Rotational Inertia and Newton's Second Law for Rotation

$\sum \vec{F} = m\vec{a}$ is an incredibly useful relationship. The rotational equivalent is $\sum \vec{\tau} = I\vec{\alpha}$.

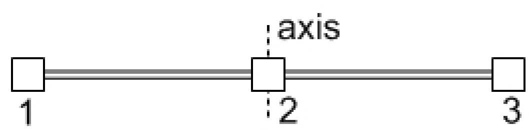
Inertia is our measure of how difficult it is to change an object's motion. For straight-line motion, an object's inertia is given by its mass. For rotational motion the **rotational inertia** depends on mass, how the mass is distributed, and even on what axis the object is rotating about. The symbol for rotational inertia is *I*, and it represents the rotational analog of mass.

In general, the rotational inertia of an object can be calculated by splitting the object into small pieces and adding up the rotational inertia of each piece using the equation:

$I = \sum_i m_i r_i^2$, where m_i represents the mass of a particular piece and r_i represents the distance of that piece from a particular axis of rotation. In practice we rarely use this method, and instead look up expressions for moments of inertia in a table, but it's nice to get some idea of how the method is used so we know where the expressions come from.

Let's try this for a system consisting of three blocks, each with a mass of $\frac{m}{3}$. The blocks are placed on a light rod of length *L* with a block at each end and one block in the center. Find an expression for the system's rotational inertia about an axis through the center of the rod.

We can find the rotational inertia of an individual block using $I = mr^2$.



Find the rotational inertia of:

Block 1:

Block 2:

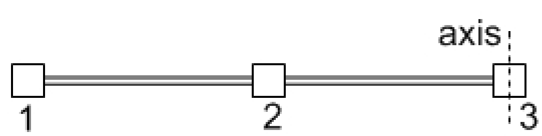
Block 3:

Add them together to find an expression for the rotational inertia of the system for this particular axis of rotation.

Let's repeat the process, but this time the axis of rotation is through one end of the rod.

Find the rotational inertia of:

Block 1:



Block 2:

Block 3:

Add them together to find an expression for the rotational inertia of the system for this particular axis of rotation.
