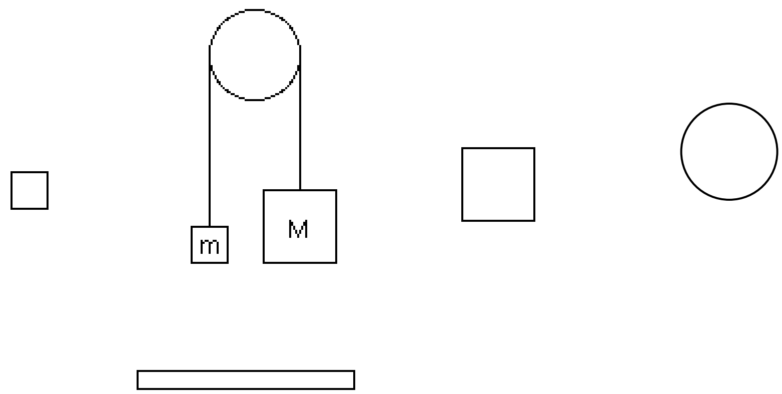
Rotational Dynamics

Let's first do a better job analyzing Atwood's machine.

An Atwood's machine is a device that has two objects connected by a string that passes over a pulley. Assume $M > m$, and that the pulley is a solid disk with a mass m_p.



Sketch free-body diagrams showing the forces acting on the objects of mass M and m and on the pulley.

For mass M, show which direction you're taking to be positive. Apply Newton's second law to obtain a relationship between M, a (the acceleration), and the forces acting on mass M.

For mass m, show which direction you're taking to be positive. Apply Newton's second law to obtain a relationship between m, a (the acceleration), and the forces acting on mass m.

For the pulley, show which direction you're taking to be positive. Apply Newton's second law to obtain a relationship between m_p, α (the angular acceleration), and the **torques** acting on the pulley.

Combine your three equations to find an expression for the acceleration a in terms of g, m, M, and m_p.

Now we'll consider a non-equilibrium situation involving a system we looked at before, with a string holding a hinged rod in a horizontal position. The string holding the rod horizontal is cut, and the rod starts to swing down toward the ground.

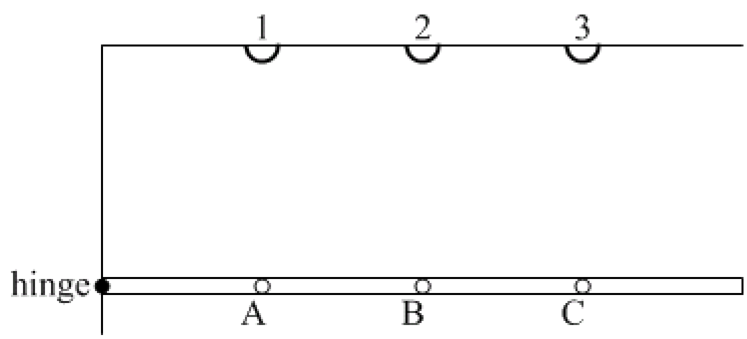
The rotational inertia of a uniform rod rotating about an axis through one end is $I = \frac{1}{3}mL^2$.

Another useful equation in this situation is that $\alpha = \frac{a_T}{r}$, where a_T is the tangential acceleration. Also, remember that the magnitude of a torque is $\tau = r\,F\sin\theta$.

Our goals here are to calculate two things. Immediately after the string is cut, when the rod is still horizontal, find:

(a) The acceleration of the rod's center-of-mass.
(b) The magnitude and direction of the force applied to the rod by the hinge.

Step 1 – Sketch a free-body diagram of the rod.



Step 2 – Apply Newton's Second Law, $\sum \vec{F} = m\vec{a}$.

Step 3 – Apply Newton's Second Law for Rotation, $\sum \vec{\tau} = I\vec{\alpha}$.

Step 4 – Solve the problem.