# Equilibrium

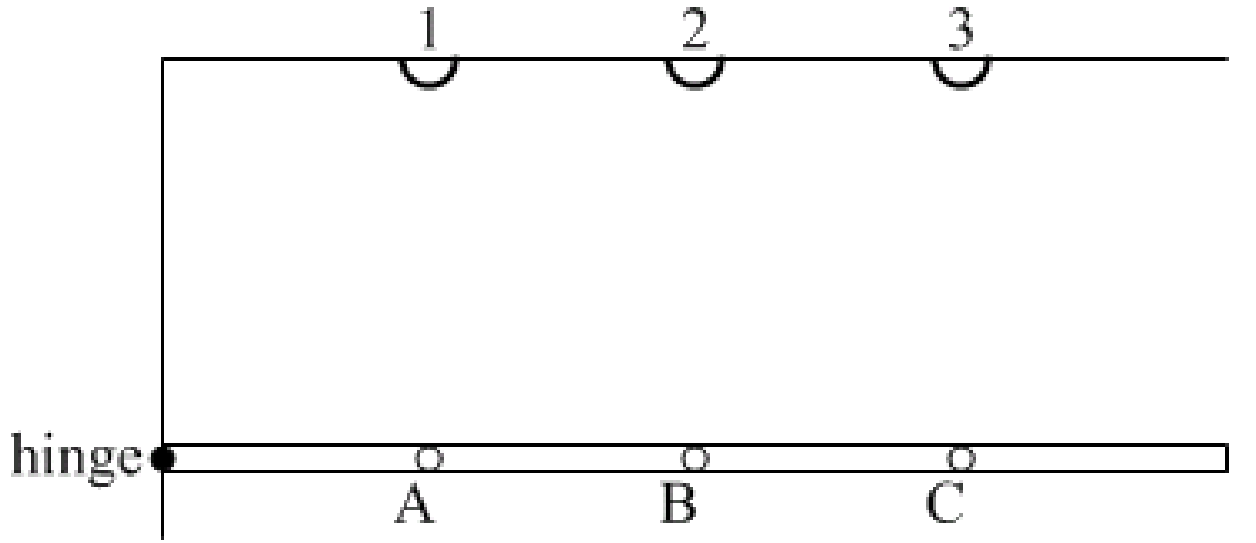
For an object to remain in equilibrium, two conditions must be met.

The object must have no net force: $\sum \vec{F} = 0$

and no net torque: $\sum \vec{\tau} = 0$

# Worksheet

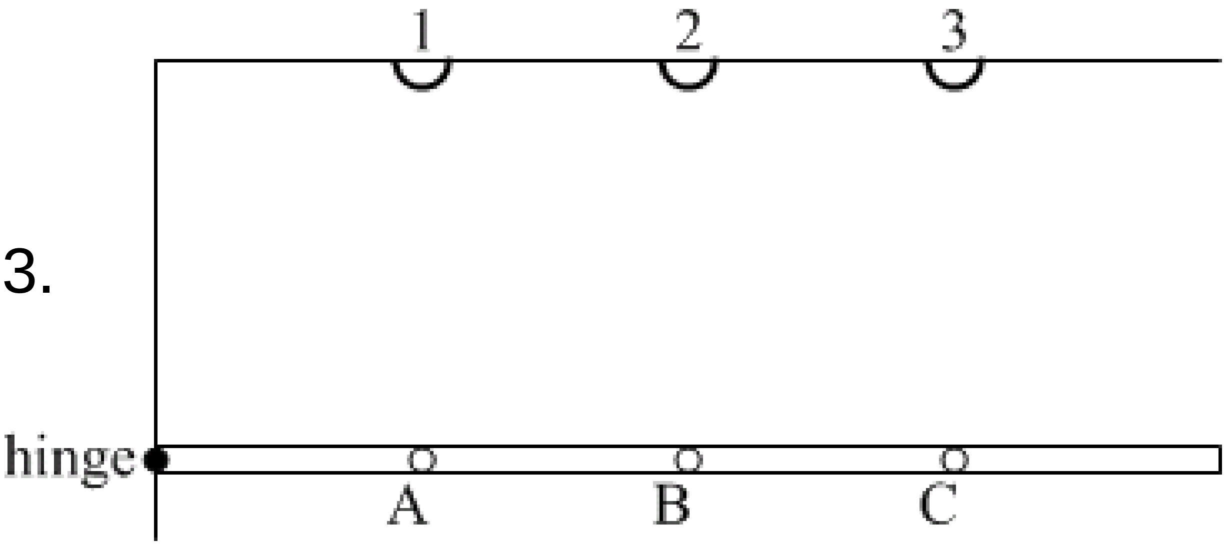
A uniform rod with a length *L* and a mass *m* is attached to a wall by a hinge at the left end. A string will hold the rod in a horizontal position; the string can be tied to one of three points, lettered A-C, on the rod. The other end of the string can be tied to one of three hooks, numbered 1-3, above the rod. This system could be a simple model of a broken arm you want to immobilize with a sling.



# Sling, part 1

How would you attach a string so the rod is held in a horizontal position but the **hinge exerts no force at all on the rod**?

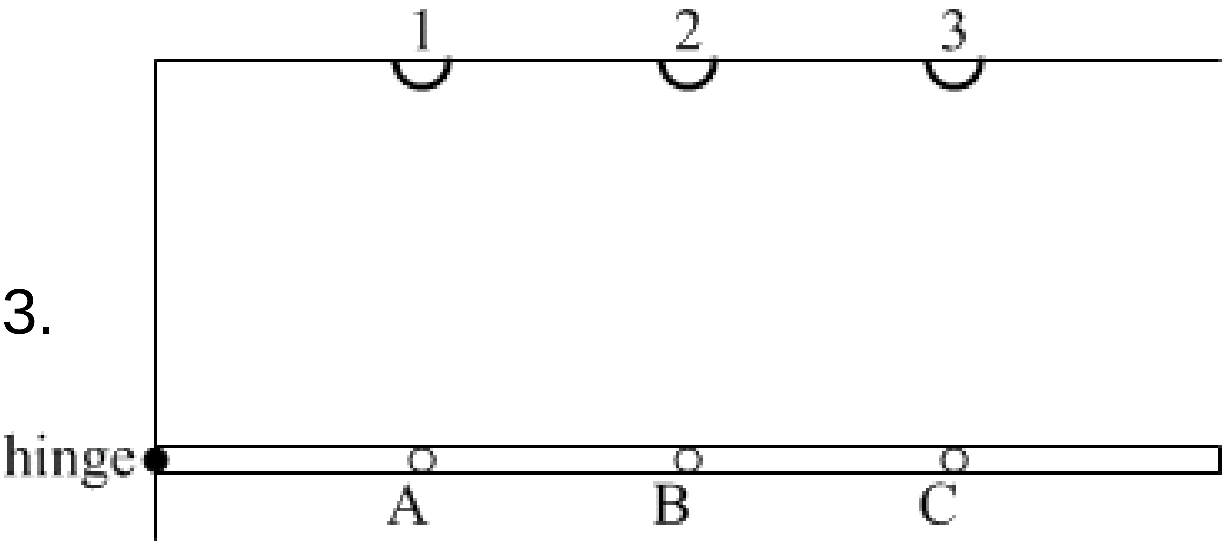
1. A → 1.
2. A → 2.
3. A → 3.
4. B → 1 or B → 3.
5. B → 2.
6. C → 1.
7. C → 2.
8. C → 3.
9. It can't be done.



# Sling, part 2

How would you attach a string so the rod is held in a horizontal position while the force exerted on the rod by the hinge **has no horizontal component, but has a non-zero vertical component directed straight up**?

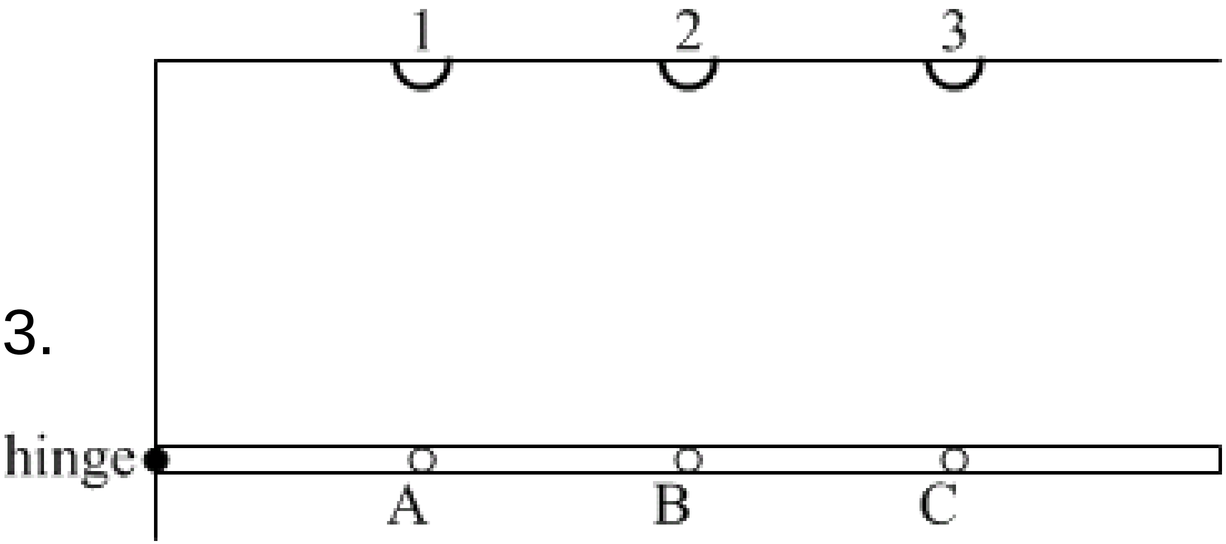
1. A → 1.
2. A → 2.
3. A → 3.
4. B → 1 or B → 3.
5. B → 2.
6. C → 1.
7. C → 2.
8. C → 3.
9. It can't be done.



# Sling, part 3

How would you attach a string so the rod is held in a horizontal position while the force exerted on the rod by the hinge **has no vertical component, but has a non-zero horizontal component**?

1. A → 1.
2. A → 2.
3. A → 3.
4. B → 1 or B → 3.
5. B → 2.
6. C → 1.
7. C → 2.
8. C → 3.
9. It can't be done.



# A balanced beam


A uniform beam sits on two identical scales. Scale A is farther from the center than scale B, but the beam remains in equilibrium.

Which scale shows a higher reading?

How far to the left could scale B be moved without the beam tipping over?

# A balanced beam


A uniform beam sits on two identical scales. Scale A is farther from the center than scale B, but the beam remains in equilibrium.

Which scale shows a higher reading?

Scale B – it is closer to the center of gravity.

How far to the left could scale B be moved without the beam tipping over?

# A balanced beam


A uniform beam sits on two identical scales. Scale A is farther from the center than scale B, but the beam remains in equilibrium.

Which scale shows a higher reading?

Scale B – it is closer to the center of gravity.

How far to the left could scale B be moved without the beam tipping over?

To the center of the beam, but no farther. The beam's center of gravity must be between the supports to be stable.

# A balanced beam


Could you place a weight on the beam without the reading on scale A changing?

Could you place a weight on the beam and cause the reading on scale A to decrease?

# A balanced beam


Could you place a weight on the beam without the reading on scale A changing?

Yes – place it directly over scale B.

Could you place a weight on the beam and cause the reading on scale A to decrease?

# A balanced beam


Could you place a weight on the beam without the reading on scale A changing?

Yes – place it directly over scale B.

Could you place a weight on the beam and cause the reading on scale A to decrease?

Yes – put it on the beam to the right of scale B.

# A balanced beam

What happens to the scale readings when scale B is moved to the right?


1. A is unchanged, B goes up.
2. A is unchanged, B goes down.
3. Both readings decrease.
4. Both readings increase.
5. A goes up, B goes down.
6. A goes down, B goes up.
7. None of the above.

# The human spine

Equilibrium ideas can be applied to the human body, including the spine. If you bend your upper body so it is horizontal, you put a lot of stress on the *lumbrosacral disk*, the disk separating the lowest vertebra from the tailbone (the *sacrum*). Picking something up when bending this way is even worse.

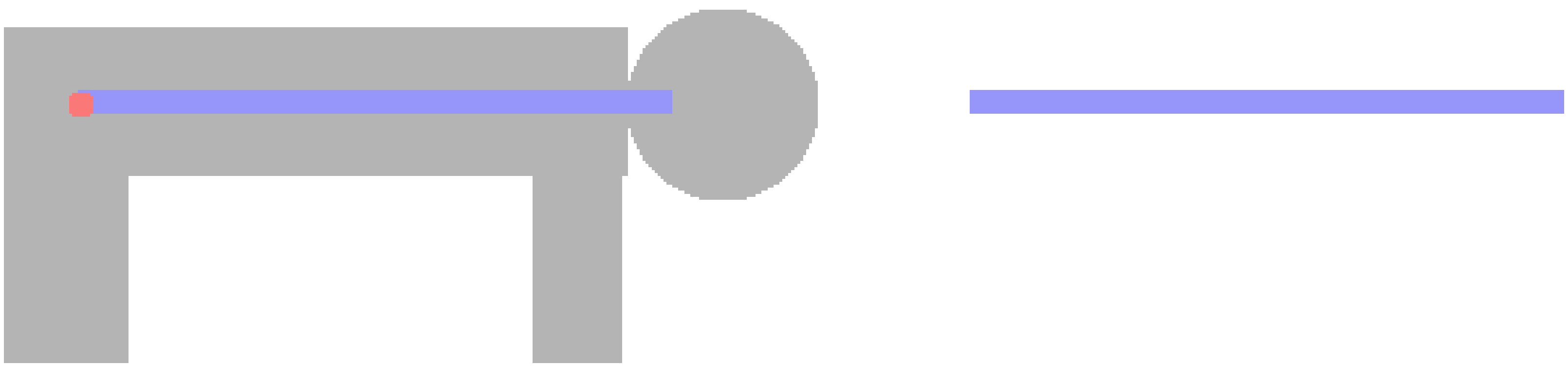
# The human spine

Simulation     Treat the spine as a pivoted bar. There are essentially three forces acting on this bar:

The force of gravity, mg, acting on the upper body (this is about 65% of the body weight).

The tension in the back muscles. This can be considered as one force T that acts at an angle of about 12° to the horizontal when the upper body is horizontal.

The support force F from the tailbone, which also acts at a small angle measured from the horizontal.



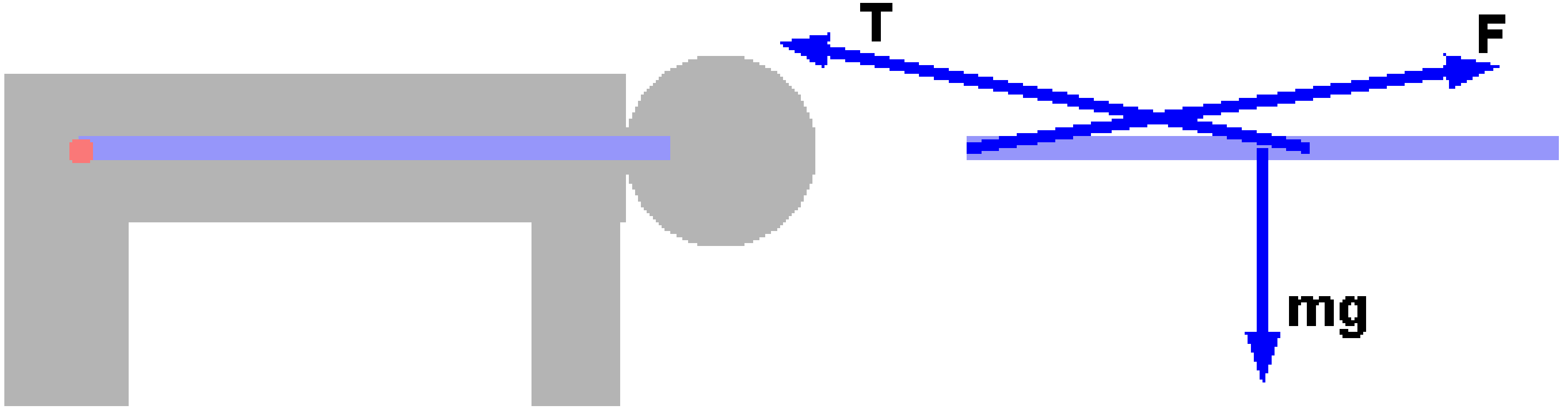
# The human spine

Treat the spine as a pivoted bar. There are essentially three forces acting on this bar:

The force of gravity, mg, acting on the upper body (this is about 65% of the body weight).

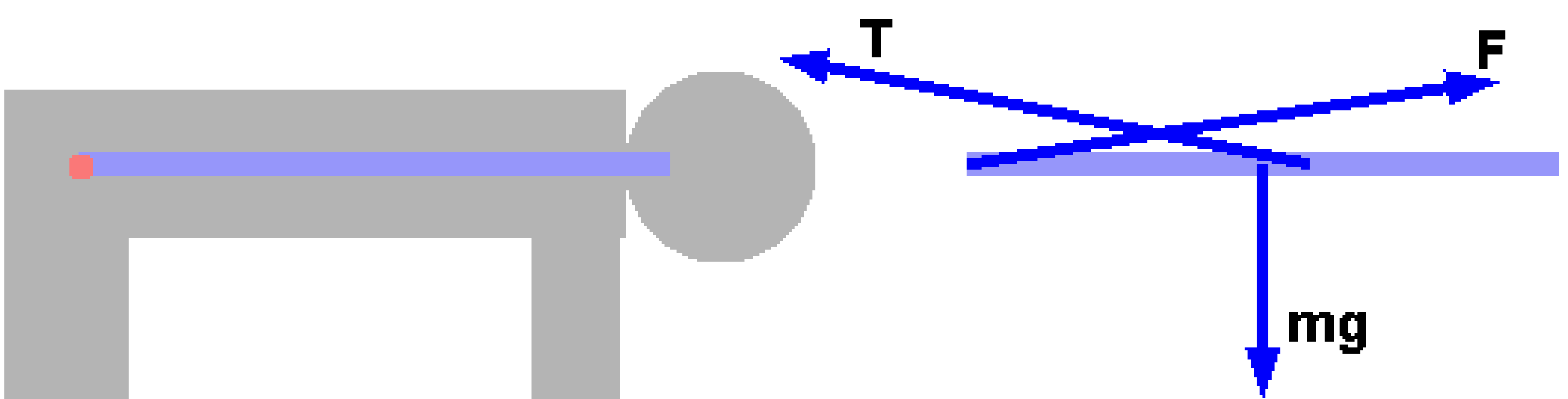
The tension in the back muscles. This can be considered as one force T that acts at an angle of about 12° to the horizontal when the upper body is horizontal.

The support force F from the tailbone, which also acts at a small angle measured from the horizontal.



# The human spine

For a person with a weight of 600 N, the upper body weighs almost 400 N. What are T and F?



# The human spine

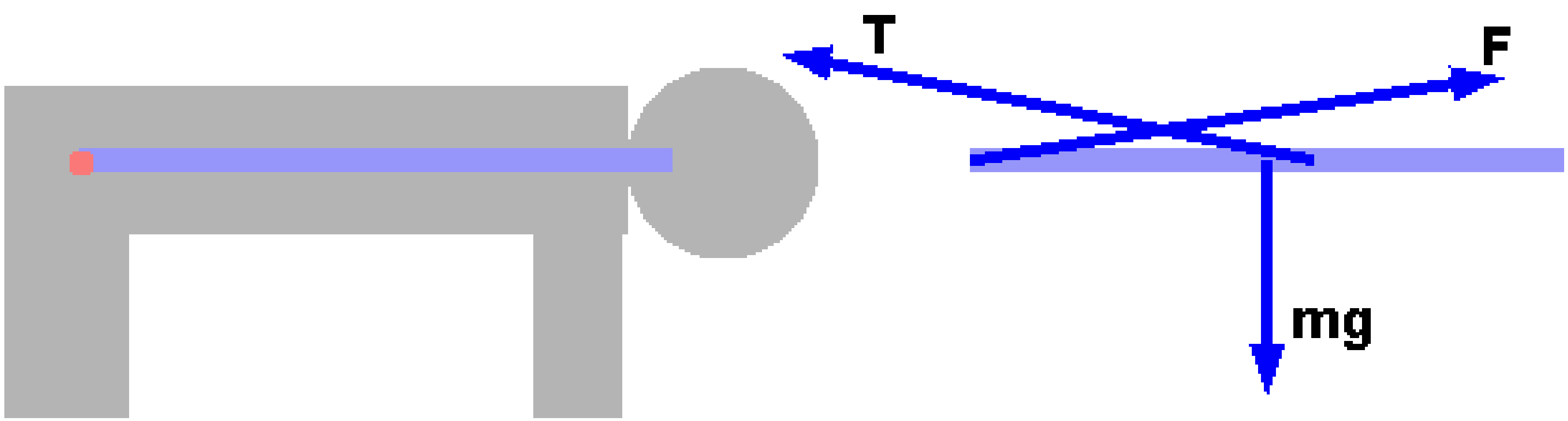
For a person with a weight of 600 N, the upper body weighs almost 400 N. What are T and F?

How far do forces get us? $\sum \vec{F} = 0$

X-direction: $+F_x - T_x = 0$

Y-direction: $+F_y + T_y - mg = 0$

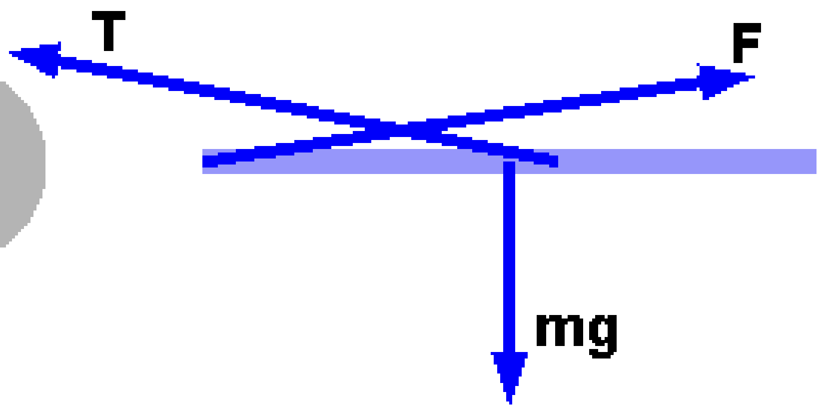
Now what?



# The human spine

Let's try summing torques: $\sum \vec{\tau} = 0$

Choose a point where one of the unknown forces passes through to sum torques.

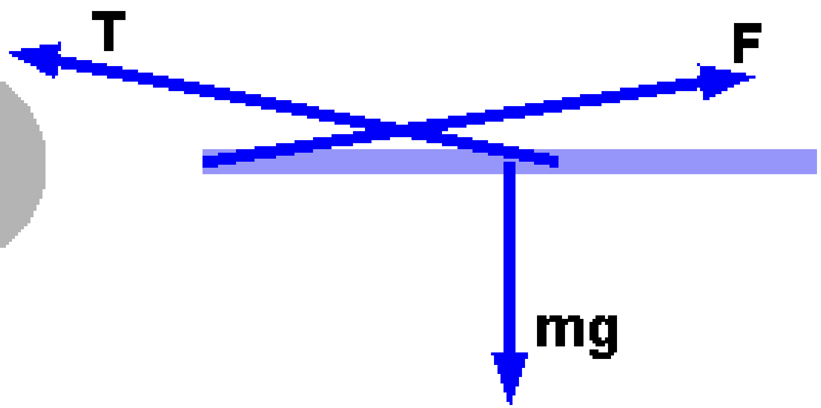


# The human spine

Let’s try summing torques: $\sum \vec{\tau} = 0$

Choose a point where one of the unknown forces passes through to sum torques.

In this case, choose the tailbone (the left end, below) as our axis for torques – this eliminates F.



# The human spine

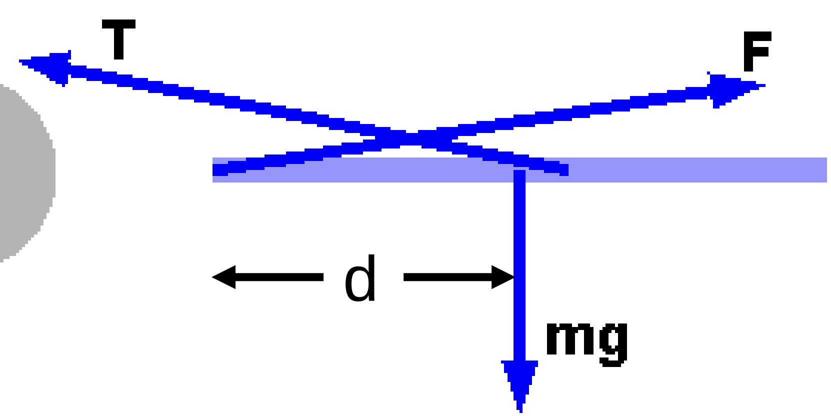
Take torques about the tailbone – I'm choosing counter-clockwise to be positive for torque. Effectively, *T* is applied about 10% farther from the tailbone than the force of gravity.

$$+(1.1d)(T)(\sin 12°) - d(mg) = 0$$

$$T = \frac{mg}{1.1(\sin 12°)}$$

If mg = 400 N, T comes out to about 1700 N!

The components of the support force *F* can be found from our force equations. F is also about 1700 N.



# The human spine

Picking up something, which has a weight of 100 N, with your arms increases both T and F by about 600 N!

The moral of the story: bend your legs instead of your back. Picking up a 100 N bag of groceries by bending at the knee produces a force of about 500 N on the bottom disk in the spine. The force is 4-5 times larger if you bend your back!

# Newton's Second Law for Rotation

The equation $\sum \vec{\tau} = I\vec{\alpha}$ is the rotational equivalent of $\sum \vec{F} = m\vec{a}$.

Torque plays the role of force.

Rotational inertia plays the role of mass.

Angular acceleration plays the role of the acceleration.

# Applying Newton's Second Law

A constant force of $F$ = 8 N is applied to a string wrapped around the outside of the pulley. The pulley is a solid disk of mass $M$ = 2.0 kg and radius $R$ = 0.50 m, and is mounted on a horizontal frictionless axle. What is the pulley's angular acceleration?

Simulation

What should we do first?

Why are we told that the pulley is a solid disk?

# Applying Newton's Second Law

A constant force of $F$ = 8 N is applied to a string wrapped around the outside of the pulley. The pulley is a solid disk of mass $M$ = 2.0 kg and radius $R$ = 0.50 m, and is mounted on a horizontal frictionless axle. What is the pulley's angular acceleration?

Simulation

What should we do first?

Draw a free-body diagram of the pulley.

Why are we told that the pulley is a solid disk?

# Applying Newton's Second Law

A constant force of $F$ = 8 N is applied to a string wrapped around the outside of the pulley. The pulley is a solid disk of mass $M$ = 2.0 kg and radius $R$ = 0.50 m, and is mounted on a horizontal frictionless axle. What is the pulley's angular acceleration?

Simulation

What should we do first?

Draw a free-body diagram of the pulley.

Why are we told that the pulley is a solid disk?

So we know what to use for the rotational inertia. $I = \frac{1}{2}MR^2$

# Applying Newton's Second Law

$$\sum \vec{\tau} = I\vec{\alpha}$$

$$RF = \frac{1}{2}MR^2\alpha$$

$$F = \frac{1}{2}MR\alpha$$

$$\alpha = \frac{2F}{MR} = \frac{2 \times (8.0\text{ N})}{(2.0\text{ kg}) \times (0.5\text{ m})} = 16\text{ rad/s}^2$$

# Two pulleys

Simulation

We take two identical pulleys, both with string wrapped around them. On the one on the left we apply an 8 N force to the string. On the one on the right we hang an object with a weight of 8 N. Which pulley has the larger angular acceleration?

# Two pulleys

We take two identical pulleys, both with string wrapped around them. On the one on the left we apply an 8 N force to the string. On the one on the right we hang an object with a weight of 8 N. Which pulley has the larger angular acceleration?

1. The one on the left
2. The one on the right
3. Neither, they're equal

# Two pulleys

For the pulley on the left, the tension in the string is 8 N.

Simulation

For the system on the right, let's draw the free-body diagram of the weight.

What does the free-body diagram tell us about the tension in the string?

# Two pulleys

For the pulley on the left, the tension in the string is 8 N.

For the system on the right, let's draw the free-body diagram of the weight.

What does the free-body diagram tell us about the tension in the string? For the weight to have a net force directed down, the tension must be less than the force of gravity. So, the tension is less than 8 N.