## Rotational Kinematics

Today marks the start of the several days we'll spend looking at rotating systems. Much of what we learning previously can be applied to such systems.

Consider a system that consists of a large block tied to a string wrapped around the outside of a rather large pulley. The pulley has a radius of 2.0 m . When the system is released from rest, the block falls with a constant acceleration of $0.5 \mathrm{~m} / \mathrm{s}^{2}$, directed down.

What is the speed of the block after 4.0 s ?

How far does the block travel in 4.0 s ?

Plot a graph of the speed of the block as a function of time, up until 4.0 s .


On the graph above, plot the speed of a point on the pulley that is on the outer edge of the pulley, 2.0 m from the center.

Also, plot the speed of a point on the pulley that is 1.0 m from the center.

We will often analyze the rotational motion of a rotating system using constantacceleration equations that have the same form as the one-dimensional constantacceleration equations we used before.

| Straight-line motion equation | Analogous rotational motion equation |
| :---: | :---: |
| $\vec{v}=\vec{v}_{i}+\vec{a} t$ | $\vec{\omega}=\vec{\omega}_{i}+\vec{\alpha} t$ |
| $\vec{x}=\vec{x}_{i}+\vec{v}_{i} t+\frac{1}{2} \vec{a} t^{2}$ | $\vec{\theta}=\vec{\theta}_{i}+\vec{\omega}_{i} t+\frac{1}{2} \vec{\alpha} t^{2}$ |
| $v^{2}=v_{i}^{2}+2 \vec{a} \Delta \vec{x}$ | $\omega^{2}=\omega_{i}^{2}+2 \vec{\alpha} \Delta \vec{\theta}$ |

Consider the following situation. While fixing the chain on your bike, you have the bike upside down. Your friend comes along and gives the front wheel, which has a radius of 30 cm , a spin. You observe that the wheel has an initial angular velocity of $2.0 \mathrm{rad} / \mathrm{s}$, and that the wheel comes to rest after 50 s .

Assume that the wheel has a constant angular acceleration. Either by using the equations above, or by using other means, determine how many revolutions the wheel makes.

## Torque

Torque is essentially the rotational equivalent of force. Let's spend some time figuring out what exactly that statement means.

If you have ever opened a door you have a basic understanding of torque. How quickly the door opens depends on the force you apply to the door, where on the door you exert the force, and in what direction you exert the force. All of these factor into the torque you exert on the door. Consider the following situations involving a revolving door.

Situation 1
A - a force $F$ is applied perpendicular to the door halfway between the center and an edge.
B - a force $F$ is applied perpendicular to the door at one edge.
C - a force $2 F$ is applied perpendicular to the door halfway between the center and one edge.
Rank these situations based on the magnitude of the torque experienced by the door, from largest to smallest. This is equivalent to ranking the situations based on their angular accelerations (i.e., ranking them by how effective the force is at getting the door to spin).

## Situation 2

In each case the magnitude of the force is the same and the force is applied to the door halfway between the center and one edge.
A - the force is directed perpendicular to the door.
B - the angle between the force and the door is $30^{\circ}$, with a component away from the center.
C - the angle between the force and the door is also $30^{\circ}$, with a component toward the center.
Rank these situations based on the magnitude of the torque experienced by the door, from largest to smallest.

## Situation 3

A - the force $F$ is directed perpendicular to the door and is applied halfway between the center and one edge.
B - a force $F$ directed perpendicular to the door is applied halfway between the center and one edge; another force $F$ in the opposite direction is applied at the edge of the door.
C - a force $F$ directed perpendicular to the door is applied halfway between the center and one edge; another force $F$ in the opposite direction is applied at the opposite edge of the door.

Rank these situations based on the magnitude of the torque experienced by the door, from largest to smallest.

The magnitude of the torque resulting from applying a force $F$ at a distance $r$ from an axis of rotation is: $\tau=r F \sin \theta$.

The angle $\theta$ represents the angle between the line of the force and the line the distance $r$ is measured along. Torque is a vector, so its direction is also important. We usually handle the direction separately, however.
Consider three equivalent ways of finding the torque applied by a particular force.
A rod of length $L$ is hinged at one end and held horizontal by means of a string attached to the other end. Find the torque associated with the tension in the string, about an axis passing through the hinge.

Method 1 - use the whole force and the angle.
First, in what direction is the torque from the tension? Clockwise or counterclockwise?


Identify the three factors in the torque equation above.
What is $F$ in this case?
What is $r$ in this case?
What is the angle between $F$ and $r$ ?
Apply the torque equation to find torque.
Method 2 - Split the force into vertical and horizontal components. Sketch this.
Vertical component
Horizontal component
What is $F$ in this case?
What is $F$ in this case?
What is $r$ in this case?
What is $r$ in this case?
What is the angle between $F$ and $r$ ? What is the angle between $F$ and $r$ ?
Apply the torque equation to find torque. Add the two torques together as vectors (account for direction with + and - signs) to find the net torque.

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[^0]:    Method 3 - The lever-arm method. Draw a line from the hinge to the string so the line meets the string at a $90^{\circ}$ angle. Measure $r$ along this line. Sketch this.
    What is $F$ in this case?
    What is $r$ in this case?
    What is the angle between $F$ and $r$ ?
    Apply the torque equation to find torque.

