

## Angular Momentum

*Let's analyze a rotational collision.*

Let's analyze a rotational collision, following the basic procedure we followed to analyze a one dimensional collision. Sarah, with mass  $m$  and velocity  $\vec{v}$ , runs toward a playground merry-go-round, which is initially at rest, and jumps on at its edge. Sarah and the merry-go-round (which has mass  $M$ , radius  $R$ , and  $I = cMR^2$ ) then spin together with a constant angular velocity  $\omega_f$ . If Sarah's initial velocity is tangent to the circular merry-go-round, what is  $\omega_f$ ?

Is linear momentum conserved here? Why or why not?

Is angular momentum conserved here? Why or why not?

One issue we have in applying angular-momentum conservation is that it's not obvious that there is any angular momentum before the collision, and there clearly is some after the collision. However, a linear momentum can be transformed into an angular momentum in the same way that a force is transformed into a torque.

If  $\tau = rF \sin \theta$  then the expression for the angular momentum is  $L =$

Write an equation representing angular momentum conservation in this case. Solve it for  $\omega_f$ .

## Rolling and Rotational Kinetic Energy

*Let's look at how to incorporate rotational kinetic energy into our energy analysis.*

The equation for rotational kinetic energy is analogous to the  $\frac{1}{2}mv^2$  equation we're used to for translational kinetic energy. Rotational kinetic energy is  $K_{rot} = \frac{1}{2}I\omega^2$ . This can be directly incorporated into our usual five-term energy equation.

Take a round object (ball, cylinder, disk, ring, etc.) and roll it down an incline. The object starts from rest at a point that is  $h$  higher than the bottom of the incline. Assuming the object rolls without slipping down the incline determine its speed at the bottom.

Sketch this situation, showing the object in two positions, one at the top of the incline and one at the bottom.

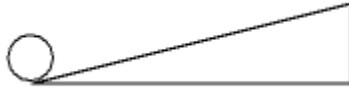
Start with the usual conservation of energy equation:  $K_i + U_i + W_{nc} = K_f + U_f$ .

Cross out all the terms that are zero in this equation.

Write out expressions for the remaining terms. Remember to account for both translational kinetic energy and rotational kinetic energy, if appropriate. Keep everything in terms of variables. We'll use as general an analysis as possible, so use  $I = cMR^2$ , where  $c$  will be determined by the shape of the object we use.

Find an expression for the speed of the object at the bottom of the incline. Note that because the object rolls without slipping we can use  $\omega = v/R$ .

Two objects of equal mass and radius are rolling along a flat surface when they encounter a gradual incline of constant slope. One object is a uniform solid sphere, and the other is a ring. Both objects roll without slipping at all times. **Briefly justify all your answers below.**



If the objects have identical velocities at the bottom of the incline, which travels farthest up the slope before rolling back down?

the sphere       the ring       they travel the same distance

If the objects instead have identical total kinetic energies at the bottom of the incline, which travels furthest up the slope before rolling back down? The total kinetic energy is the sum of the translational and rotational kinetic energies.

the sphere       the ring       they travel the same distance

Sketch the free-body diagram of one of these objects as it rolls without slipping up the slope. If you include a force of friction clearly indicate whether it is a kinetic force of friction or a static force of friction.

If the objects have identical velocities at the bottom of the incline, which object experiences a larger frictional force as it rolls without slipping up the incline?

the sphere       the ring  
 neither, they have equal (non-zero) frictional forces  
 neither one of the objects has a frictional force