

Work done by individual forces

We can determine the work done by a particular force.

$$W = F\Delta r \cos \theta$$

The net work done on an object is the sum of the work done by each of the individual forces acting on the object.

$$W_{\text{net}} = W_{\text{hand}} + W_{\text{gravity}} + W_{\text{friction}}$$

First let's look at W_{hand} for some simple situations.

Work, example 2

You raise the 10 N object 0.5 m vertically. The object starts and ends at rest. How much work do you do on the object?

1. 5 J
2. More than 5 J
3. Less than 5 J



- Your hand exerts an upward force of ~ 10 N through an upward displacement of 0.5 m so $W_{\text{hand}} \sim 5$ J. But should you worry about the unbalanced forces needed to start and stop the motion?
- If gravity and the hand are the only two forces,
$$W_{\text{gravity}} + W_{\text{hand}} = \Delta K$$
- $W_{\text{gravity}} = (mg)(\Delta y)(-1)$ is exactly -5 J, and ΔK is exactly zero.
- Therefore, $W_{\text{hand}} = \text{exactly } + 5$ J, and any work done to start and stop the motion must cancel out!
- Of course, this assumes that there are no other forces acting, such as friction or air resistance. If you lifted the object by pulling it up from the bottom of a barrel of crude oil, you would have to do a lot more work...

Energy Conservation

Is there a general principle associated with energy that applies to all three* fundamental forces in nature?

*three fundamental forces: gravity; electroweak force; strong force; [and maybe dark energy/quintessence..]

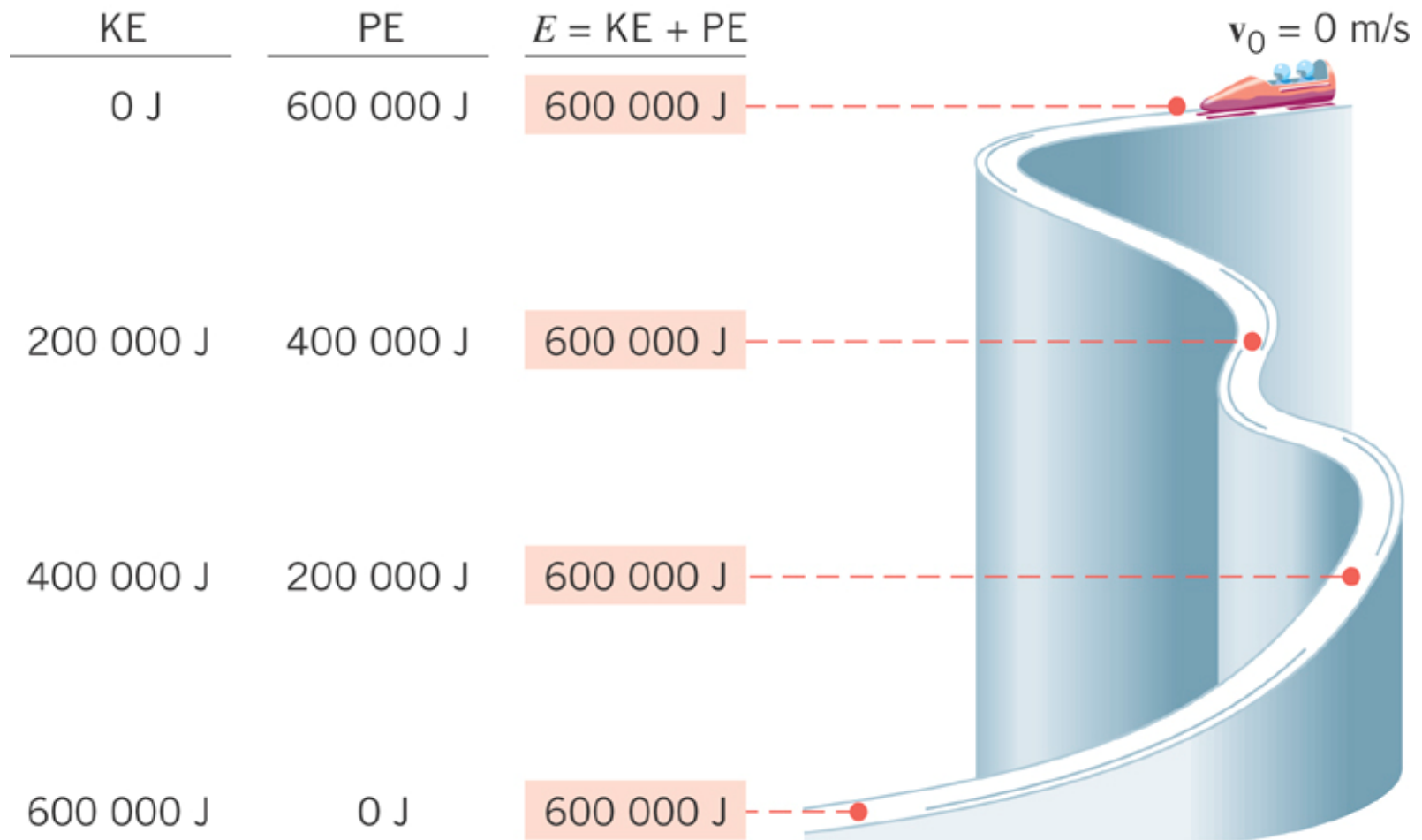
Gravity is an example of a ***conservative force***, with the associated energy determined solely by the position. It is also ***reversible***, i.e. **if you go back to the starting point, you recover all the mechanical energy – the speed is the same.**

Forces that are not conservative: Thrust exerted by rocket motors and **forces that lead to a permanent irreversible loss in energy:** friction, air resistance...

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

The total mechanical energy ($E = KE + PE$) of an object remains constant as the object moves, provided that the net work done by ***external non-conservative forces*** is zero.

6.5 The Conservation of Mechanical Energy



THE LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed, but can only be converted from one form to another.

$$U_i + K_i + W_{nc} = U_f + K_f$$

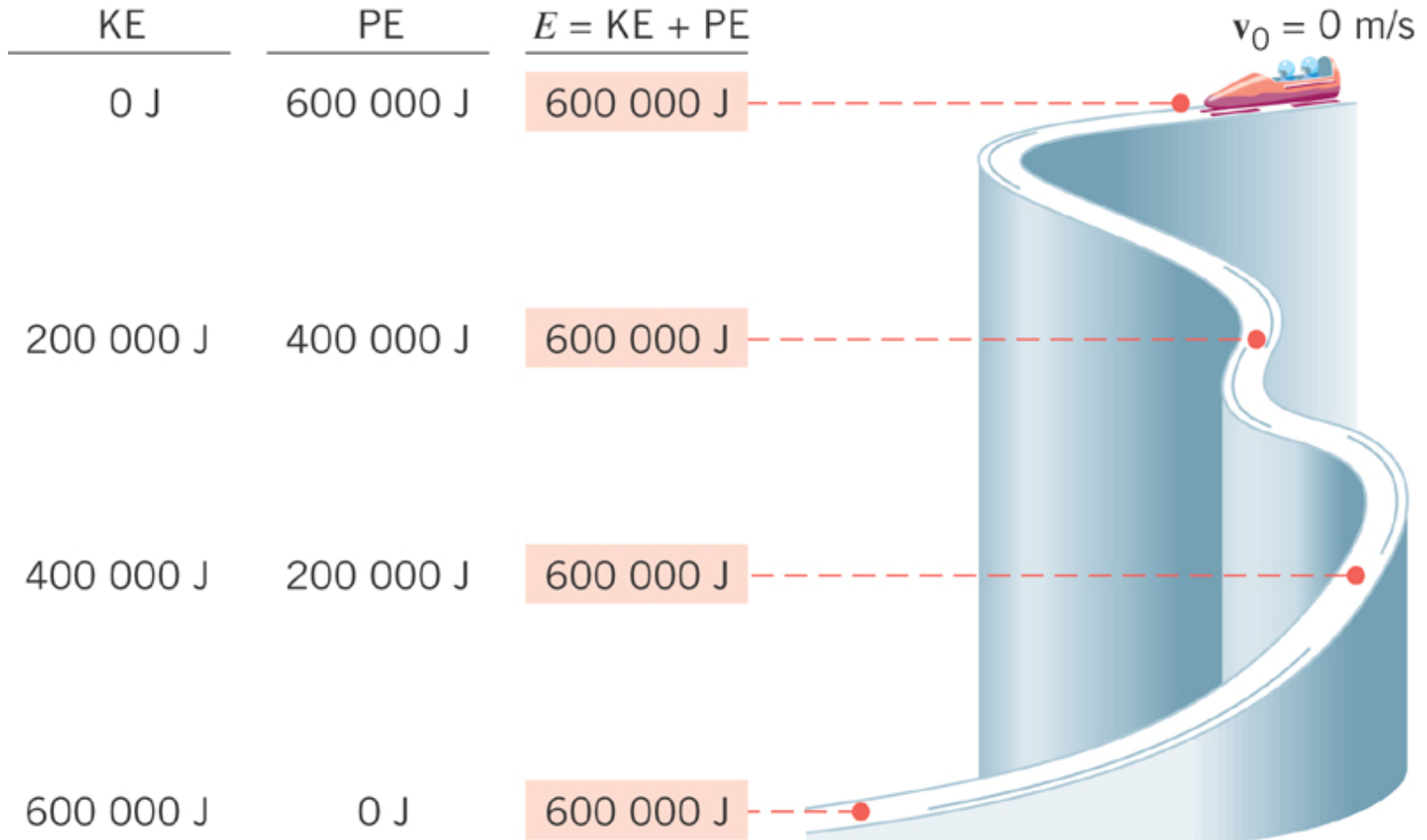
U_i and U_f are the initial and final potential energy

K_i and K_f are the initial and final kinetic energy

W_{nc} is the work done by non-conservative forces

W_{nc} can be negative (e.g., kinetic friction) or positive (e.g., you pulling on a string)

6.5 The Conservation of Mechanical Energy



$$U_i + \cancel{K_i} + \cancel{W_{nc}} = \cancel{U_f} + K_f$$

Crush the can

A heavy weight is dropped from a height of 1.2 m on a soda can, crushing it.

What is conserved in this process?

1. Both mechanical energy and total energy
2. Mechanical energy, but not total energy
3. Total energy, but not mechanical energy
4. Neither mechanical energy nor total energy
5. It depends on whether it's Coke or Pepsi



Worksheet

Define a zero level.

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Ground level

Write out the five-term equation. Cross terms out.

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$$v_f = \sqrt{2gh}$$

Worksheet

A block with a mass of 1.0 kg is released from rest from the top of a ramp that has the shape of a 3-4-5 triangle. The ramp measures 1.8 m high by 2.4 m wide, with the hypotenuse of the ramp measuring 3.0 m. What is the speed of the block when it reaches the bottom, assuming there is no friction between the block and the ramp?

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Eliminate terms that are zero.

$$U_i = K_f$$

Three balls

Three identical balls are launched with the same initial speed from the top of a cliff overlooking flat ground. Ball A is launched horizontally. Ball B has an initial velocity directed 20 degrees below the horizontal. Ball C has an initial velocity directed 40 degrees above the horizontal. Which ball hits the ground with the highest speed?

1. Ball A
2. Ball B
3. Ball C
4. Equal for all three



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$$U_i + K_i = K_f$$

U_i is the same for all three balls.

K_i is the same for all three balls.

K_f must therefore be the same for all three balls.

A race

Two identical carts are released from rest at the same time. Cart A travels down a straight incline, while cart B travels down a path that dips below that of cart A and then returns to the same level as that of cart A. Which cart wins the race?

1. Cart A
2. Cart B
3. It's a tie



Lesson

Energy is a great way to relate positions and speeds, but it does not give us direct information about time.

Energy Conservation

What does friction or air resistance do?

Certain forces, like friction or air resistance, are *Irreversible*, i.e. if you go back to the starting point, you will NOT recover all the mechanical energy.

Such forces are ***non-conservative***.

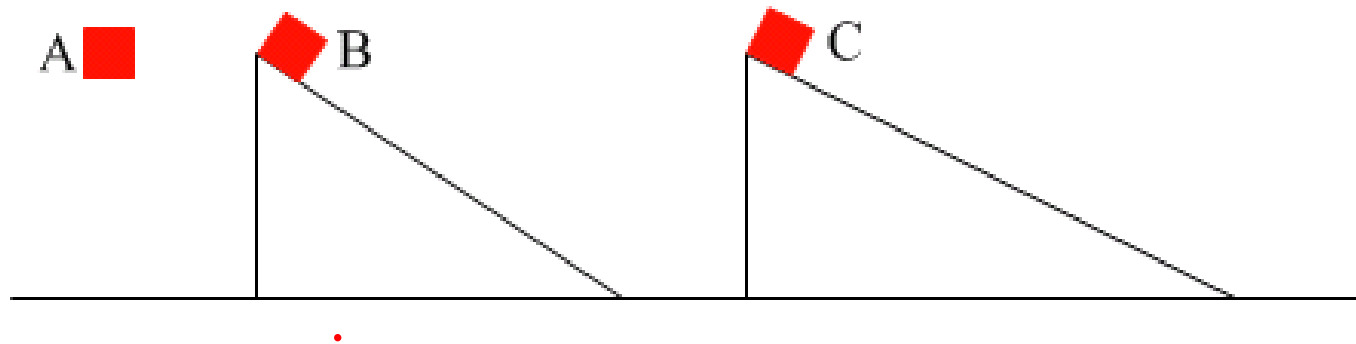
Unsolved problem: At the atomic level, air resistance, friction arise from the conservative force called electromagnetism.

No one knows why air resistance and friction, which arise at the atomic level from the electromagnetic force, leads to irreversibility. This seems to be a fundamental law of nature discovered by experiments.

Blocks

Three identical blocks are initially the same height above the floor, and are released from rest. Block A falls straight down, while blocks B and C travel down frictionless ramps. Ramp B is steeper than ramp C. Rank the blocks based on their kinetic energy as they reach the floor.

1. $A > B > C$
2. $A > B = C$
3. $A = B = C$



Force analysis, or energy analysis?

We now have two powerful ways of analyzing physical situations.

1. Analyze forces, apply Newton's Second Law, and apply constant-acceleration equations.
2. Use energy conservation.

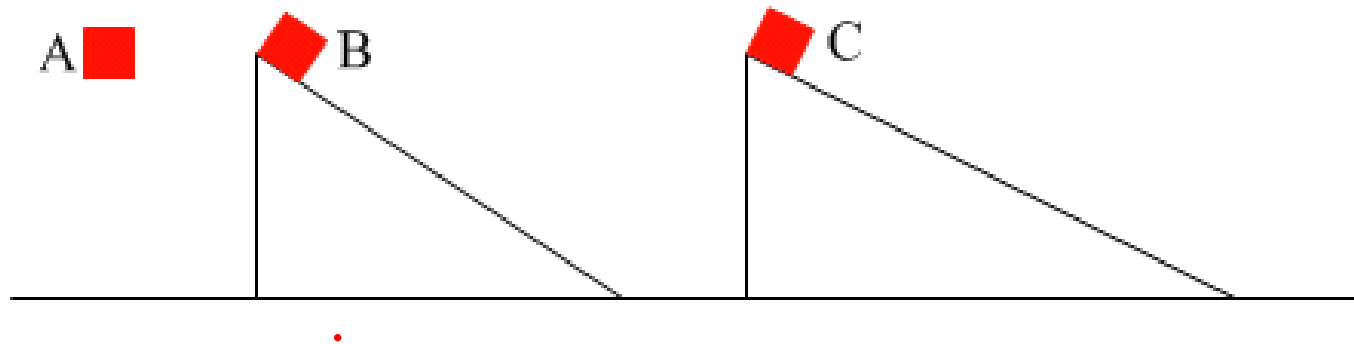
Which method do you use to answer:

- How do the speeds of the blocks compare?
- What is the final speed of block C?
- How long does it take block C to reach the floor?

Blocks, with friction

Three identical blocks are initially the same height above the floor, and are released from rest. Block A falls straight down, while blocks B and C travel down ramps. Ramp B is steeper than ramp C, but the coefficient of friction is the same for the ramps. Rank the blocks based on their kinetic energy as they reach the floor.

1. $A > B > C$
2. $A > B = C$
3. $A = B = C$



Worksheet from last time

1. Define a zero level for gravitational potential energy.

The bottom of the ramp.

2. Write out the five-term energy-conservation equation.

$$U_i + K_i + W_{nc} = U_f + K_f$$

Eliminate terms that are zero.

$$U_i = K_f$$

3. Substitute expressions for the remaining terms.

$$mgh = \frac{1}{2}mv_f^2$$

4. Solve: $v_f = \sqrt{2gh} = \sqrt{2 \times (10 \text{ m/s}^2) \times 1.8 \text{ m}} = 6.0 \text{ m/s}$

Accounting for friction

It turns out that we can't neglect friction for the block, because we find that the block's speed at the bottom of the ramp is 2.0 m/s less than the value we calculated above. Use the energy-conservation equation to find a numerical value for the work done by friction on the block.

So, the speed at the bottom is only 4.0 m/s, not 6.0 m/s.

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$$U_i + W_{nc} = K_f$$

$$W_{nc} = K_f - U_i = \frac{1}{2}mv_f^2 - mgh$$

$$W_{nc} = \frac{1}{2}(1.0 \text{ kg}) \times (4.0 \text{ m/s})^2 - (1.0 \text{ kg}) \times (10 \text{ m/s}^2) \times (1.8 \text{ m})$$

$$W_{nc} = 8.0 \text{ J} - 18 \text{ J} = -10 \text{ J}$$

Finding the coefficient

$$W_{nc} = F \Delta r \cos \phi = -F_K \times 3.0 \text{ m}$$

$$W_{nc} = -\mu_K F_N \times 3.0 \text{ m}$$

$$\mu_K = \frac{W_{nc}}{-F_N \times 3.0 \text{ m}} = \frac{-10 \text{ J}}{-F_N \times 3.0 \text{ m}}$$

$$\mu_K = \frac{10 \text{ J}}{Mg \cos \theta \times 3.0 \text{ m}} = \frac{10 \text{ J}}{(10 \text{ N}) \times \frac{4}{5} \times 3.0 \text{ m}}$$

$$\mu_K = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

Today's worksheet, page 3

Let's look at the block on the U-shaped track.

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3. Substitute expressions for the remaining terms.

$$mgh_i + W_{nc} = mgh_f$$

$$W_{nc} = mg(h_f - h_i)$$

DEFINITION OF AVERAGE POWER

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work.

$$\bar{P} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t}$$

$$\text{joule/s} = \text{watt (W)}$$

$$\bar{P} = \frac{\text{Change in energy}}{\text{Time}}$$

1 horsepower = 550 foot · pounds/second = 745.7 watts

$$\bar{P} = F \bar{v}$$

Power of a human being

An interesting calculation is the average power output of a human being. This can be determined from the amount of energy we consume in a day in the way of food. Most of us take in something like 2500 "calories" in a day, although what we call calories is really a kilocalorie. Use this as our energy output per day [*Why is this a valid assumption?*].

Take the 2.5×10^6 cal and convert to joules, using the conversion factor:

$$1 \text{ cal} = 4.186 \text{ J}$$

This gives roughly 1×10^7 J. Figuring out our average power output, we simply divide the energy by the number of seconds in a day, 86400, which gives a bit more than 100 W.

Table 6.4 Human Metabolic Rates^a

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

^aFor a young 70-kg male.