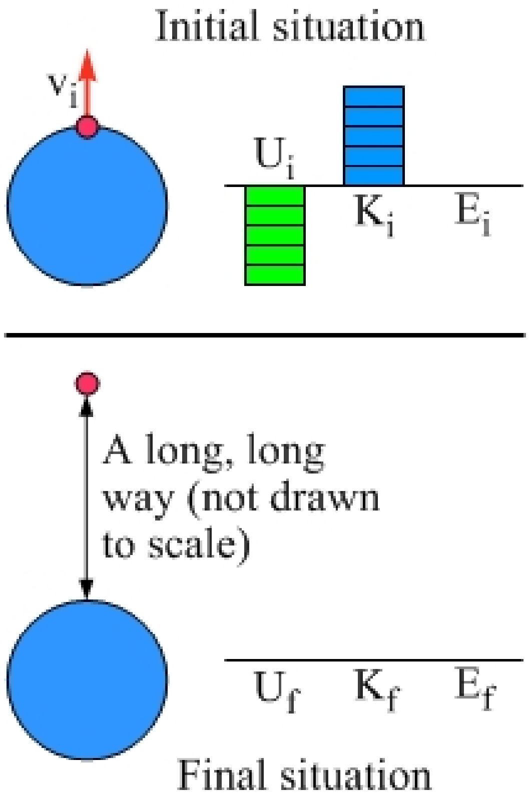
$$U_i + K_i + W_{nc} = U_f + K_f$$

Which terms can we cross out immediately?

Assume no resistive forces, so $W_{nc} = 0$

Assume the object barely makes it to infinity, so both $U_f$ and $K_f$ are zero.

This leaves: $U_i + K_i = 0$



# Escape speed

$$U_i + K_i = 0$$

If the total mechanical energy is negative, the object comes back. If it is positive, it never comes back.

$$-\frac{GmM}{R} + \frac{1}{2} m v_{escape}^2 = 0$$

The mass of the object, *m*, does not matter. Solving for the escape speed gives:

$$v_{escape} = \sqrt{\frac{2GM}{R}}$$

*M* is the mass of the planet; *R* is the planet's radius.

For the Earth, we get $v_{escape}$ = 11.2 km/s.

# Orbits and Energy

Simulation

Consider an object in orbit around a much larger object.

A circular orbit is a very special case, requiring a particular speed. A little faster, or a little slower for our object, and the orbit is elliptical.

If the speed is simply $\sqrt{2}$ times the speed for a circular orbit, the object goes off in a parabolic path and never comes back. If the speed is even larger, the path is hyperbolic, but the object still doesn't come back.