Rolling without Slipping

Now we'll analyze a rolling-without-slipping situation.

A cylinder of mass M and radius R has a string wrapped around it, with the string coming off the cylinder above the cylinder. The string is pulled to the right with a force of magnitude F. Our goal is to determine the acceleration of the cylinder if the cylinder rolls without slipping along the horizontal surface.

Sketch this situation and draw a free-body diagram for the cylinder, showing all the forces acting on it.

Step 1 - Apply Newton's Second Law, $\sum \vec{F} = m\vec{a}$.

Step 2 - Apply Newton's Second Law for Rotation, $\sum \vec{\tau} = I\vec{\alpha}$. We usually take torques about the center of the cylinder.

Step 3 – Substitute the rotational inertia of the cylinder, $I = \frac{1}{2}MR^2$, into your torque equation.

Step 4 – For rolling without slipping, we can use the relationship $\alpha = \frac{a}{R}$. Substitute that into your equation from step 3.

Step 5 – Combine your equations to find an expression for the cylinder's acceleration, and to find the magnitude and direction of the force of static friction acting on the cylinder.

Bowling Ball Physics

For all these exercises, use $g = 10$ m/s^2. We will assume that a bowling ball is a uniform solid sphere with a mass of $M = 5$ kg and a radius of $R = 10$ cm.

Situation 1 – At the bowling alley. The coefficients of friction between the ball and the floor are: $\mu_K = 0.20$ and $\mu_S = 0.25$.

When you release the ball, it is in contact with the floor. The ball's initial velocity is 4.0 m/s, and the ball is not rotating at all, initially. Our goal here is to determine how much time passes, after you release the ball, before the ball is rolling without slipping.

Sketch the free-body diagram of the ball, immediately after you release it.

Find the ball's acceleration.

Find the ball's angular acceleration.

Find the time from the instant you release the ball until the ball starts rolls without slipping.

When the ball is rolling without slipping, what does the free-body diagram look like?

Situation 2 – Down the incline.
In this situation, you release the ball from rest at the top of an incline, and the ball rolls without slipping down the incline. The angle of the incline is θ with respect to the horizontal. Keep your answers in terms of M, the mass of the ball, and g, the acceleration due to gravity.

Sketch the free-body diagram of the ball, for the situation of the ball rolling without slipping down the incline.

Apply Newton's second law:

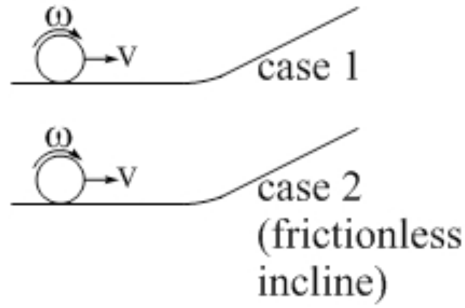
Apply Newton's second law for rotation:

Put your two equations together to solve for the ball's acceleration. Express the acceleration in terms of g and θ.

Find the force of friction that acts on the ball. Express the force of friction in terms of M, g and θ.

Situation 3 – Up the incline.

Case 1 and case 2 show two situations of a bowling ball traveling up an incline. In both cases, the ball is initially rolling without slipping with the same constant velocity along a horizontal surface before reaching the incline. In case 1, the ball then rolls without slipping up the incline. In case 2, the ball travels up a frictionless incline.



Draw two free-body diagrams. For case 1, show all the forces being applied to the ball as the ball rolls without slipping up the incline. If a force of friction acts on the ball, show clearly the direction of the force of friction as well as whether it is static friction (F_S) or kinetic friction (F_K). For case 2, show the forces being applied to the ball as the ball travels up the frictionless incline.

In which case does the ball travel farther up the incline before reversing direction?

[ ] case 1 [ ] case 2 [ ] equal in both cases

Justify your answer above conceptually, by referring to your free-body diagrams.

If you were to calculate the acceleration of the ball in case 1, and the force of friction acting on the ball in case 1, how would your answers compare to the answers you obtained for the situation of the ball rolling without slipping down the hill, on the previous page?