Newton's Second Law

Our version of Newton's Second Law: $\vec{F}_{net} = m\vec{a}$

Re-write it: $\vec{F}_{net} = m\vec{a} = m\dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\Delta(m\vec{v})}{\Delta t}$

That last equality is true only if mass is constant, but the expression is actually generally true.

Newton's version of Newton's Second Law: $\vec{F}_{net} = \dfrac{\Delta(m\vec{v})}{\Delta t}$

Newton's version is more general than ours, because Newton's version also accounts for changes in mass (e.g., what happens with a rocket).

Impulse

Newton's version of Newton's Second Law: $\vec{F}_{net} = \dfrac{\Delta(m\vec{v})}{\Delta t}$

Re-arrange to get the impulse equation, where impulse is the product of the net force and the time interval over which the force is applied.

Impulse: $\vec{F}_{net}\,\Delta t = \Delta(m\vec{v})$

Let's think of a name for the quantity $m\vec{v}$, which appears on the right side of the equation.

Impulse

Newton's version of Newton's Second Law: $\vec{F}_{net} = \frac{\Delta(m\vec{v})}{\Delta t}$

Re-arrange to get the impulse equation, where impulse is the product of the net force and the time interval over which the force is applied.

Impulse: $\vec{F}_{net}\,\Delta t = \Delta(m\vec{v})$

Let's think of a name for the quantity $m\vec{v}$, which appears on the right side of the equation.

Let's call this momentum.

Momentum and Impulse

Momentum is a vector, pointing in the direction of the

_____________.

Momentum and Impulse

Momentum is a vector, pointing in the direction of the velocity.

The symbol for momentum is $\vec{p}$: $\vec{p} = m\vec{v}$

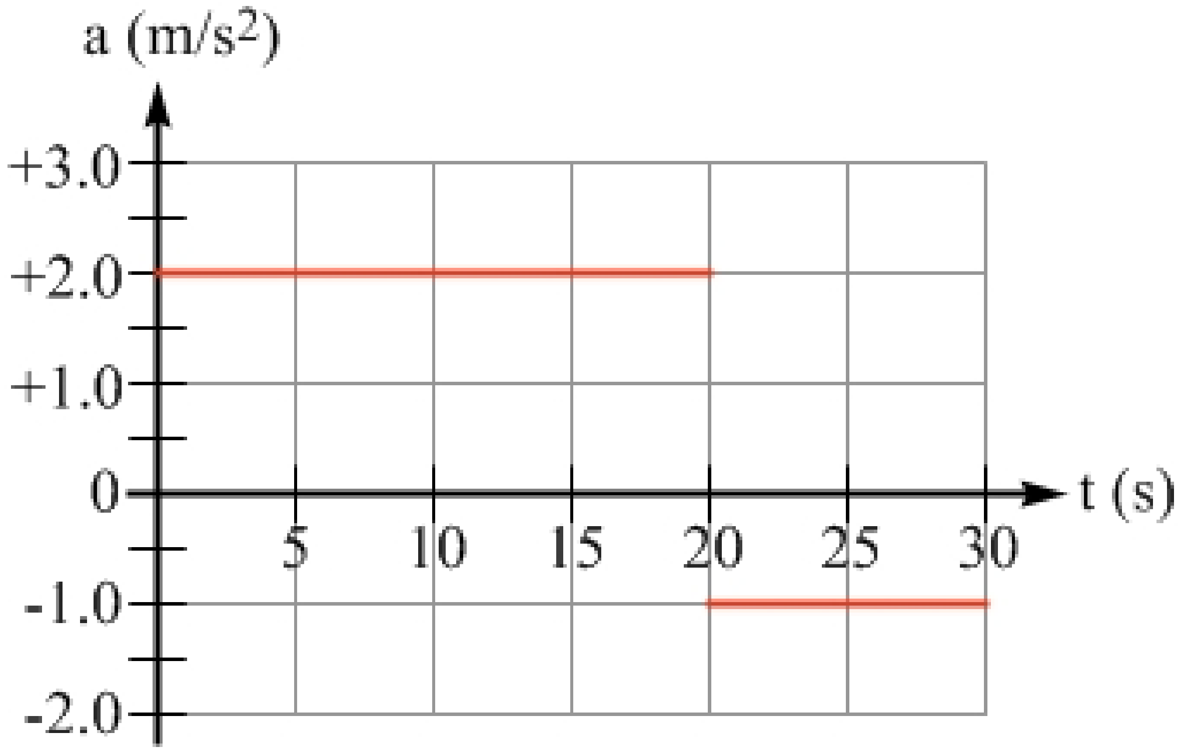
An object's momentum changes when it experiences a net force.

The change in momentum is equal to the impulse.

$$\Delta\vec{p} = \vec{F}_{net}\,\Delta t$$

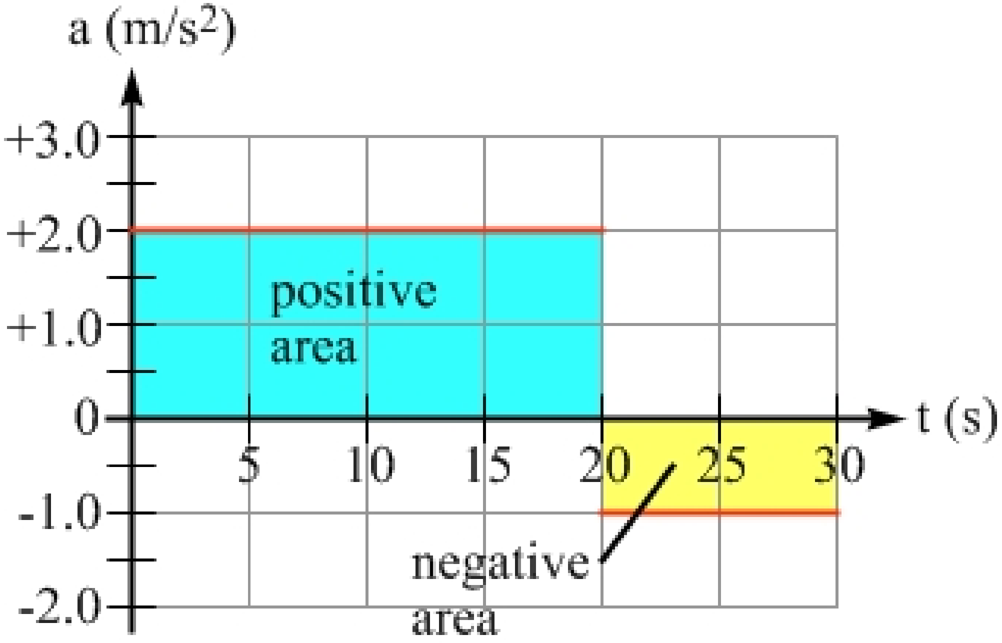
Review: Acceleration vs. time graph

Here's a graph of acceleration vs. time for an object.



The acceleration vs. time graph

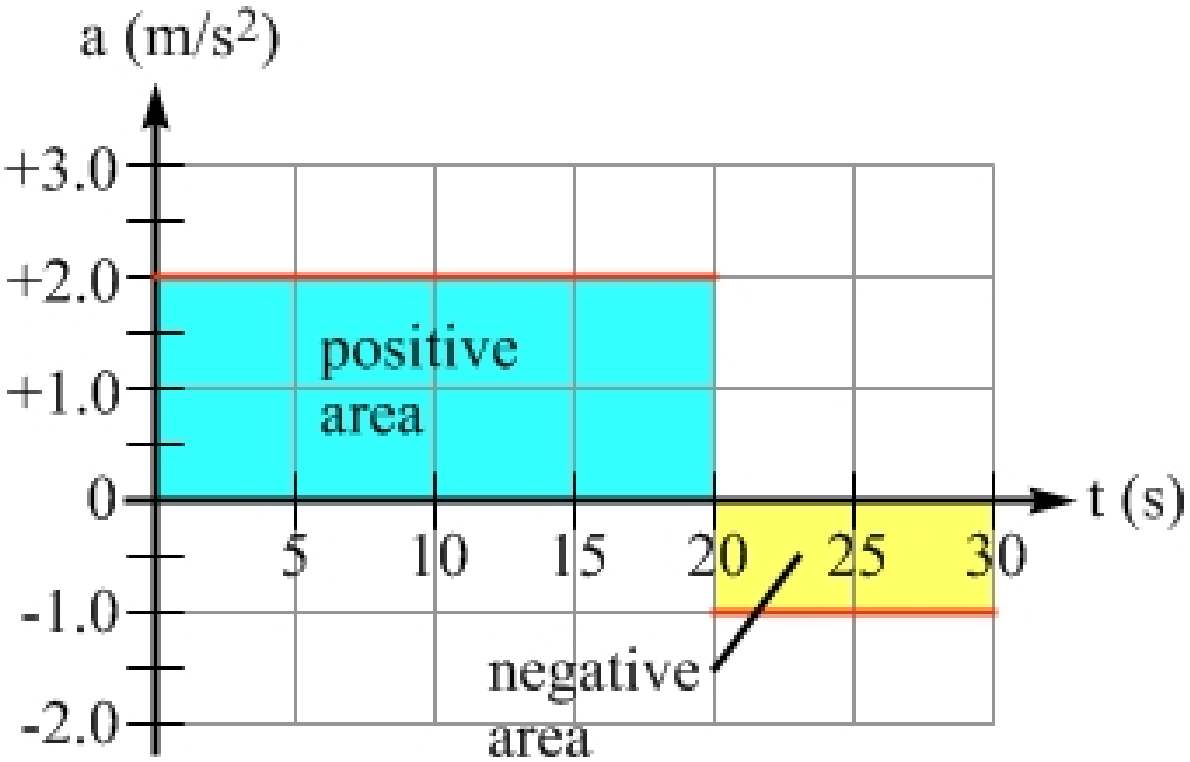
The area under the acceleration versus time graph represents __________?



1. The position.
2. The displacement.
3. The velocity.
4. The change in velocity.
5. None of the above.

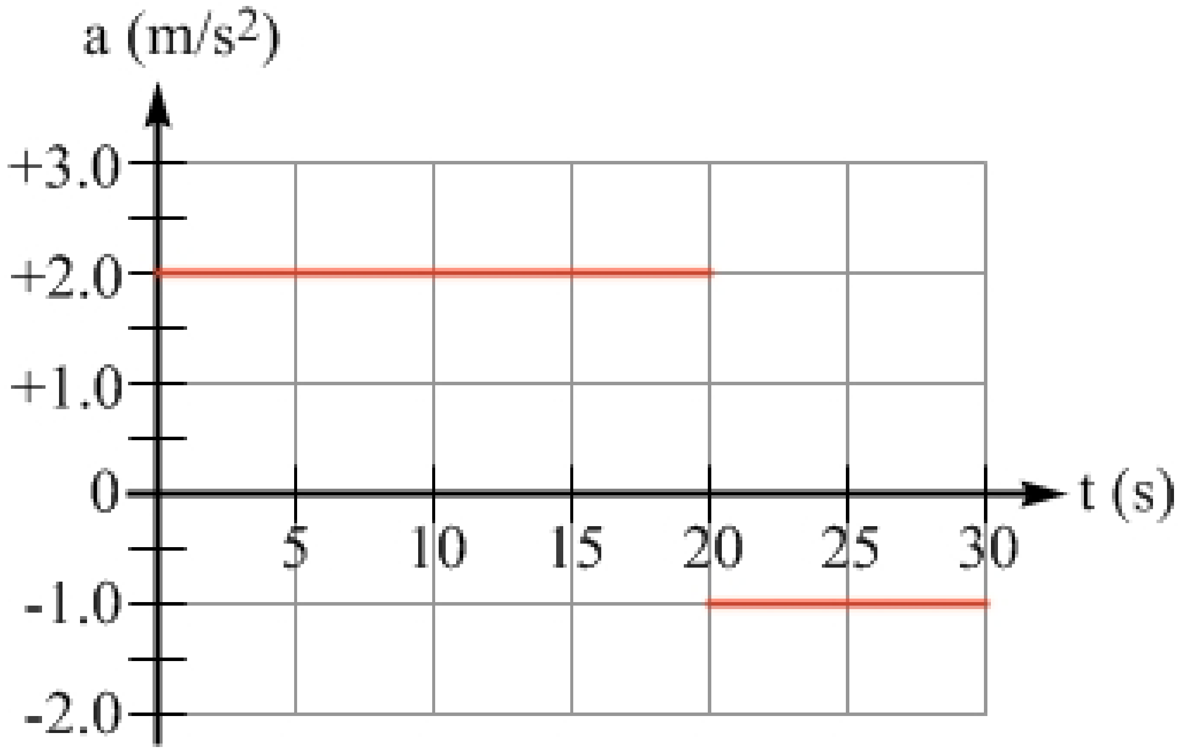
Review: Acceleration vs. time graph

The area under the acceleration versus time graph represents the change in velocity.



Acceleration to net force

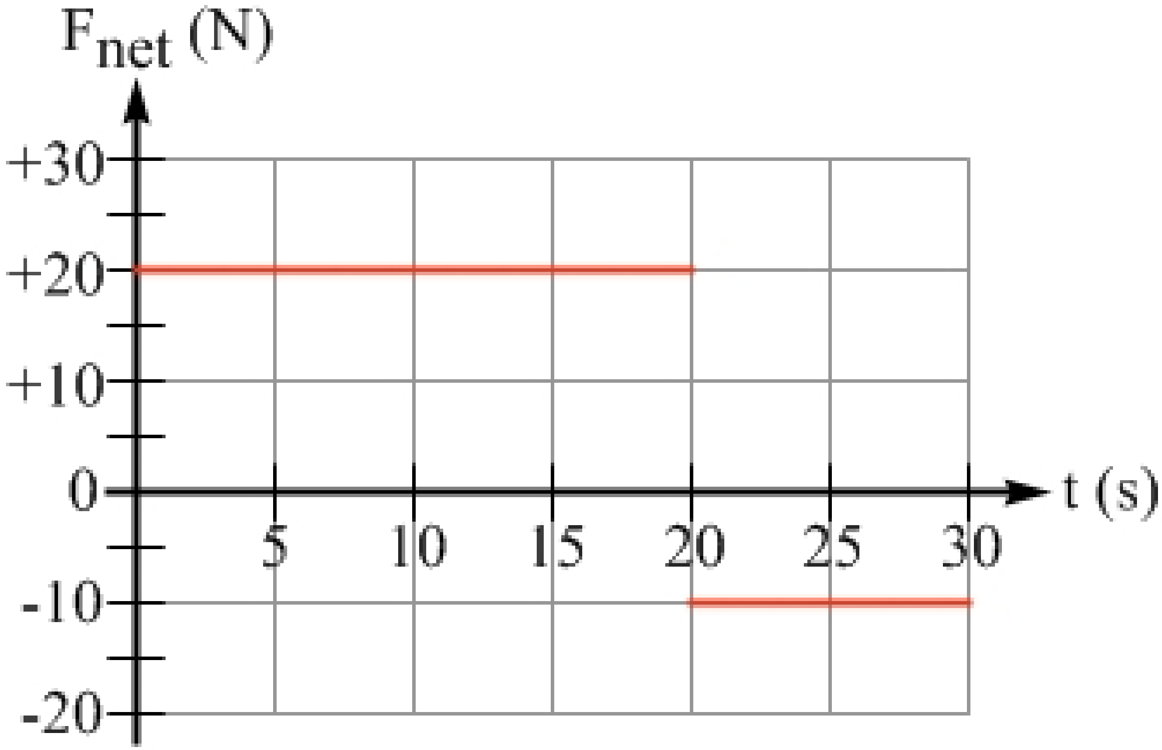
The object's mass is 10 kg. How do you convert the acceleration vs. time graph to a net force vs. time graph?



Acceleration to net force

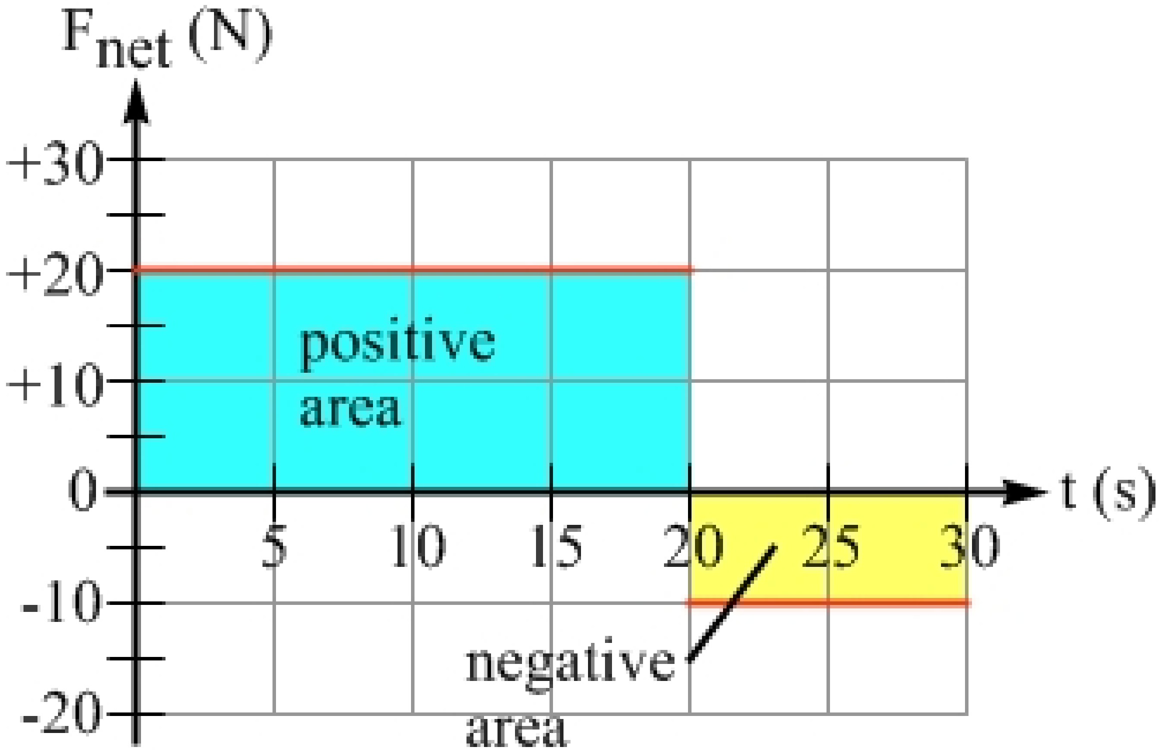
The object’s mass is 10 kg. How do you convert the acceleration vs. time graph to a net force vs. time graph?

Multiply the y-axis by the mass.



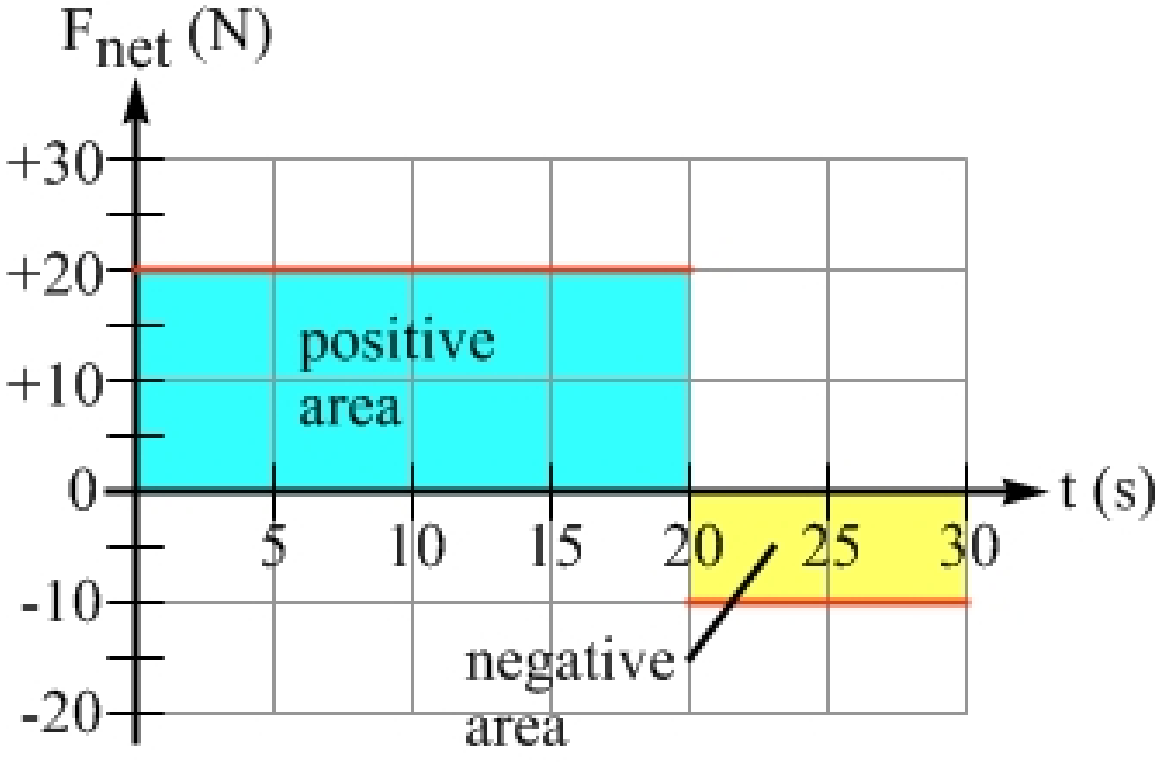
The net force vs. time graph

The area under the net force vs. time graph represents ____________ ?



The net force vs. time graph

The area under the net force vs. time graph represents the change in momentum (also known as the impulse).



Worksheet

Spend a few minutes on the first side of the worksheet.

Worksheet – the table

Time (s)	Momentum (kg m/s)
0	+2
1	+6
2	+4
3	+2
4	0
5	-2
6	-4
7	0
8	+4

Conservation

What do we mean when we say that a quantity is conserved?

Conservation

What do we mean when we say that a quantity is conserved?

The quantity has the same value at all times.

Momentum conservation

Momentum changes when a net force acts on a system. The longer the net force acts, the larger the change in momentum.

What happens when no net force acts on a system?

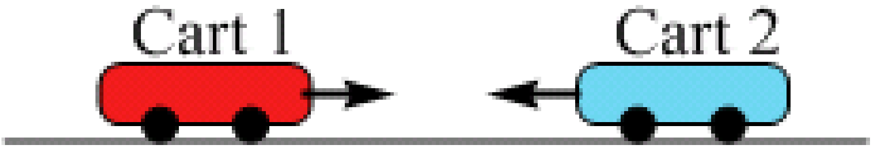
Momentum conservation

Momentum changes when a net force acts on a system. The longer the net force acts, the larger the change in momentum.

What happens when no net force acts on a system?

The momentum of the system is conserved.

Colliding carts



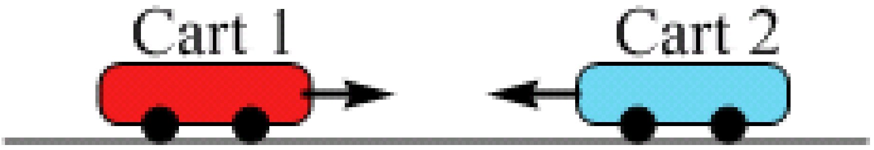
Two carts collide on a track.

Cart 1 has a mass of 0.5 kg and a velocity of 20 cm/s to the right.

Cart 2 has a mass of 0.5 kg and a velocity of 20 cm/s to the left.

What is the net momentum of the two-cart system?

Colliding carts



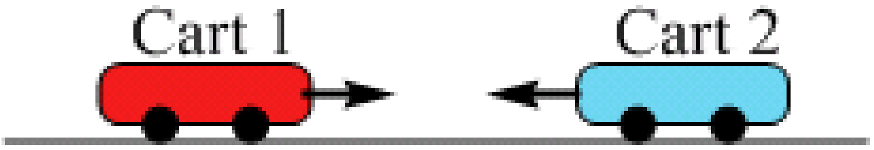
Two carts collide on a track.

Cart 1 has a mass of 0.5 kg and a velocity of 20 cm/s to the right.

Cart 2 has a mass of 0.5 kg and a velocity of 20 cm/s to the left.

What is the net momentum of the two-cart system?
Zero – momentum is a vector, so the momenta cancel.

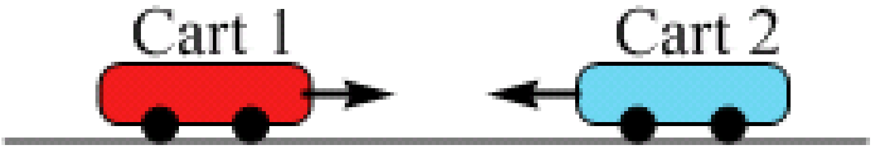
Colliding carts



No net external force acts on the two-cart system. Therefore, the momentum of the system should be conserved.

Is the momentum of cart 1 conserved in the collision?

Colliding carts



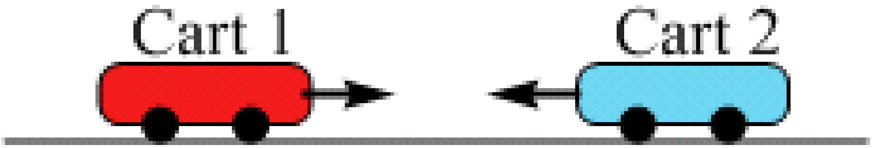
No net external force acts on the two-cart system. Therefore, the momentum of the system should be conserved.

Is the momentum of cart 1 conserved in the collision?

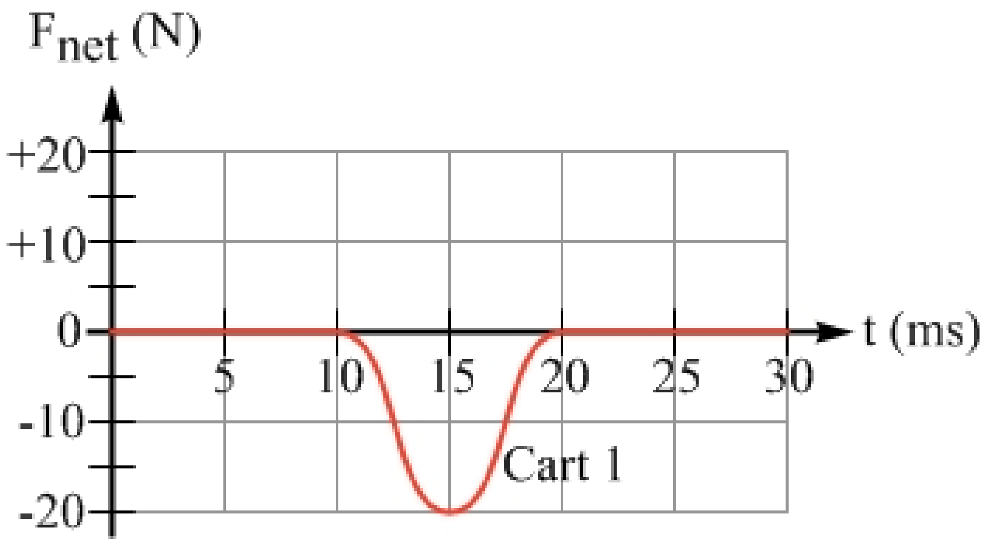
No, because cart 2 applies a force to cart 1 during the collision.

Similarly, the momentum of cart 2 also changes.

Colliding carts

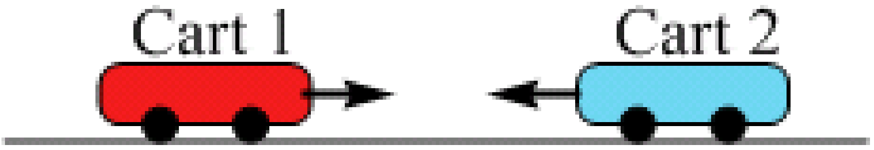


The change in momentum for cart 1 is the area under the net force vs. time graph for cart 1.

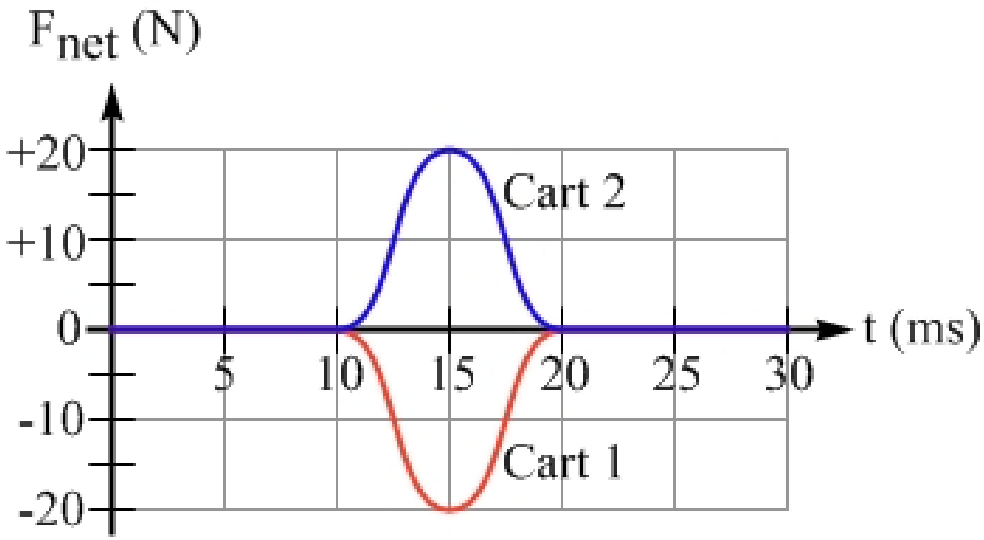


How does the area under the net force vs. time graph for cart 2 compare to that for cart 1?

Colliding carts



Newton's Third Law: The force applied by cart 1 on cart 2 is equal and opposite to the force applied by 2 on 1.



The two areas have the same magnitude, but opposite sign.

Momentum conservation

The fact that momentum is conserved for the two-cart system is a consequence of Newton’s Third Law. The areas under the two force vs. time graphs must always be equal in magnitude, but opposite in sign.

Cart 1’s change in momentum is always equal-and-opposite to cart 2’s change in magnitude.

The momentum of the two-cart system must be conserved – for all collisions in which the net external force is equal.

This also applies to cars colliding on the mean streets of Springfield, and football players, etc.

The happy ball and the sad ball

The happy ball is a bouncy rubber ball. The sad ball looks identical but stops dead on impact. The balls are rolled, one at a time, down a ramp toward a wooden block. Only one ball knocks the block over – which?

1. The happy ball.
2. The sad ball.

Analyze the collision

1. The happy ball

Momentum before the collision	Momentum after the collision
Happy ball: +mv	Happy ball: ___
Block: ___	Block: ___
System: ___	System: ___

Analyze the collision

1. The happy ball

Momentum before the collision	Momentum after the collision
Happy ball: +mv	Happy ball: ____
Block: 0	Block: ____
System: +mv	System: ____

Analyze the collision

1. The happy ball

Momentum before the collision	Momentum after the collision
Happy ball: +mv	Happy ball: -mv
Block: 0	Block: ____
System: +mv	System: +mv

Analyze the collision

1. The happy ball

Momentum before the collision	Momentum after the collision
Happy ball: +mv	Happy ball: -mv
Block: 0	Block: +2mv
System: +mv	System: +mv

Analyze the collision

2. The sad ball

Momentum before the collision	Momentum after the collision
Sad ball: +mv	Sad ball: ____
Block: 0	Block: ____
System: +mv	System: ____

Analyze the collision

2. The sad ball

Momentum before the collision	Momentum after the collision
Sad ball: +mv	Sad ball: 0
Block: 0	Block: ___
System: +mv	System: +mv

Analyze the collision

2. The sad ball

Momentum before the collision	Momentum after the collision
Sad ball: +mv	Sad ball: 0
Block: 0	Block: +mv
System: +mv	System: +mv

The result?

The happy ball transfers twice as much momentum to the block as the sad ball does.

It is critical to remember that momentum is a vector when applying momentum conservation.

Kinetic energy

Kinetic energy is energy associated with motion.

$$K = \frac{1}{2}mv^2$$

An explosion

A spring-loaded cart is placed back-to-back with another cart on a track. The carts are initially at rest. The spring is released, and the carts move off in opposite directions. Consider the momentum and kinetic energy of the system. What is conserved in this process?

1. Momentum but not kinetic energy.
2. Kinetic energy but not momentum.
3. Both momentum and kinetic energy.
4. Neither momentum nor kinetic energy.

An explosion

The system's momentum is conserved, but the system's kinetic energy is not conserved.

What is the system's momentum at all times?

An explosion

The system's momentum is conserved, but the system's kinetic energy is not conserved.

What is the system's momentum at all times?

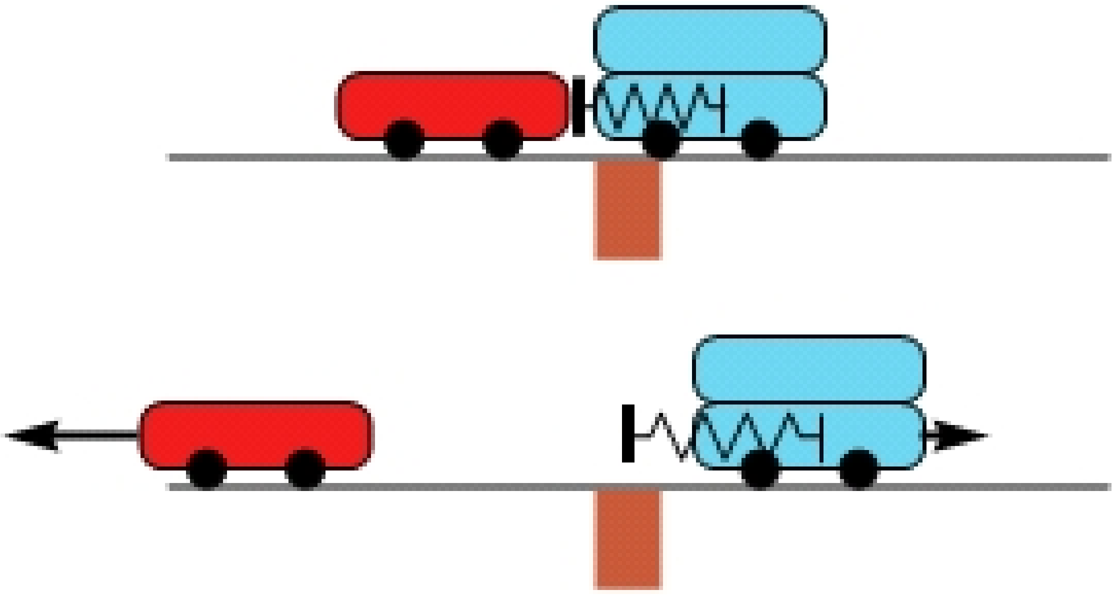
Zero.

Is there anything about the two-cart system that is always at rest, consistent with the system's momentum being zero all the time?

Center-of-mass

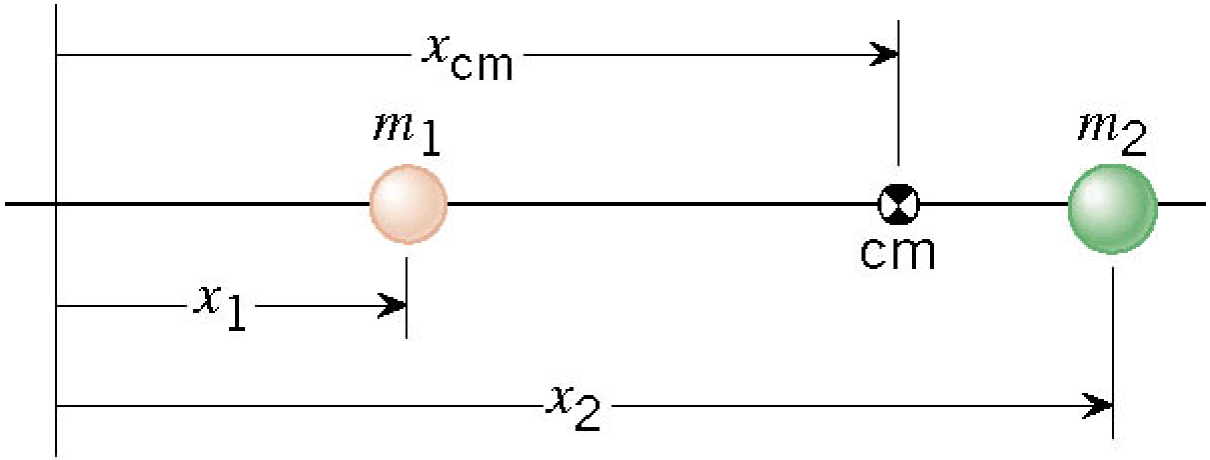
The system's center-of-mass remains at rest.

Let's investigate this further.



Center-of-mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

When we represented a complicated extended object, like a car of mass *m*, by a 'point', we chose the location of the point to be the center-of-mass. Simulation

Velocity of the center-of-mass

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

Apply Newton's Laws to the object, treated as a point mass located at the center-of-mass.

Velocity of the center-of-mass

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

When there is no net **external** force on the system, the center-of-mass has zero acceleration.

For stationary objects, in the absence of external forces:

One or more parts of the system can move, but other parts must move in response to keep the center-of-mass in the same place.

Internal forces cannot change the position of the center-of-mass.

Tossing an object

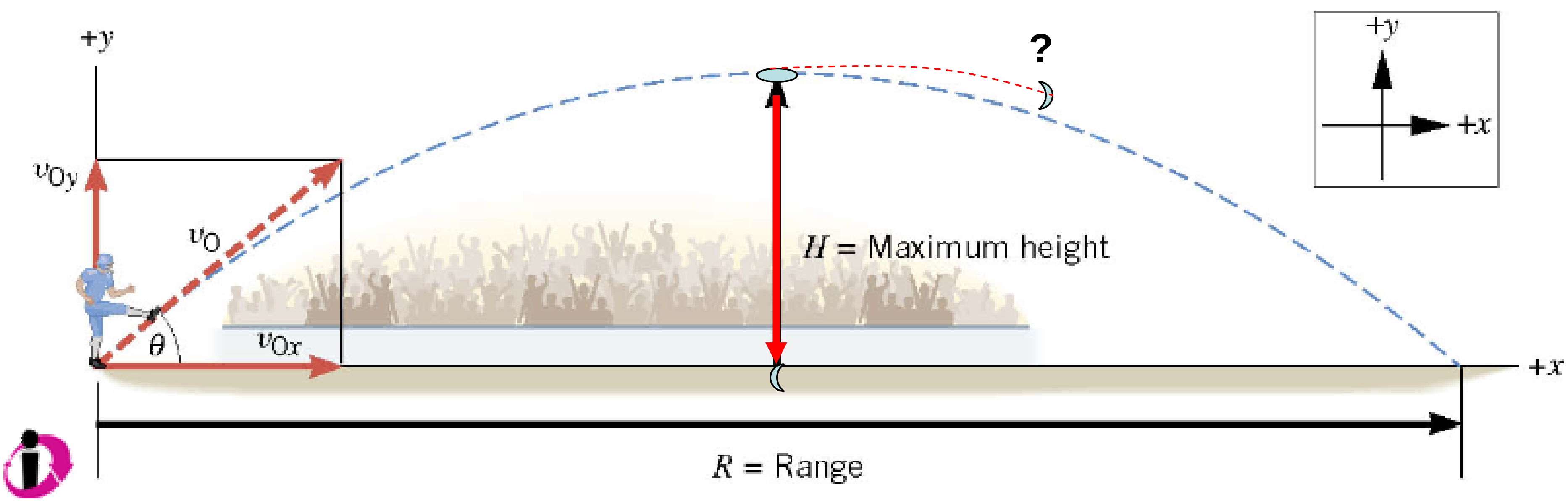
A baton is thrown at some angle to the horizontal. Describe the path followed by the baton's center-of-mass.

Baton simulation

An exploding football

A placekicker kicks a football that would otherwise travel a total distance of 50 m. At the top of the flight, the ball explodes into two equal pieces. One piece stops, and then falls straight down. Ignore air resistance. The distance from the kicker at which the second piece lands is:

(1) 0 m (2) 25 m (3) 50 m (4) 75 m (5) 100 m



Collisions

The two basic rules for analyzing collisions:

1. The momentum of the system is conserved in a collision.

2. Do not assume that the system's kinetic energy is conserved unless you have a good reason to do so.

Classifying collisions

We can classify collisions based on what happens to the kinetic energy.

Type of collision	**Kinetic energy**	
Super-elastic	More kinetic energy afterwards.	
Elastic	Kinetic energy is conserved.	
Inelastic	Less kinetic energy afterwards.	
Completely inelastic	Less kinetic energy, and the objects stick together.	

Elasticity

The elasticity of a collision is the ratio of the relative velocities of the two colliding objects after the collision to the relative velocity before the collision.

$$k = \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$$

Classifying collisions

We can classify collisions based on what happens to the kinetic energy, or in terms of the elasticity.

Type of collision	Kinetic energy	Elasticity
Super-elastic	More kinetic energy afterwards.	$k > 1$
Elastic	Kinetic energy is conserved.	$k = 1$
Inelastic	Less kinetic energy afterwards.	$0 < k < 1$
Completely inelastic	Less kinetic energy, and the objects stick together.	$k = 0$

Example collisions

Collision simulation

A completely inelastic collision

A cart with a mass *m* and a velocity of 2.0 m/s to the right collides with a stationary cart of mass 5*m*. The collision is completely inelastic (the carts stick together after the collision).

What is the velocity of the two-cart system after the collision?

A completely inelastic collision

A cart with a mass *m* and a velocity of 2.0 m/s to the right collides with a stationary cart of mass 5*m*. The collision is completely inelastic (the carts stick together after the collision).

What is the velocity of the two-cart system after the collision?

Apply momentum conservation.

Momentum before the collision = momentum afterwards

$$m(+2.0 \text{ m/s}) + 0 = 6m(v_f)$$

$$v_f = \frac{m(+2.0 \text{ m/s})}{6m} = \frac{+2.0 \text{ m/s}}{6} = +0.33 \text{ m/s}$$

A completely elastic collision

A cart with a mass *m* and a velocity of 2.0 m/s to the right collides with a stationary cart of mass 5*m*. The collision is completely elastic (kinetic energy is conserved).

What is the velocity of each cart after the collision?

A completely elastic collision

A cart with a mass *m* and a velocity of 2.0 m/s to the right collides with a stationary cart of mass 5*m*. The collision is completely elastic (kinetic energy is conserved).

What is the velocity of each cart after the collision?

Momentum before the collision = momentum afterwards

$$m(+2.0 \text{ m/s}) + 0 = m(v_{1f}) + 5m(v_{2f})$$

A completely elastic collision

A cart with a mass *m* and a velocity of 2.0 m/s to the right collides with a stationary cart of mass 5*m*. The collision is completely elastic (kinetic energy is conserved).

What is the velocity of each cart after the collision?

Momentum before the collision = momentum afterwards

$$m(+2.0\ \text{m/s}) + 0 = m(v_{1f}) + 5m(v_{2f})$$

Kinetic energy beforehand = kinetic energy afterwards

$$\frac{1}{2}m(+2.0\ \text{m/s})^2 + 0 = \frac{1}{2}m(v_{1f})^2 + \frac{1}{2}5m(v_{2f})^2$$

A completely elastic collision

An alternate method.

Momentum before the collision = momentum afterwards

$$m(+2.0\ \text{m/s}) + 0 = m(v_{1f}) + 5m(v_{2f})$$

Elasticity $$+2.0\ \text{m/s} = v_{2f} - v_{1f}$$

A completely elastic collision

An alternate method.

Momentum before the collision = momentum afterwards

$$m(+2.0\ \text{m/s}) + 0 = m(v_{1f}) + 5m(v_{2f})$$

Elasticity $$+2.0\ \text{m/s} = v_{2f} - v_{1f}$$

Momentum $$+2.0\ \text{m/s} = v_{1f} + 5v_{2f}$$

A completely elastic collision

An alternate method.

Momentum before the collision = momentum afterwards

$$m(+2.0\ \text{m/s}) + 0 = m(v_{1f}) + 5m(v_{2f})$$

Elasticity $$+2.0\ \text{m/s} = v_{2f} - v_{1f}$$

Momentum $$+2.0\ \text{m/s} = v_{1f} + 5v_{2f}$$

Add $$+4.0\ \text{m/s} = 6v_{2f}$$

A completely elastic collision

An alternate method.

Momentum before the collision = momentum afterwards

$$m(+2.0\ \text{m/s}) + 0 = m(v_{1f}) + 5m(v_{2f})$$

Elasticity $\quad +2.0\ \text{m/s} = v_{2f} - v_{1f}$

Momentum $\quad +2.0\ \text{m/s} = v_{1f} + 5v_{2f}$

Add $\quad +4.0\ \text{m/s} = 6v_{2f}$

Solve $\quad v_{2f} = 0.67\ \text{m/s} \quad$ and $\quad v_{1f} = -1.33\ \text{m/s}$

Evil twin

You are driving at high speed when you see your evil twin, driving an identical car, coming directly toward you. You both slam on your brakes, but it's too late to stop and there is about to be a collision. At the last instant, you spot a very solid immovable object by the side of the road. Which is better for you, to hit your evil twin or to swerve and hit the immovable object instead?

Assume the speed when you collide is the same whether you hit your evil twin or the immovable object, and that your evil twin is going at the same speed you are. Either collision is a head-on collision.

It is better for you (and just consider your outcome) if you

1. Hit the immovable object
2. Hit your evil twin
3. It doesn’t matter, they’re equivalent

Comparisons

Impulse: $\Delta\vec{p} = \vec{F}_{net}\,\Delta t$ (a vector equation)

The change in momentum is the net force multiplied by the time interval over which the net force acts.

Impulse is the area under the net force vs. time graph.

Net work: $\Delta K = \left(F_{net}\cos\theta\right)\Delta r$ (a scalar equation)

The change in kinetic energy is the component of the net force acting along the displacement, multiplied by the displacement over which the net force acts.

Net work is the area under the net force vs. position graph.

Net work

An object's change in kinetic energy is equal to the net work.

The work-kinetic energy theorem: $\Delta K = F_{net} \Delta r \cos\theta$

The net work is:

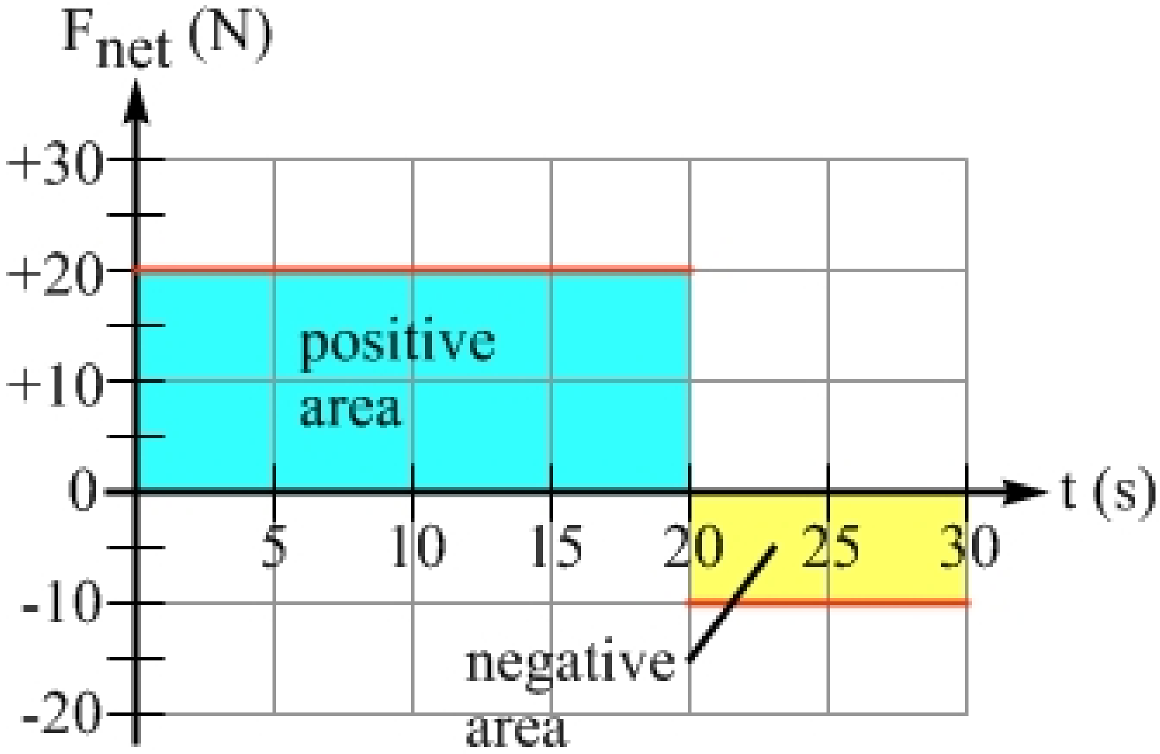
• zero when the net force is perpendicular to the displacement.

• positive when the net force has a component in the direction of the displacement.

• negative when the net force has a component opposite to the direction of the displacement.

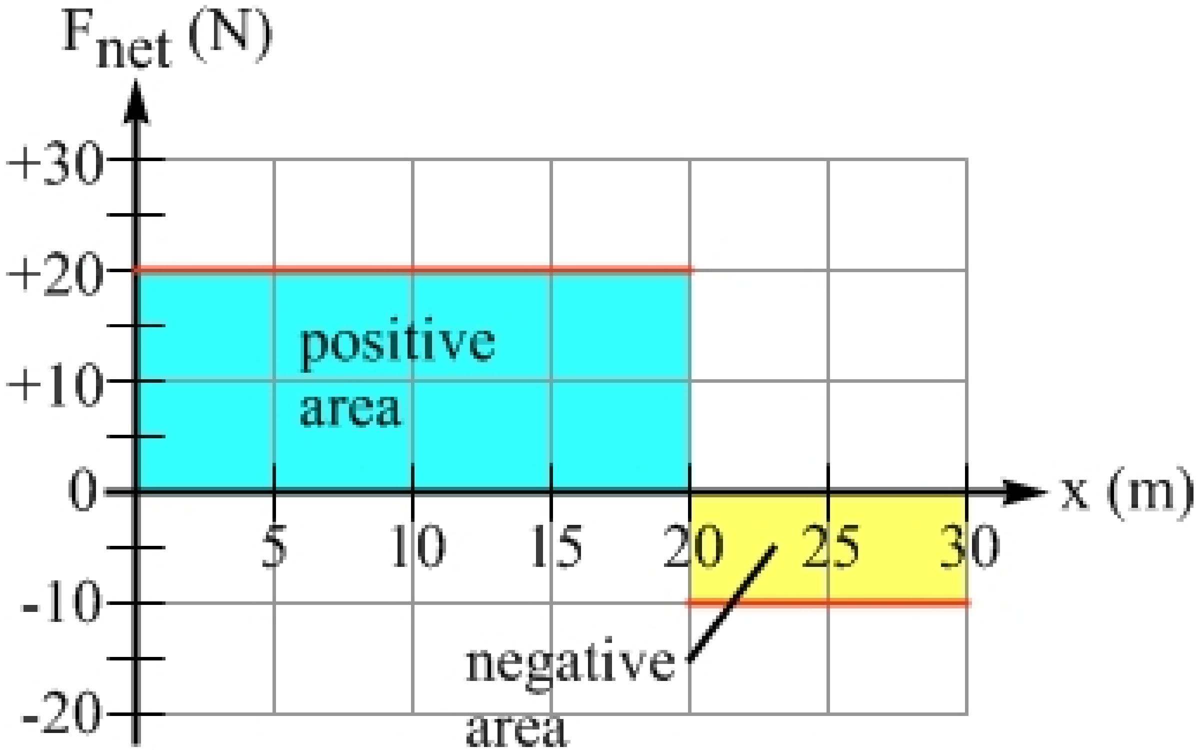
The net force vs. time graph

The area under the net force vs. time graph represents the change in momentum (also known as the impulse).



The net force vs. position graph

The area under the net force vs. position graph represents the change in kinetic energy (also known as the net work).



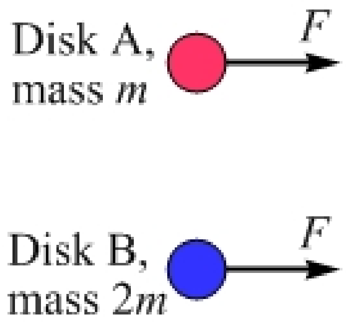
Worksheet

Spend a few minutes on the first side of the worksheet.

Worksheet – the table

Position (m)	Kinetic energy (J)
0	+4
1	+6
2	+8
3	+4
4	0
5	-4
6	-8
7	-6
8	-4

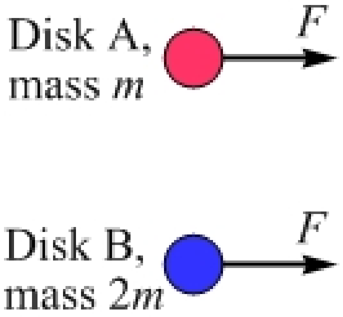
Two disks



Two disks are initially at rest. The mass of disk B is two times larger than that of disk A. The two disks then experience equal net forces *F*. These net forces are applied for the same amount of time. After the net forces are removed:

1. The disks have the same momentum and kinetic energy.
2. The disks have equal momentum; disk A has more kinetic energy.
3. The disks have equal momentum; disk B has more kinetic energy.
4. The disks have equal kinetic energy; disk A has more momentum.
5. The disks have equal kinetic energy; disk B has more momentum.

Two disks, scenario 2



Two disks are initially at rest. The mass of disk B is two times larger than that of disk A. The two disks then experience equal net forces *F*. These net forces are applied over equal displacements. After the net forces are removed:

1. The disks have the same momentum and kinetic energy.
2. The disks have equal momentum; disk A has more kinetic energy.
3. The disks have equal momentum; disk B has more kinetic energy.
4. The disks have equal kinetic energy; disk A has more momentum.
5. The disks have equal kinetic energy; disk B has more momentum.

Work done by individual forces

We can determine the work done by a particular force.

$$W = F\Delta r \cos\theta$$

The net work done on an object is the sum of the work done by each of the individual forces acting on the object.

Work, example 1

You hold an object weighing 10 N so that it is at rest. How much work do you do on the object? The work you do is:

1. Zero.
2. Positive.
3. Negative.

Work, example 2

You raise the 10 N object 0.5 m vertically. The object starts and ends at rest. How much work do you do on the object?

1. 5 J
2. More than 5 J
3. Less than 5 J

Work, example 3

You move the 10 N object 2 m horizontally. The object starts and ends with the same speed.

How much work do you do on the object?

1. 0
2. 20 J
3. some positive value, but not 20 J
4. some negative value

Work, example 4

A car traveling at an initial speed v on a flat road comes to a stop in a distance L once the brakes are applied. Assuming the friction force that brings the car to rest does not change, how far does the car travel once the brakes are applied if the initial speed is $2v$?

1. $\sqrt{2}\,L$
2. $2L$
3. $3L$
4. $4L$

Tossing a ball

Let's apply the work - kinetic energy relation to a ball thrown straight up from an initial height of *y* = 0 that reaches a maximum height *y* = *h* before falling back down to *y* = 0.

At maximum height, the velocity is zero.

For the up part of the trip, we get:

$$W_{net} = F_{net} \Delta y \cos\theta = \Delta K$$

$$-mgh = 0 - \frac{1}{2} m v_0^2$$

This gives $mgh = \frac{1}{2} m v_0^2$

On the way down, the force of gravity does positive work, so the kinetic energy increases.

Tossing a ball

mgh represents some kind of energy.

On the way up, the kinetic energy is transformed to *mgh*, and on the way down, the *mgh* energy is transformed back to kinetic energy.

What kind of energy is this *mgh* energy?

Tossing a ball

mgh represents some kind of energy.

On the way up, the kinetic energy is transformed to *mgh*, and on the way down, the *mgh* energy is transformed back to kinetic energy.

What kind of energy is this *mgh* energy?

Gravitational potential energy. Kinetic energy is energy associated with motion. Potential energy is energy associated with position.

We can talk about the work done by gravity or, equivalently, we can use gravitational potential energy.