Newton's Second Law for Rotation

The equation $\sum \vec{\tau} = I\vec{\alpha}$ is the rotational equivalent of $\sum \vec{F} = m\vec{a}$.

Torque plays the role of force.

Rotational inertia plays the role of mass.

Angular acceleration plays the role of the acceleration.

Rolling

- We can view rolling motion as a superposition of pure rotation and pure translation.

- For rolling without slipping, the rotational speed of the outside of the wheel equals the translational speed.

- The net instantaneous velocity at the bottom of the wheel is zero, while at the top it is twice the translational velocity of the wheel.

- A point on the edge of the wheel traces out a cycloid.

An accelerating car

You are driving your front-wheel drive car on Comm. Ave. You are stopped at a red light, and when the light turns green you accelerate smoothly so that there is no slipping between your car tires and the road. During the acceleration period, in what direction is the force of friction from the road acting on your front tires? Is it static friction or kinetic friction?

1. The frictional force is kinetic friction acting in the direction you are traveling.
2. The frictional force is kinetic friction acting opposite to the direction you are traveling.
3. The frictional force is static friction acting in the direction you are traveling.
4. The frictional force is static friction acting opposite to the direction you are traveling.

An accelerating car

Car simulation

Let's first turn friction off. With no friction at all, pushing down on the accelerator makes the front wheels spin clockwise. They spin on the frictionless surface, the rear wheels do nothing, and the car goes nowhere.

Friction on the front wheels opposes the spinning, so it must point in the direction the car wants to go. For the front wheels to roll without slipping, the friction must be static.

An accelerating car

During the acceleration period, in what direction is the force of friction from the road acting on your rear tires? Is it static friction or kinetic friction?

1. The frictional force is kinetic friction acting in the direction you are traveling.
2. The frictional force is kinetic friction acting opposite to the direction you are traveling.
3. The frictional force is static friction acting in the direction you are traveling.
4. The frictional force is static friction acting opposite to the direction you are traveling.

An accelerating car

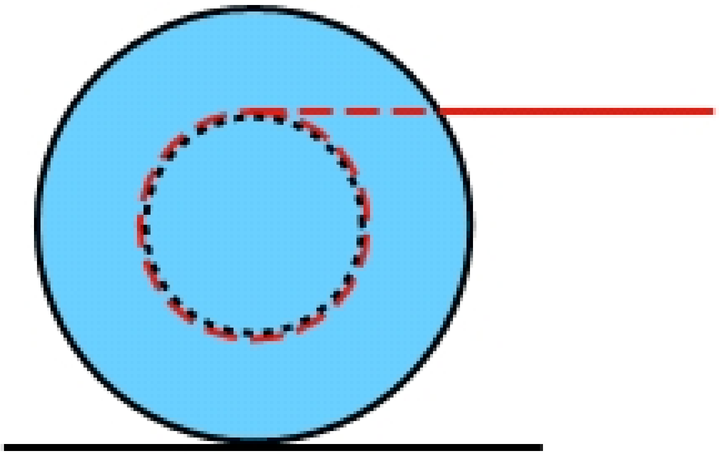
If we turn on friction to the front wheels only, the car accelerates forward with the back wheels dragging along the road without spinning. Friction opposes this motion, so it must point opposite to the way the car is going. Again, it must be static friction.

The static friction force acting on the front wheels is the force that accelerates the car forward. It is much larger than the friction force on the rear wheels, which just has to give the rear wheels the correct angular acceleration.

Big yo-yo

A large yo-yo stands on a table. A rope wrapped around the yo-yo's axle is pulled horizontally to the right, with the rope coming off the yo-yo above the axle. In which direction does the yo-yo move?

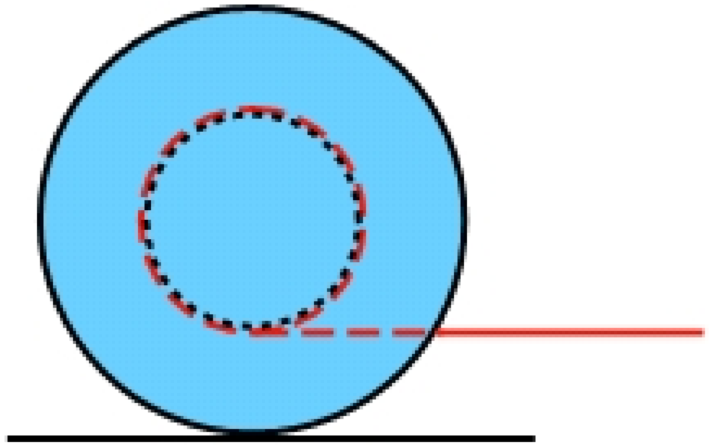
1. to the right
2. to the left
3. it won't move



Big yo-yo, again

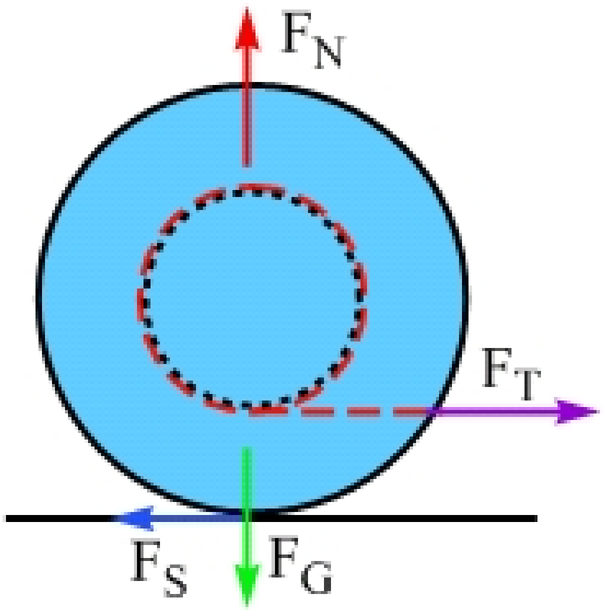
The situation is repeated but with the rope coming off the yo-yo below the axle. If the rope is pulled to the right, which way will the yo-yo move now?

1. to the right
2. to the left
3. it won't move



Analyzing the yo-yo

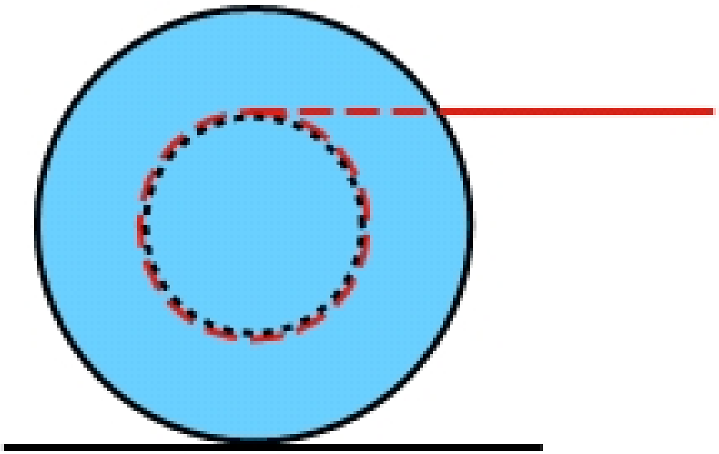
The key to determining which way the yo-yo moves is to look at the torque due to the tension about the point where the yo-yo contacts the surface. All the other forces pass through that point, and do not give rise to any torques about the contact point.



Big yo-yo

If the yo-yo's axle is half the radius of the yo-yo, and the yo-yo moves a distance *L* to the right when the rope is pulled from above the axle, how far does the end of the rope move?

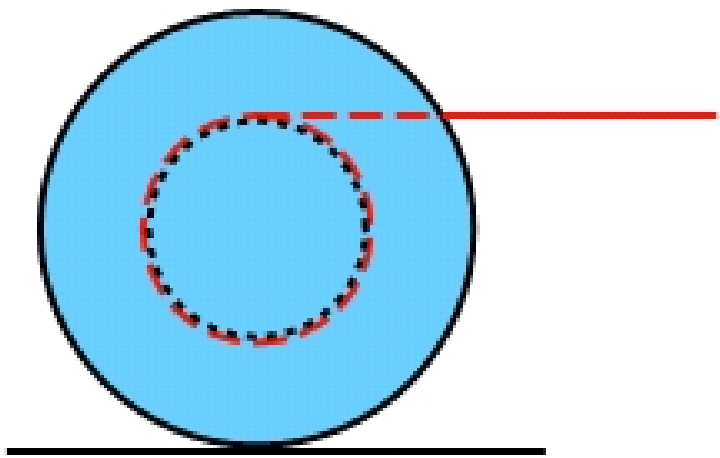
1. it doesn't move at all
2. *L*/3
3. *L*/2
4. 2*L*/3
5. *L*
6. 4*L*/3
7. 3*L*/2
8. 5*L*/3
9. 2*L*



The distance moved by the rope

If the axle is half the yo-yo's radius, a point on the outer edge of the axle has a rotational speed equal to half the yo-yo's translational speed. Let’s call the translational speed *v*. Above the axle, where the rope is unwinding, the net velocity is 1.5 *v*, because the rotational and translational velocities have the same direction.

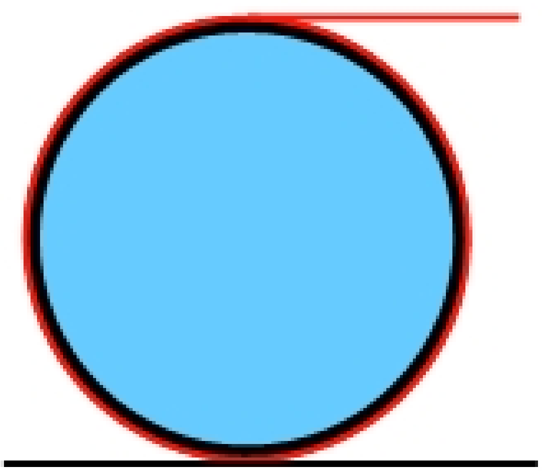
If the yo-yo moves a distance *L*, the end of the rope moves a distance 1.5 *L*.



An accelerating cylinder

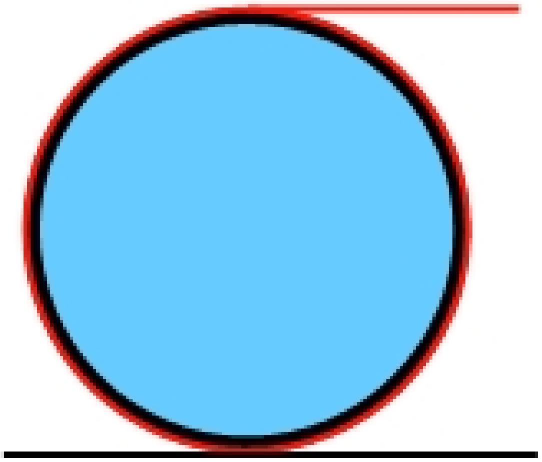
A cylinder of mass *M* and radius *R* has a string wrapped around it, with the string coming off the cylinder above the cylinder. If the string is pulled to the right with a force *F*, what is the acceleration of the cylinder if the cylinder rolls without slipping?

1. $a = \frac{F}{m}$
2. $a < \frac{F}{m}$
3. $a > \frac{F}{m}$



An accelerating cylinder

We would expect the acceleration to be something other than F/M only if there is another horizontal force acting on the cylinder. Is there such a force?

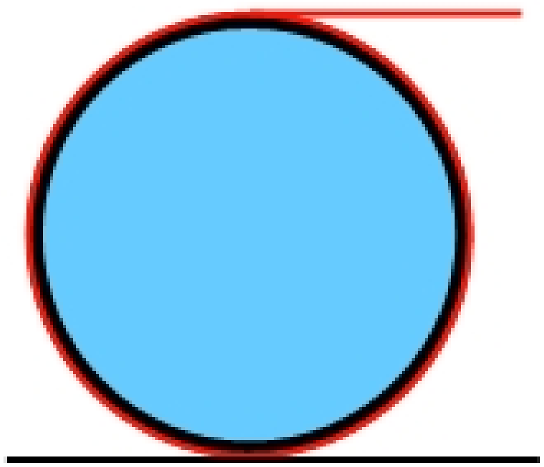


An accelerating cylinder

We would expect the acceleration to be something other than *F*/*M* only if there is another horizontal force acting on the cylinder. Is there such a force?

Yes, a force of static friction. Let's draw it in as pointing to the left and solve for the acceleration.

Simulation



An accelerating cylinder

Take positive to the right, and clockwise positive for torque.

The normal force cancels *Mg* vertically. Apply Newton's Second Law for horizontal forces, and for torques:

Forces

$$\sum \vec{F}_x = M\vec{a}$$

$$+F - F_S = Ma$$

Torques

$$\sum \vec{\tau} = I\alpha$$

*

An accelerating cylinder

Take positive to the right, and clockwise positive for torque.

The normal force cancels *Mg* vertically. Apply Newton's Second Law for horizontal forces, and for torques:

Forces	**Torques**
$\sum \vec{F}_x = M\vec{a}$	$\sum \vec{\tau} = I\alpha$
$+F - F_S = Ma$	$+RF + RF_S = I\alpha$

*

An accelerating cylinder

Take positive to the right, and clockwise positive for torque.

The normal force cancels *Mg* vertically. Apply Newton's Second Law for horizontal forces, and for torques:

Forces	**Torques**
$\sum \vec{F}_x = M\vec{a}$	$\sum \vec{\tau} = I\alpha$
$+F - F_S = Ma$	$+RF + RF_S = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$
	$F + F_S = \frac{1}{2}Ma$

* Only for rolling without slipping can we use $\alpha = \frac{a}{R}$

An accelerating cylinder

Forces **Torques**

$$+F - F_S = Ma \qquad F + F_S = \frac{1}{2}Ma$$

Adding these two equations gives $2F = \frac{3}{2}Ma$,

which leads to the somewhat surprising result $a = \frac{4F}{3M}$.

We can make sense of this by solving for the force of static friction.

$$F_S = -\frac{1}{4}Ma = -\frac{1}{3}F$$

The minus sign means the friction force is opposite in direction to the way we drew it. It really points the same way as *F*.