A ballistic pendulum

A ballistic pendulum is a device used to measure the speed of a bullet. A bullet of mass m is fired at a block of wood (mass M) hanging from a string. The bullet embeds itself in the block, and causes the combined block plus bullet system to swing up a height h. What is v_0, the speed of the bullet before it hits the block?

We will work backwards to find an expression for v_0.

Mechanical energy conservation?

Define the zero level for gravitational potential energy so both the bullet and the block have zero initial gravitational potential energy. Is it correct to set the bullet's kinetic energy before the collision to the bullet + block's gravitational potential energy when the bullet + block reach their highest point after the collision?

1. Yes
2. No

Mechanical energy conservation?

Mechanical energy is not conserved in the collision.

A completely inelastic collision occurs, in which some (in fact, most!) of the mechanical energy is transformed into thermal energy when the bullet embeds itself in the block.

Is anything conserved during the collision?

Yes, momentum is conserved during the small time period during which the collision occurs.

Mechanical energy conservation!

Mechanical energy is conserved in the pendulum motion after the collision.

Work backwards, starting with the swing of the pendulum just after the collision until it reaches its maximum height, *h*.

Write out the five-term energy-conservation equation.

$$U_i + K_i + W_{nc} = U_f + K_f$$

Eliminate terms that are zero.

$$K_i = U_f$$

Substitute expressions for these terms:

$$\frac{1}{2}(M+m)v_i^2 = (M+m)gh$$

Mechanical energy conservation!

$$\frac{1}{2}(M+m)v_i^2 = (M+m)gh$$

Solve for the speed of the pendulum at its lowest point. We get the familiar expression:

$$v_i = \sqrt{2gh}$$

This is the final velocity of the system after the collision. In our collision analysis, then, we have:

$$v_f = \sqrt{2gh}$$

Analyzing the collision

Apply momentum conservation to the collision.

Momentum beforehand = momentum afterwards

$$mv_0 = (M+m)v_f$$

Bring in our previous result: $v_f = \sqrt{2gh}$

$$mv_0 = (M+m)\times\sqrt{2gh}$$

Solve for the speed of the bullet before the collision:

$$v_0 = \frac{(M+m)\times\sqrt{2gh}}{m}$$

A numerical example

If we use the following:

Mass of the bullet: m = 30 grams

Mass of the block: M = 870 grams

Maximum height: h = 0.74 m

We find that: $v_0 = \frac{(M+m)\times\sqrt{2gh}}{m} = 114$ m/s

How much mechanical energy is lost?

Look at the ratio of the kinetic energy after the collision to the kinetic energy before the collision:

$$\frac{K_f}{K_0} = \frac{\frac{1}{2}(M+m)v_f^2}{\frac{1}{2}mv_0^2} = \frac{(M+m)v_f \times v_f}{mv_0 \times v_0}$$

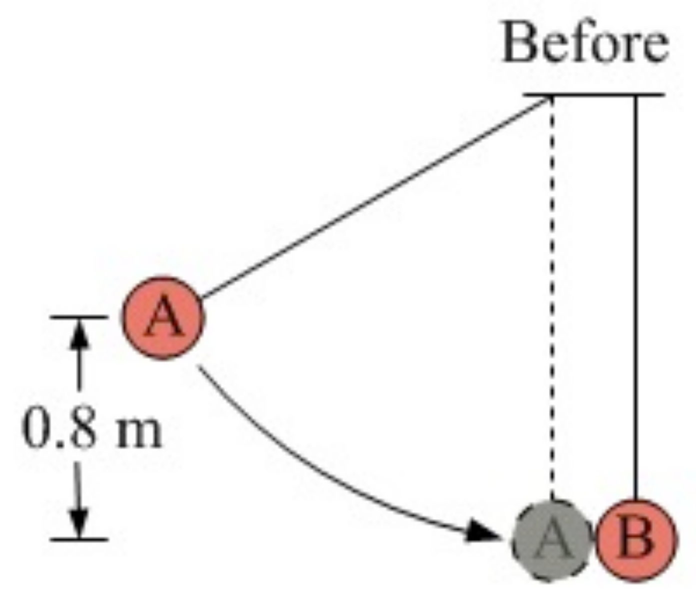
From momentum conservation: $mv_0 = (M+m)v_f$

$$\frac{K_f}{K_0} = \frac{v_f}{v_0} = \frac{m}{M+m}$$

In our numerical example, this ratio is 0.033. 3.3% of the mechanical energy remains. 96.7% is lost!

Worksheet, page 2

Two balls hang from strings of the same length. Ball A, with a mass of 4 kg, is swung back to a point 0.8 m above its equilibrium position. Ball A is released from rest and swings down and hits ball B. After the collision ball A rebounds to a height of 0.2 m above its equilibrium position, and ball B swings up to a height of 0.05 m.



Speed of ball A, before

How fast is ball A going, just before the collision?

Apply energy conservation.

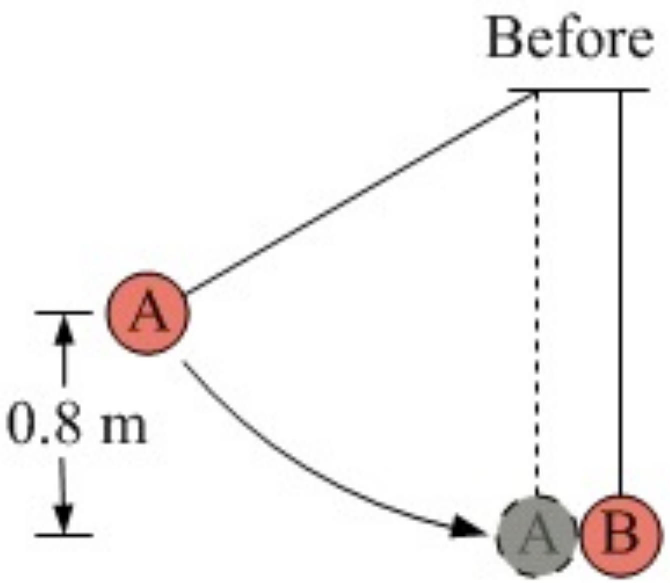
$$U_i + K_i + W_{nc} = U_f + K_f$$

Eliminate three of the terms.

$$U_i = K_f$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{16\ \mathrm{m}^2/\mathrm{s}^2} = 4.0\ \mathrm{m/s}$$



Speed of the balls, after the collision

We can use the same equation afterwards.

For ball A afterwards:

$$v = \sqrt{2gh} = \sqrt{4.0\ \mathrm{m}^2/\mathrm{s}^2} = 2.0\ \mathrm{m/s}$$

For ball B afterwards:

$$v = \sqrt{2gh} = \sqrt{1.0\ \mathrm{m}^2/\mathrm{s}^2} = 1.0\ \mathrm{m/s}$$

What is the mass of ball B?

Find the mass of ball B.

1. 4 kg
2. 8 kg
3. 12 kg
4. 16 kg
5. 24 kg
6. None of the above

Find the mass of ball B

Apply momentum conservation.

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

$$m_A v_{Ai} + 0 = m_A v_{Af} + m_B v_{Bf}$$

How do we account for the fact that momentum is a vector?

Find the mass of ball B

Apply momentum conservation.

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

$$m_A v_{Ai} + 0 = m_A v_{Af} + m_B v_{Bf}$$

How do we account for the fact that momentum is a vector?

Choose a positive direction (to the right), so the velocity of ball A after the collision is negative.

$$m_B = \frac{m_A v_{Ai} - m_A v_{Af}}{v_{Bf}} = \frac{(4\ \text{kg}) \times (+4\ \text{m/s}) - (4\ \text{kg}) \times (-2\ \text{m/s})}{+1\ \text{m/s}}$$

$$m_B = 16\ \text{kg} + 8\ \text{kg} = 24\ \text{kg}$$

What kind of collision?

Relative speed before the collision: 4 m/s

Relative speed after the collision: 3 m/s

Elasticity: $k = \dfrac{3}{4}$.

This is less than 1, so the collision is inelastic.

What kind of collision?

Kinetic energy before the collision: 32 J

Kinetic energy after the collision: 8 J + 12 J = 20 J

The kinetic energy is smaller after the collision, so the collision is inelastic. It is not completely inelastic, because the two balls do not stick together after the collision.