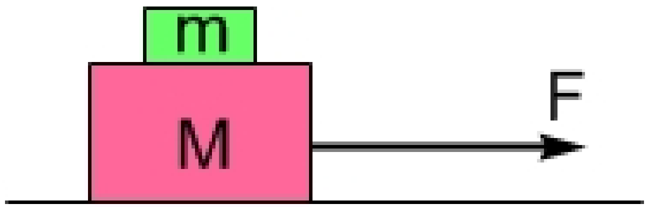
A general method for solving problems involving forces

- Draw a diagram of the situation.
- Draw one or more free-body diagrams showing all the forces acting on the object(s).
- Choose a coordinate system. It is often most convenient to align one of your coordinate axes with the direction of the acceleration.
- Break the forces up into their *x* and *y* components.
- Apply Newton's Second Law in (usually) both directions.

Two boxes (see the worksheet)

A small box of mass m = 1 kg sits on top of a large box of mass M = 2 kg on a flat table. The coefficients of friction between all surfaces in contact are:

$\mu_K = 0.40$ and $\mu_S = 0.50$

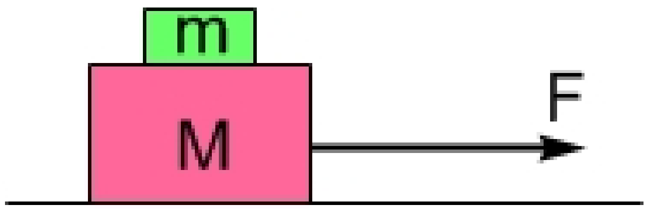


Use $g = 10$ m/s^2

A string of negligible mass is tied to the large box and a horizontal force F = 21 N is applied to the string so that the two boxes accelerate together to the right, with the small box maintaining its position on top of the large box at all times.

Free-body diagram for the two-box system

Treat the two boxes as one system, and sketch the free-body diagram of the system.



Sketch the free-body diagram of the small box.

Sketch the free-body diagram of the large box.

How many forces?

How many forces should be included on the free-body diagram of the large box?

Press the number corresponding to the number of forces you think should be shown on the box.

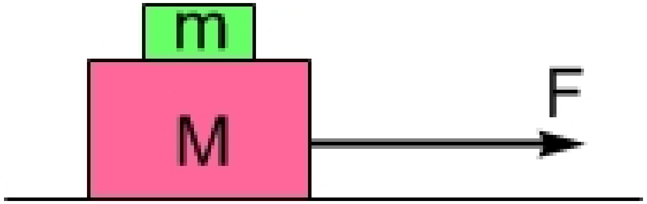
Doing the calculations

Let's say that M = 2.0 kg and m = 1.0 kg, and we'll use the approximation that g = 10 m/s^2 to simplify the calculations.

The coefficients of friction between all surfaces in contact are:

$\mu_K = 0.40$ and $\mu_S = 0.50$

If the force applied to the string is F = 21 N, the boxes accelerate together to the right with the small box maintaining its position on top of the large box. What is the acceleration of the system?



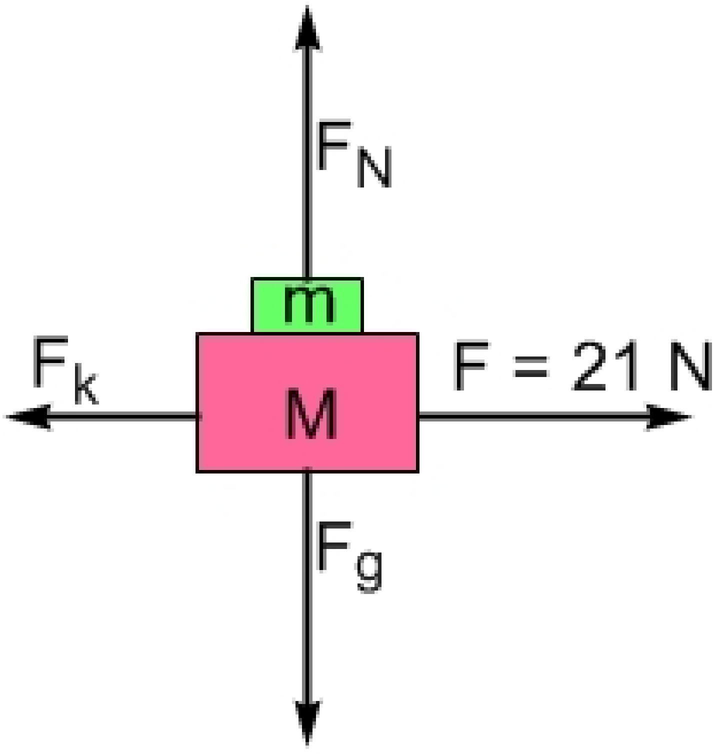
Apply Newton's Second Law

To what?

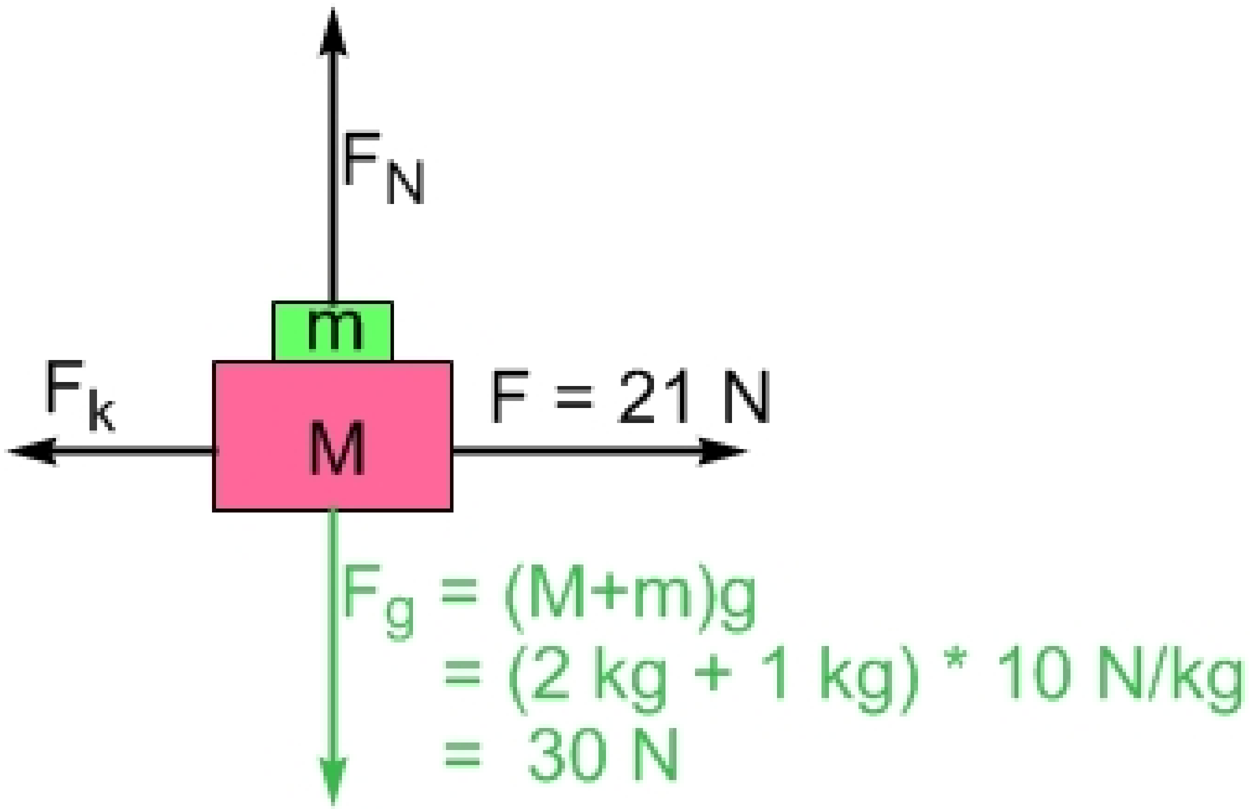
Apply Newton's Second Law

To what?

The two-box system.



Find the force of gravity



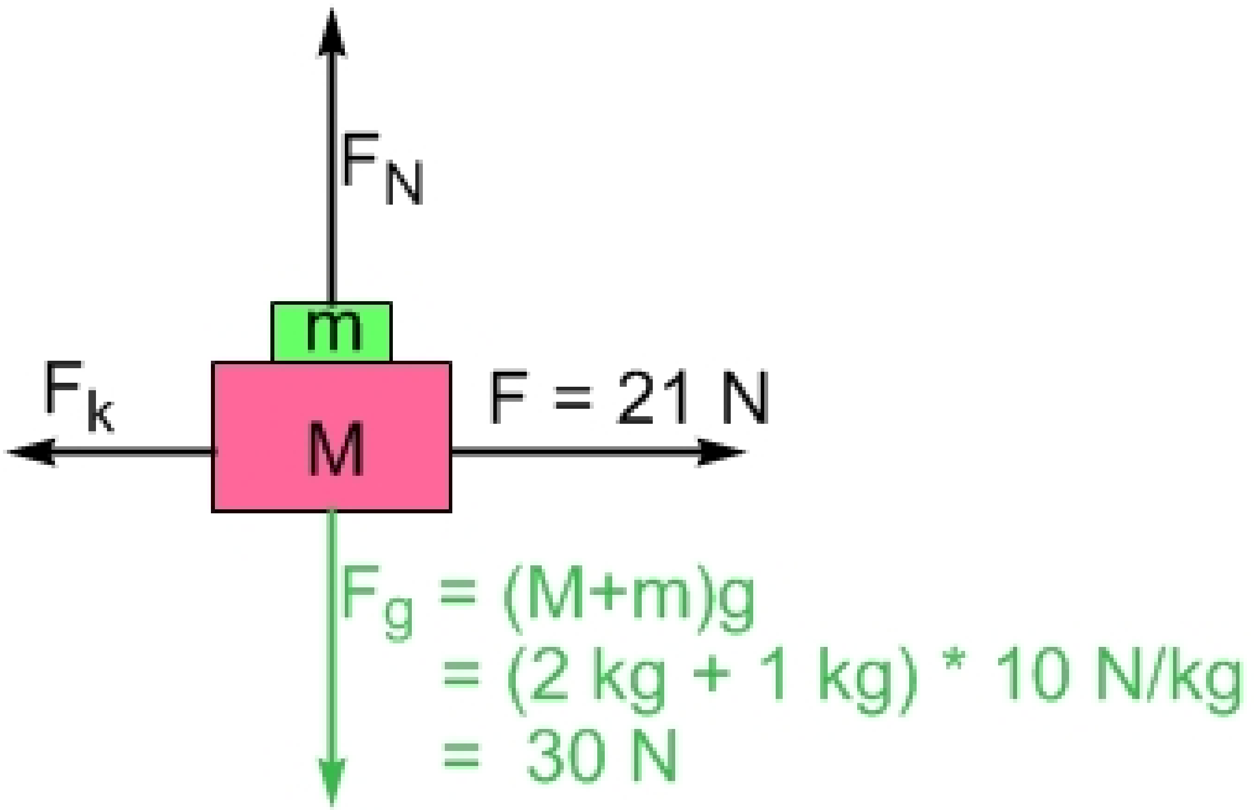
Apply Newton's Second Law

x-direction

$$\sum \vec{F}_x = (m + M)\vec{a}_x$$

y-direction

$$\sum \vec{F}_y = (m + M)\vec{a}_y$$



Apply Newton's Second Law

x-direction

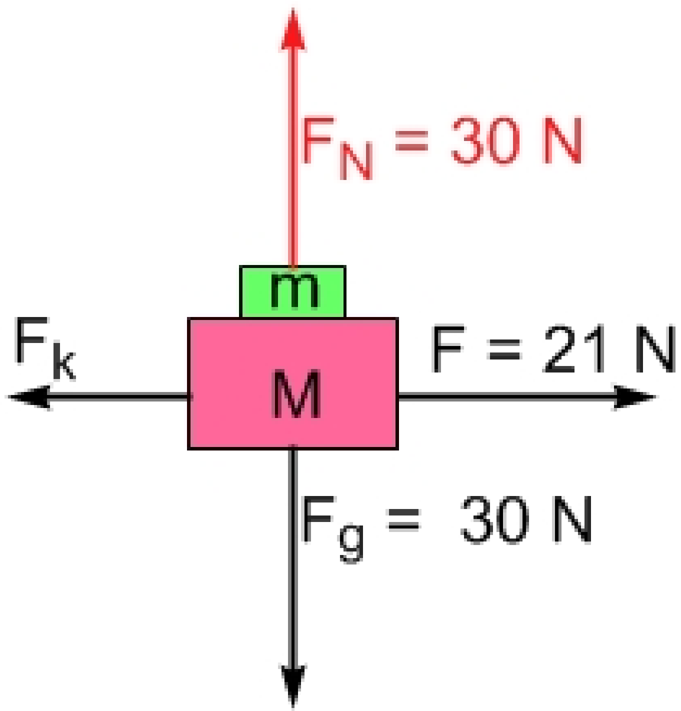
$$\sum \vec{F}_x = (m+M)\vec{a}_x$$

y-direction

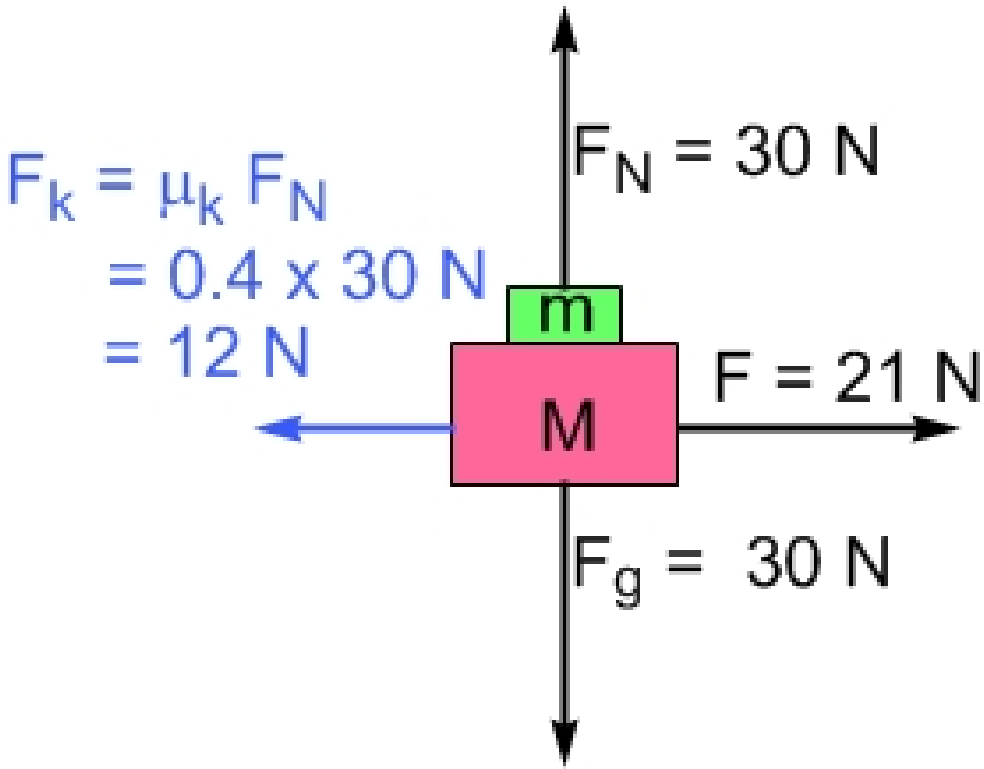
$$\sum \vec{F}_y = (m+M)\vec{a}_y$$

$$+F_N - F_g = 0$$

$$F_N = F_g = 30\ \mathrm{N}$$



Find the force of kinetic friction



Apply Newton's Second Law

x-direction

$$\sum \vec{F}_x = (m+M)\vec{a}_x$$

$$+F - F_K = (m+M)\vec{a}_x$$

$$\vec{a}_x = \frac{+F - F_K}{m+M}$$

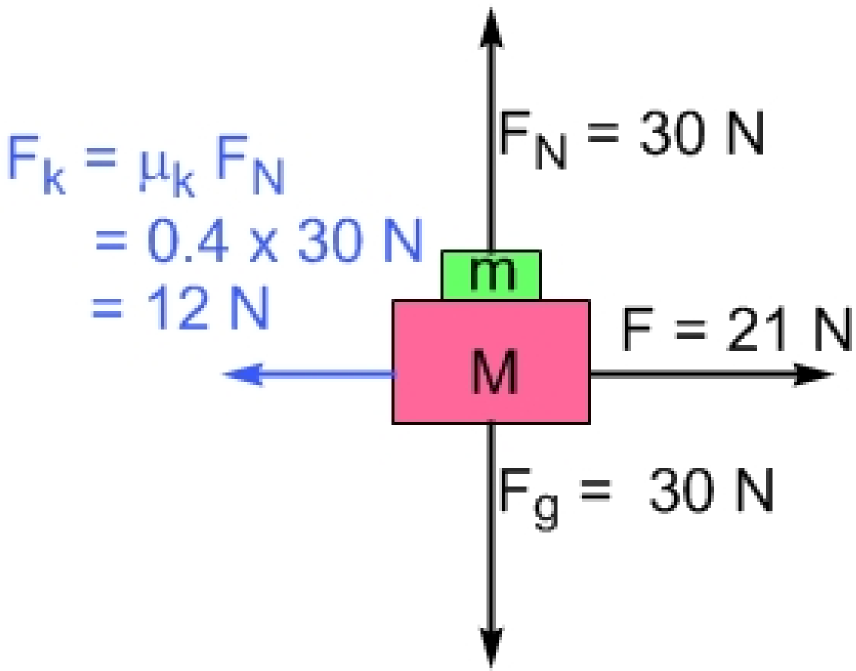
$$\vec{a}_x = \frac{+21\text{ N} - 12\text{ N}}{m+M}$$

$$\vec{a}_x = \frac{+9.0\text{ N}}{3.0\text{ kg}} = +3.0\text{ m/s}^2$$

y-direction

$$\sum \vec{F}_y = (m+M)\vec{a}_y$$

$$F_N = F_g = 30\text{ N}$$



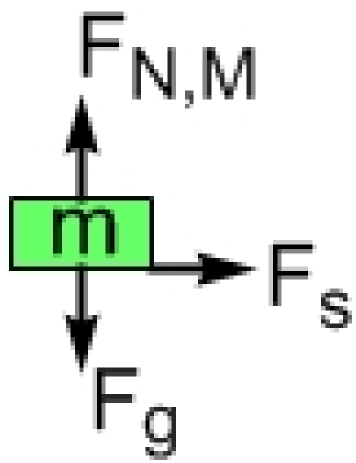
Force of friction on the small box?

Find the magnitude of the force of friction acting on the small box in this situation.

It may or may not help to know that the normal force applied by the large box on the small box has a magnitude of 10 N.

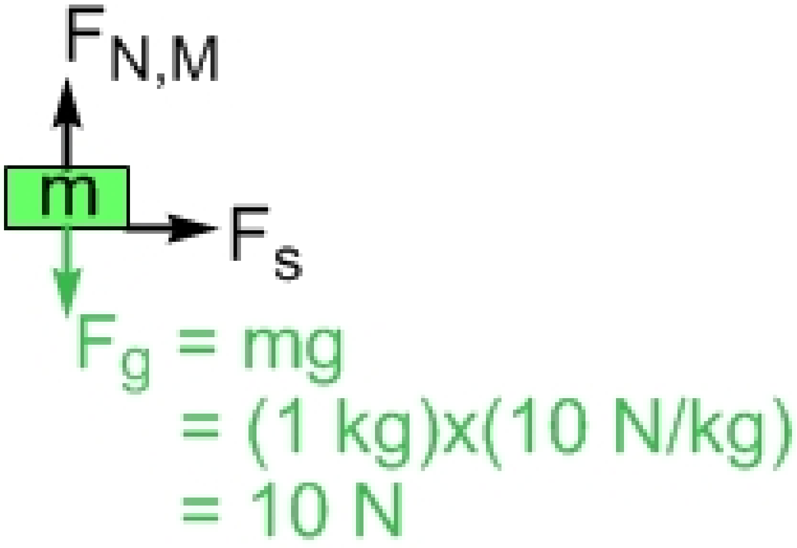
Free-body diagram of the small box

Find the magnitude of the force of friction acting on the small box in this situation. It may or may not help to know that the normal force applied by the large box on the small box has a magnitude of 10 N.



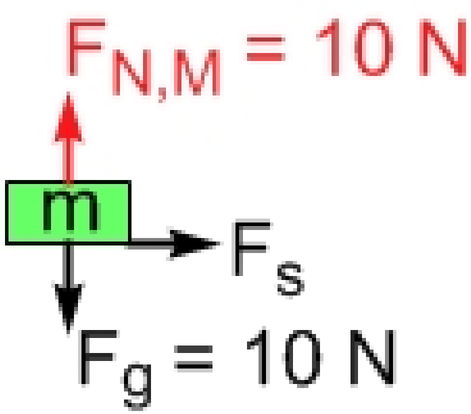
Free-body diagram of the small box

Find the magnitude of the force of friction acting on the small box in this situation. It may or may not help to know that the normal force applied by the large box on the small box has a magnitude of 10 N.



Free-body diagram of the small box

Find the magnitude of the force of friction acting on the small box in this situation. It may or may not help to know that the normal force applied by the large box on the small box has a magnitude of 10 N.



$$\sum \vec{F}_y = m\vec{a}_y = 0$$

$$+F_{N,M} - F_g = 0$$

$$F_{N,M} = F_g = 10\ \text{N}$$

The force of friction on the small box

The magnitude of the force of friction acting on the small box is ________ N.

Press the number that is equal to the number of newtons you think the force is.

Apply Newton's Second Law

Remember that this force of friction is a static force.

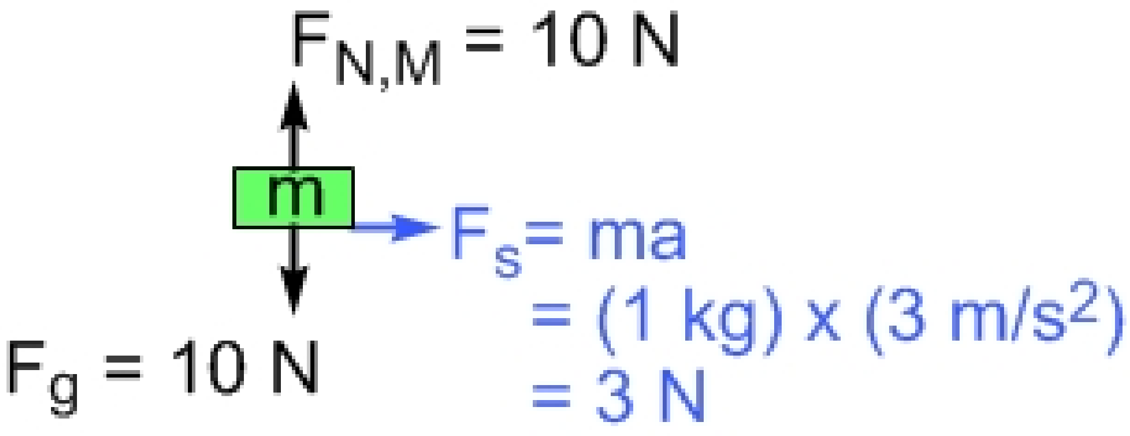
$$\sum \vec{F}_x = m\vec{a}_x$$

Apply Newton's Second Law

Remember that this force of friction is a static force.

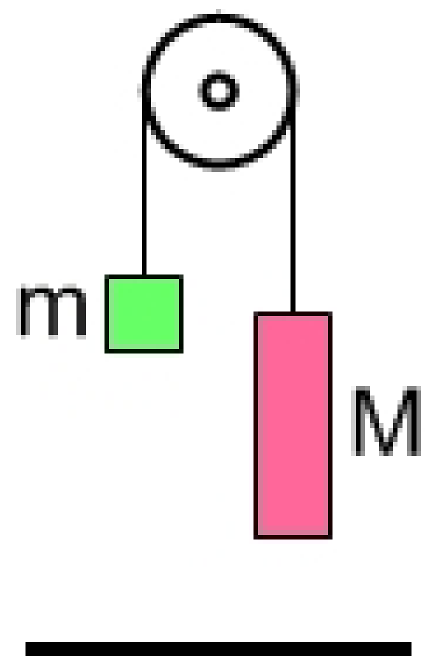
$$\sum \vec{F}_x = m\vec{a}_x$$

$$+F_S = m\vec{a}_x = (1.0\ \text{kg}) \times (3.0\ \text{m/s}^2) = 3.0\ \text{N}$$



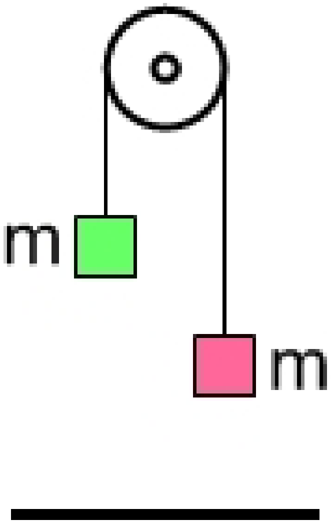
Atwood’s machine

Atwood’s machine involves one pulley, and two objects connected by a string that passes over the pulley. In general, the two objects have different masses.



Offset Atwood's machine

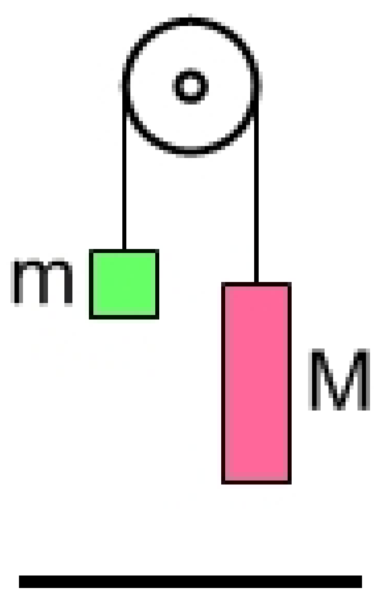
What if we hang objects of equal mass on the Atwood's machine, but when we release the system from rest one object is higher than the other? Neglect the mass of the string.



1. The object that is higher moves down; the other moves up.
2. The object that is higher moves up; the other moves down.
3. Nothing happens.

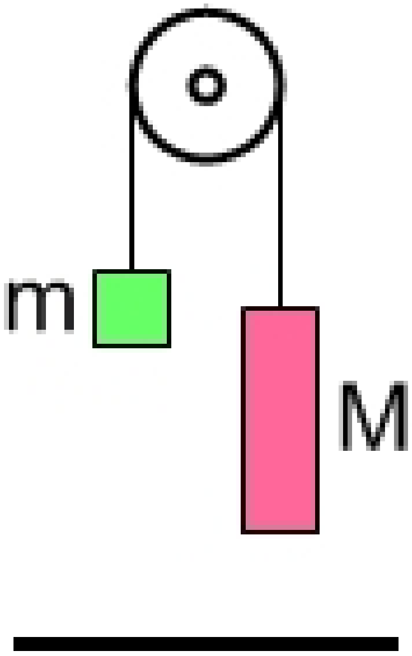
Dealing with pulleys

For now, we will assume that the pulley is frictionless and massless. We will do a more realistic analysis later in the course. The main implication of our assumption is that the tension in the string is the same on both sides of the pulley. The pulley simply re-directs the force of tension.



Unbalanced Atwood's machine

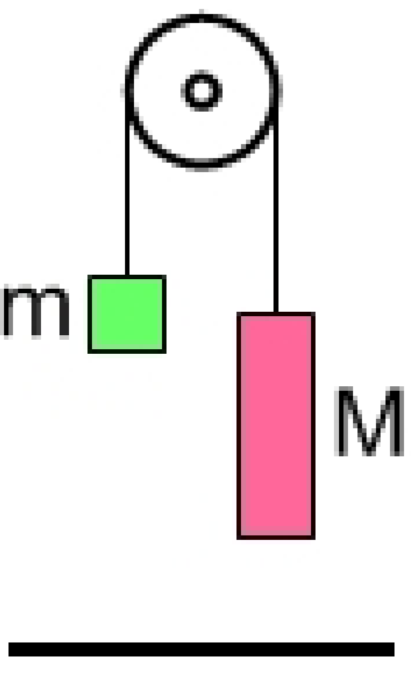
We hang objects of different mass on the Atwood's machine. When we release the system from rest, which object will have a larger acceleration? Consider the magnitude of the acceleration only.



1. The larger-mass object.
2. The smaller-mass object.
3. Neither – the accelerations have the same magnitude.

Analyzing the lighter object

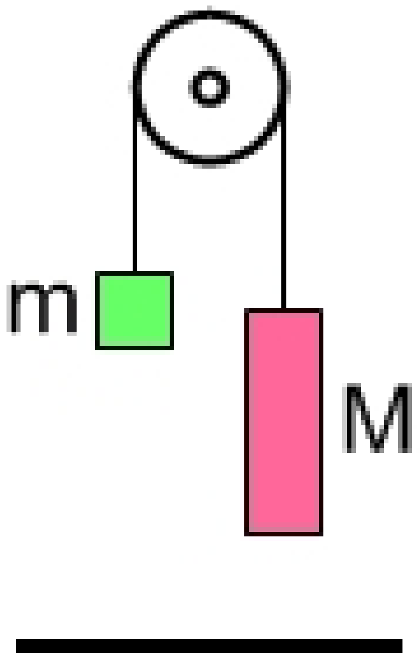
Sketch a free-body diagram for the lighter object.



Analyzing the lighter object

Sketch a free-body diagram for the lighter object.

Choose a positive direction, and apply Newton’s Second Law.



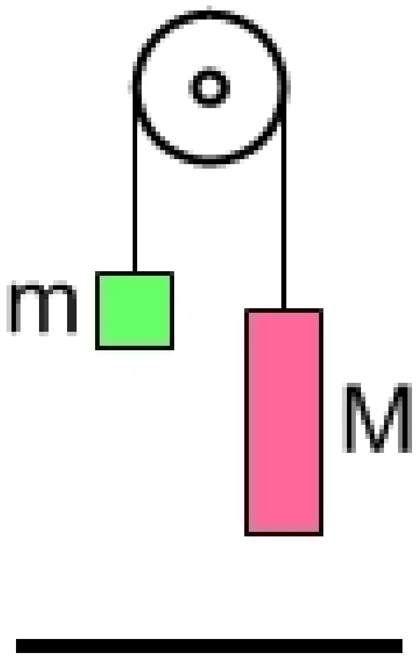
Analyzing the lighter object

Sketch a free-body diagram for the lighter object.

Choose a positive direction, and apply Newton's Second Law. Let's choose positive to be up, in the direction of the acceleration.

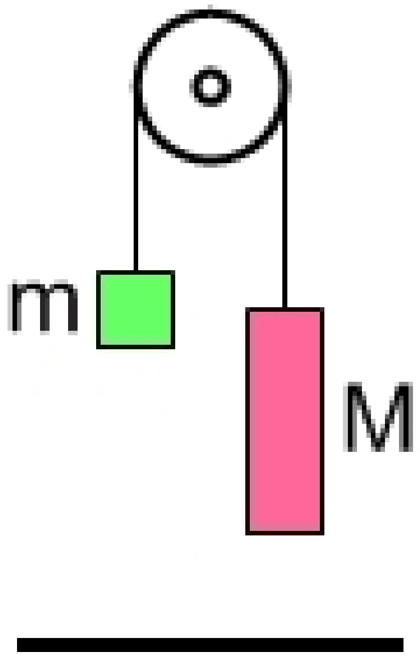
$$\sum \vec{F} = m\vec{a}$$

$$+F_T - mg = +ma$$



Analyzing the heavier object

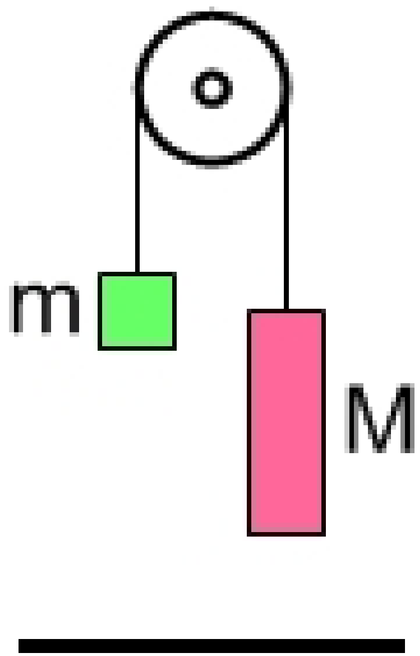
Sketch a free-body diagram for the heavier object.



Analyzing the heavier object

Sketch a free-body diagram for the heavier object.

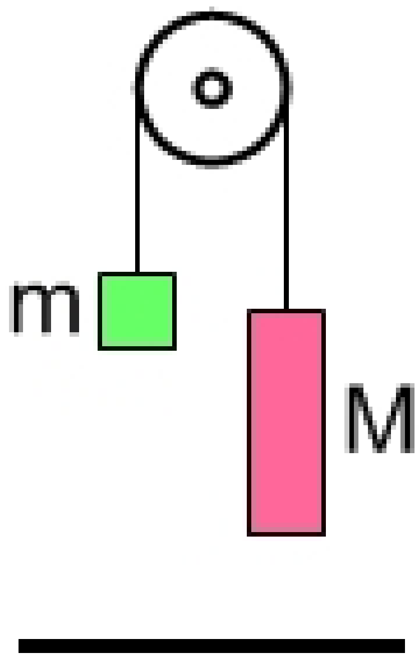
Choose a positive direction, and apply Newton's Second Law.



Analyzing the heavier object

Sketch a free-body diagram for the heavier object.

Choose a positive direction, and apply Newton's Second Law. Choose positive down this time, to match the object's acceleration.



$$\sum \vec{F} = M\vec{a}$$

$$+Mg - F_T = +Ma$$

Combine the equations

Lighter object: $+F_T - mg = +ma$

Heavier object: $+Mg - F_T = +Ma$

Combine the equations

Lighter object: $+F_T - mg = +ma$

Heavier object: $+Mg - F_T = +Ma$

Add the equations: $+Mg - mg = +Ma + ma$

$$+Mg - mg = (M + m)a$$

$$a = \frac{Mg - mg}{M + m}$$

Check your answer

$$a = \frac{Mg - mg}{M + m}$$

What happens if $M = m$?

What happens if $M >> m$?

What happens if $M << m$?

If the answers are reasonable, we can be confident in our result.

Check your answer

$$a = \frac{Mg - mg}{M + m}$$

What happens if $M = m$? $a = 0$

What happens if $M >> m$?

What happens if $M << m$?

If the answers are reasonable, we can be confident in our result.

Check your answer

$$a = \frac{Mg - mg}{M + m}$$

What happens if $M = m$? $a = 0$

What happens if $M >> m$? $a = +g$

What happens if $M << m$?

If the answers are reasonable, we can be confident in our result.

Check your answer

$$a = \frac{Mg - mg}{M + m}$$

What happens if $M = m$? $a = 0$

What happens if $M >> m$? $a = +g$

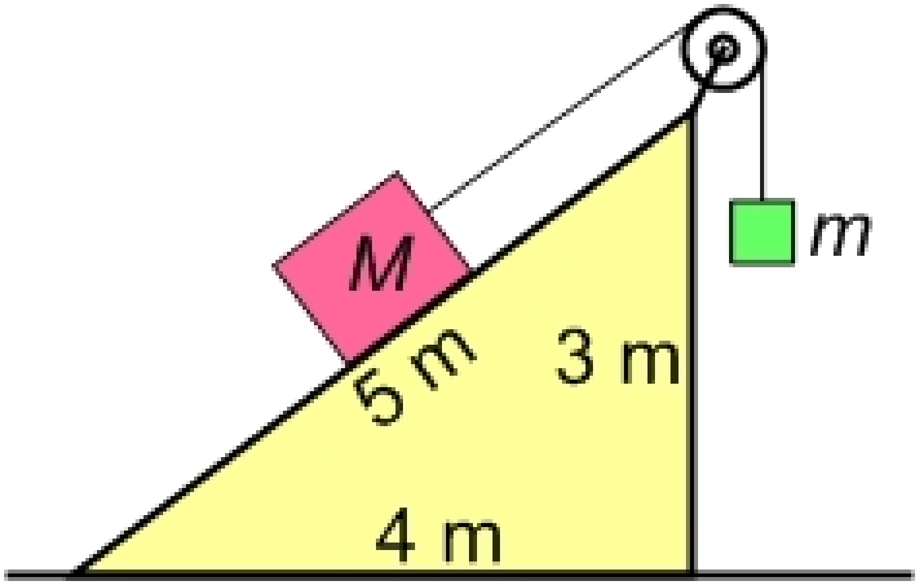
What happens if $M << m$? $a = -g$

If the answers are reasonable, we can be confident in our result.

Inclined Atwood's machine

For this problem, let's assume $g = 10\ m/s^2$.

A red block with a mass $M = 10$ kg is placed on a ramp that has the shape of the 3-4-5 triangle, with a height of 3.0 m and a width of 4.0 m. The red block is connected to a green box of mass m by a string that passes over a pulley at the top of the incline.



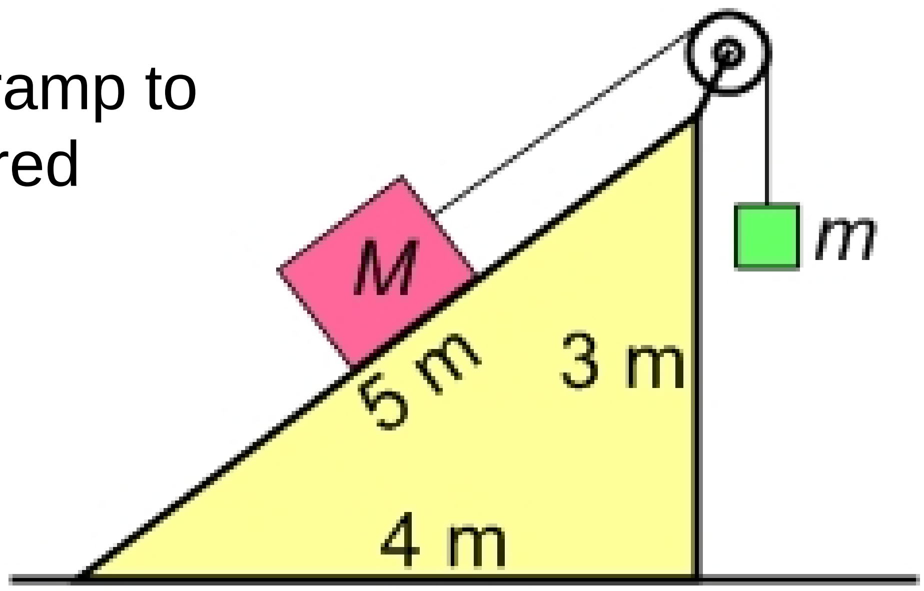
Inclined Atwood's machine

g = 10 m/s^2 and M = 10 kg.

Initially, let's assume the ramp is frictionless.

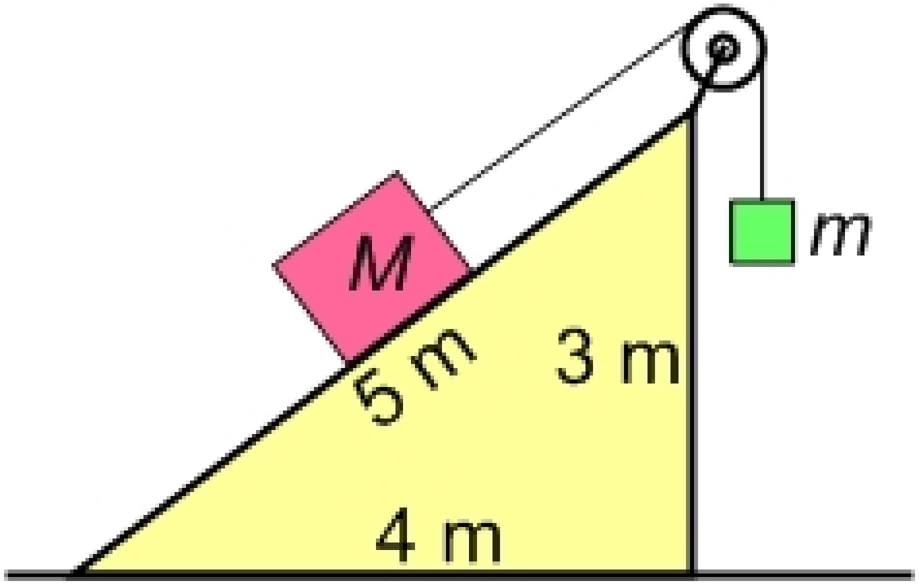
What does m need to be for the system to remain at rest?

Choose down the ramp to be positive for the red block.



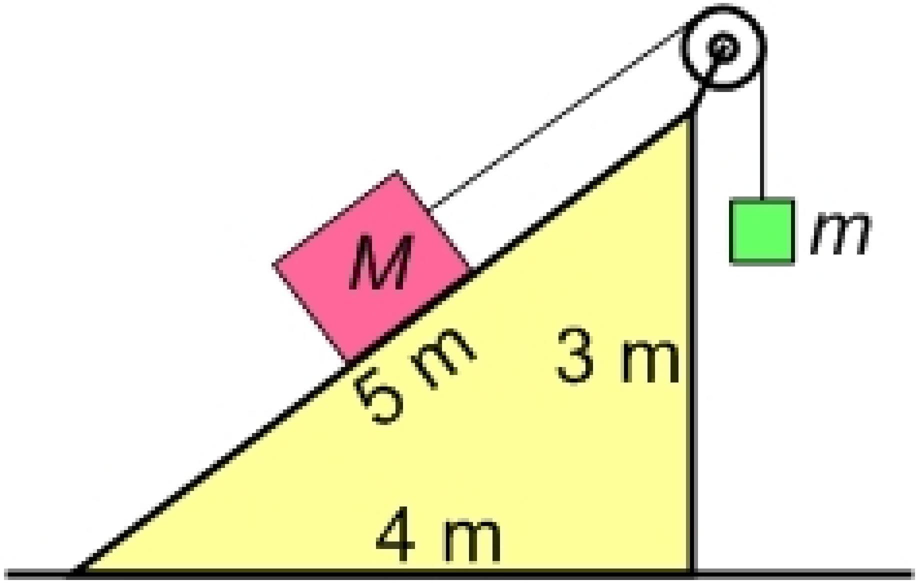
Worksheet

Follow the usual process, outlined on the worksheet, to find the value of *m* that keeps the system at rest.



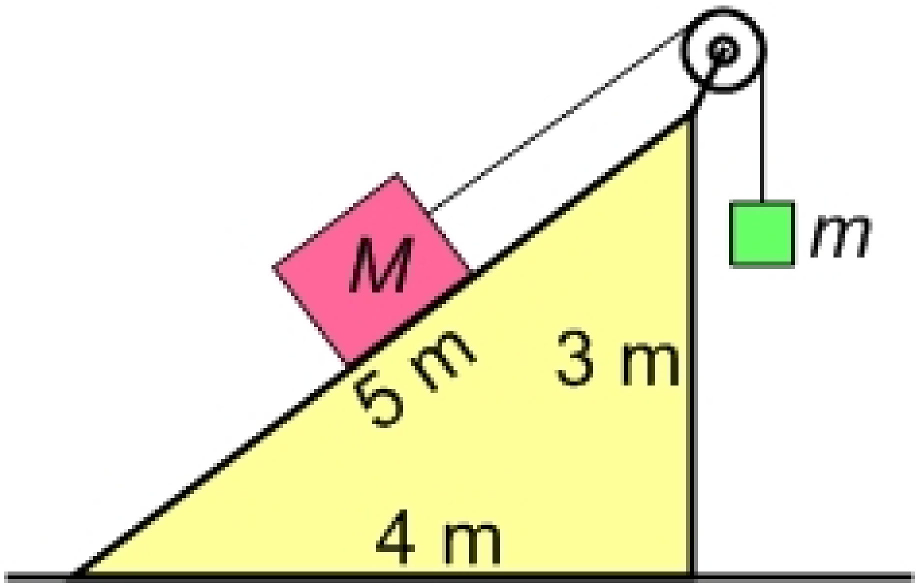
The value of *m*

What did you find for the value of *m*? You should find it to be an integer number of kilograms. Enter that integer into your clicker as your answer.



Analyzing the small box

With down the slope positive for the large block, take up to be positive for the small box (this won't matter much until later).



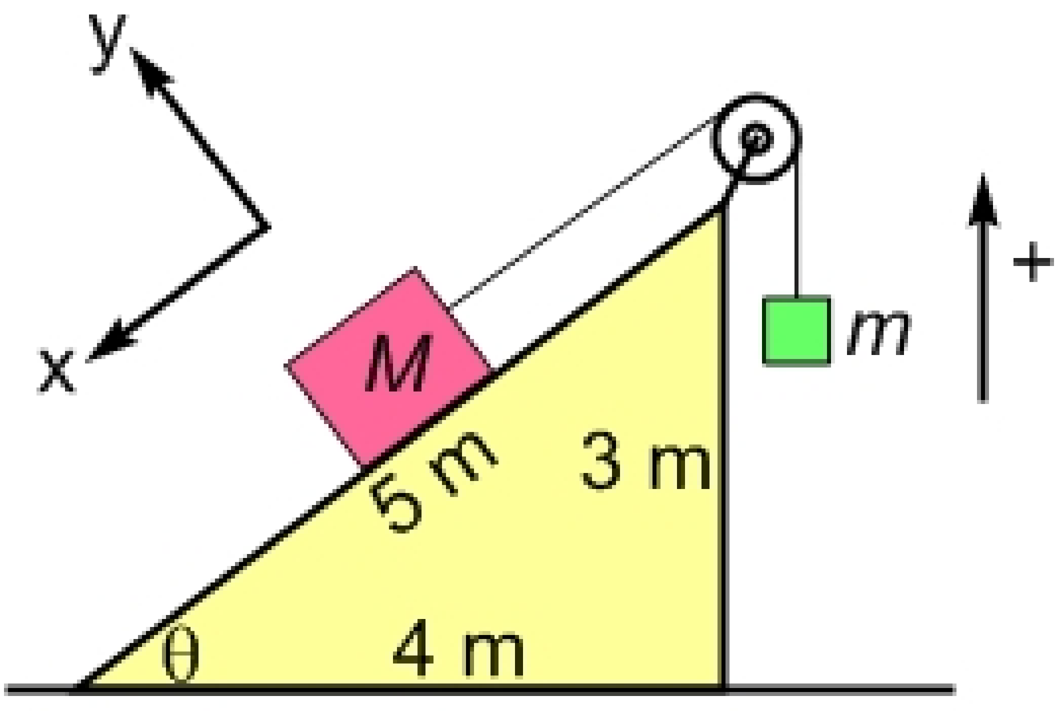
$$\sum \vec{F} = m\vec{a}$$

$$+F_T - mg = 0$$

$$F_T = mg$$

Analyzing the large block

With no friction, we can ignore the *y*-direction, perpendicular to the slope.



$$\sum \vec{F}_x = M\vec{a}_x$$

$$+Mg\sin\theta - F_T = 0$$

$$F_T = Mg\sin\theta$$

Combining the equations

Small box: $F_T = mg$

Large block: $F_T = Mg\sin\theta$

Therefore: $mg = Mg\sin\theta \longrightarrow m = M\sin\theta$

Geometry: $\sin\theta = ?$

Final result:

Combining the equations

Small box: $F_T = mg$

Large block: $F_T = Mg\sin\theta$

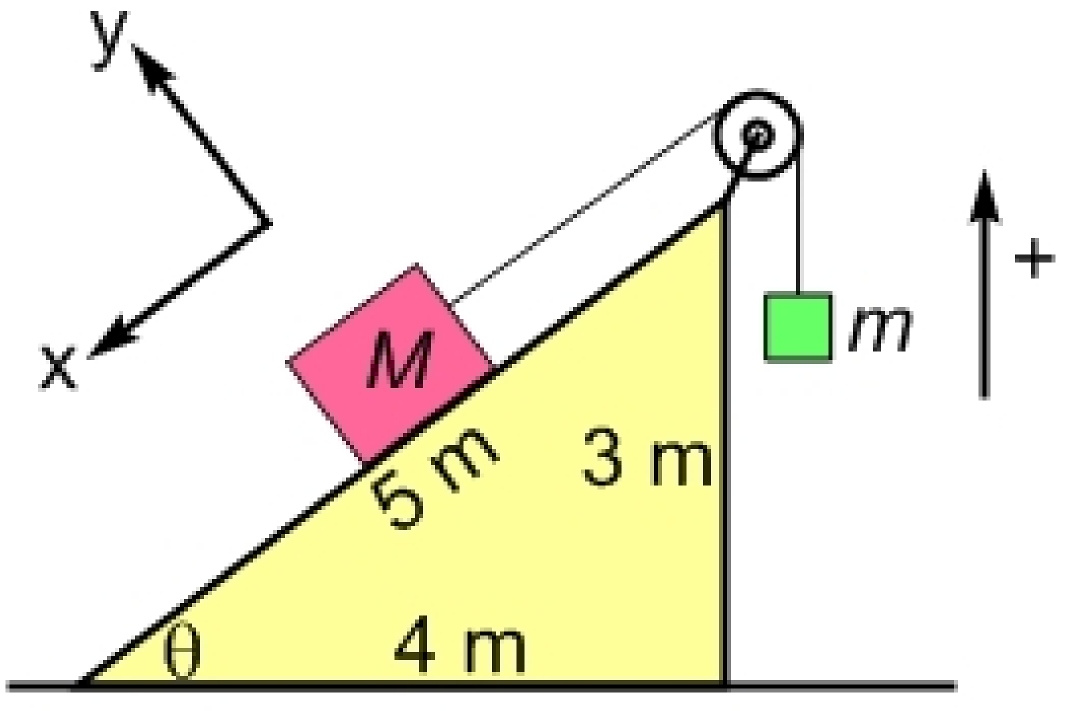
Therefore: $mg = Mg\sin\theta \longrightarrow m = M\sin\theta$

Geometry: $\sin\theta = \dfrac{3}{5}$

Final result: $m = \dfrac{3}{5}M = \dfrac{3}{5}\times 10\text{ kg} = 6\text{ kg}$

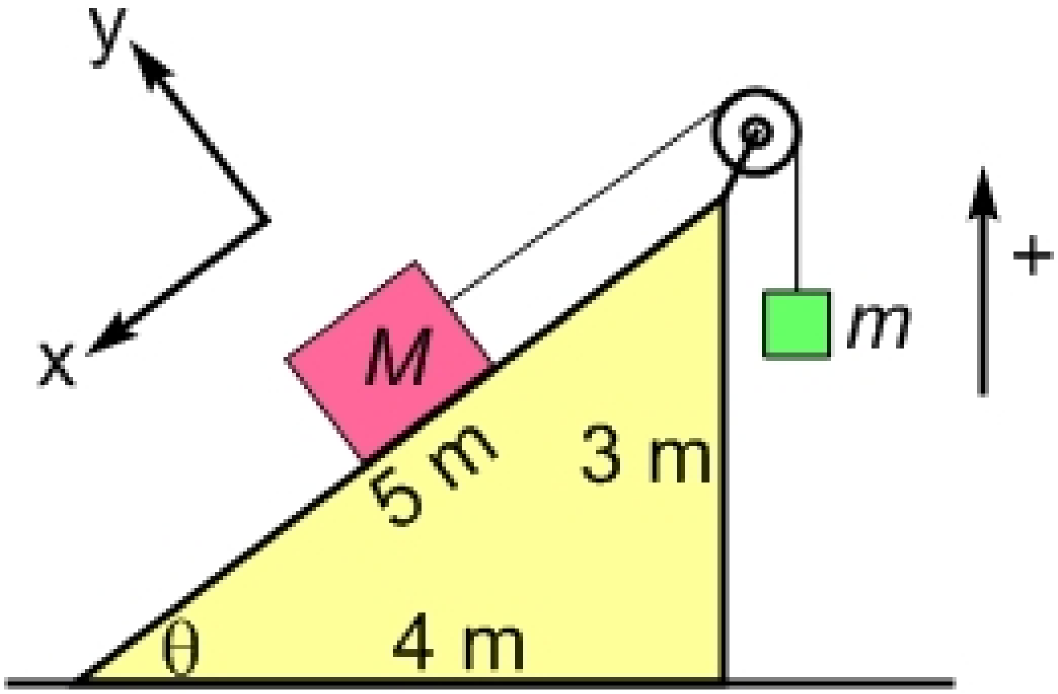
Including friction

Now let's make the problem more realistic and add some friction. The coefficients of friction for the interaction between the red block and the ramp are: $\mu_S = 0.50$ and $\mu_K = 0.25$.



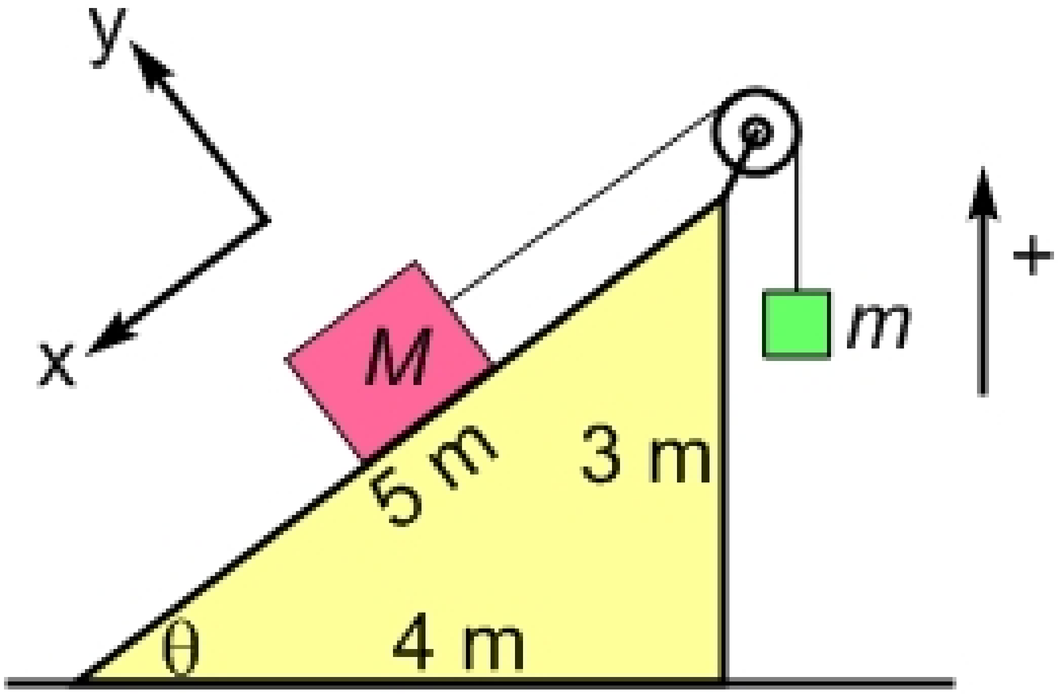
Including friction

With friction, we usually need to know the normal force acting on the large block. What is it?



Find the normal force

Apply Newton's Second Law in the *y*-direction for the large block.



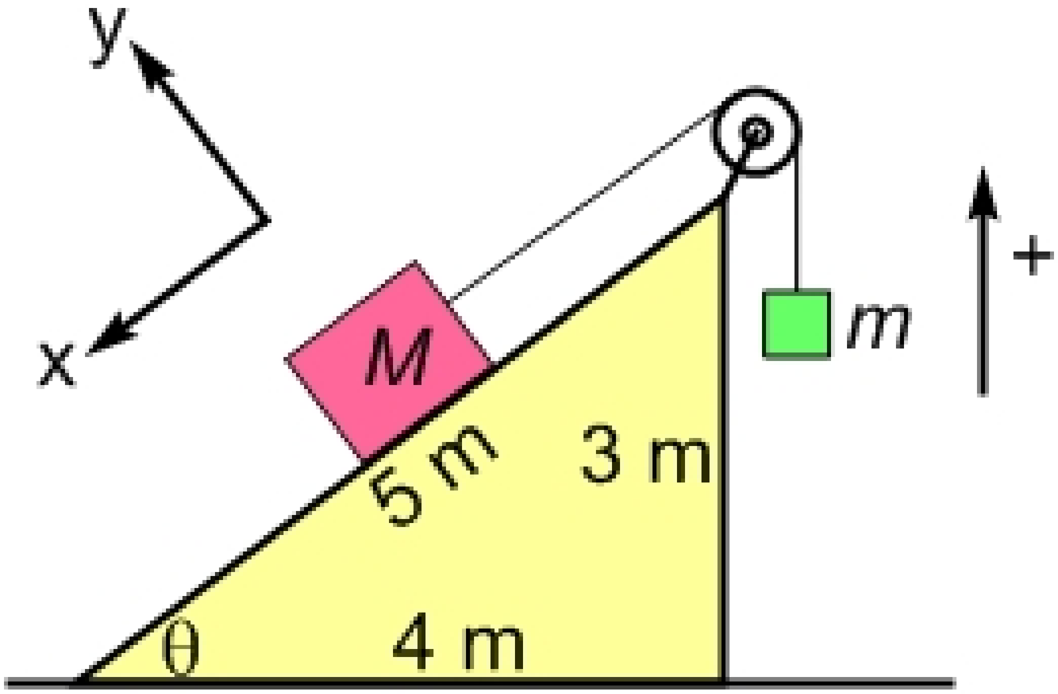
$$\sum \vec{F}_y = M\vec{a}_y$$

$$+F_N - Mg\cos\theta = 0$$

$$F_N = Mg\cos\theta$$

Find the normal force

Apply Newton's Second Law in the *y*-direction for the large block.



$$\sum \vec{F}_y = M\vec{a}_y$$

$$+F_N - Mg\cos\theta = 0$$

$$F_N = Mg\cos\theta$$

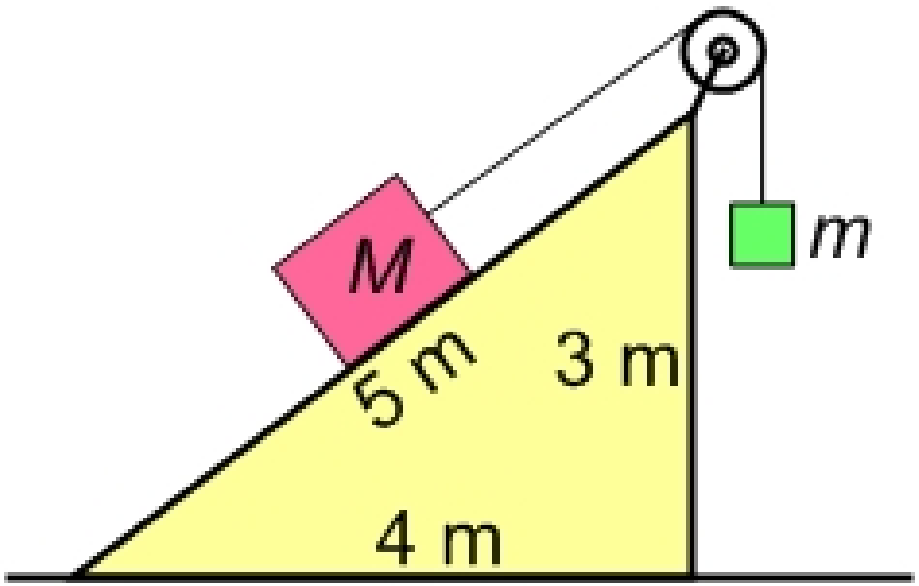
$$F_N = \frac{4}{5}Mg$$

$$F_N = \frac{4}{5}(10\text{ kg})\times(10\text{ m/s}^2) = 80\text{ N}$$

The value of the force of friction

With no friction, m = 6 kg kept the system at rest. Turning friction on, with μ_S = 0.50 and μ_K = 0.25, and m still 6 kg, what is the magnitude of the force of friction acting on the large block if the system remains at rest?

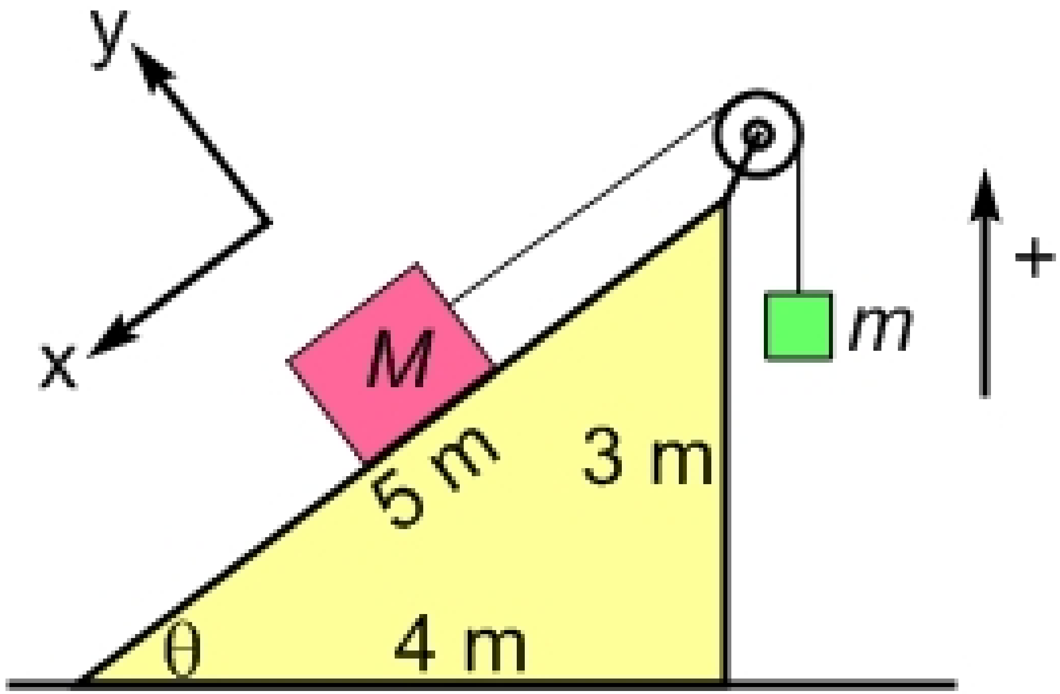
1. Zero
2. 20 N
3. 40 N
4. 60 N
5. 80 N



Now, adjust *m*

What happens as *m* is gradually increased? What changes, if anything, on the free-body diagrams?

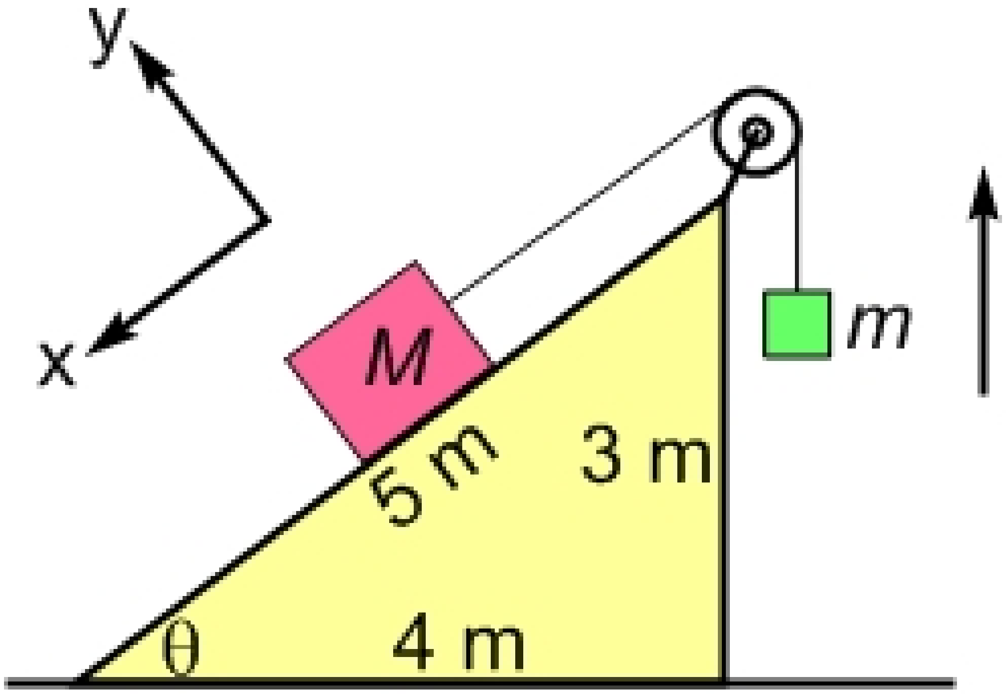
What is the maximum possible value *m* can be if we want the system to remain at rest?



Analyzing the small box

This is the same as before!

With down the slope positive for the large block, take up to be positive for the small box (this won't matter much until later).



$$\sum \vec{F} = m\vec{a}$$

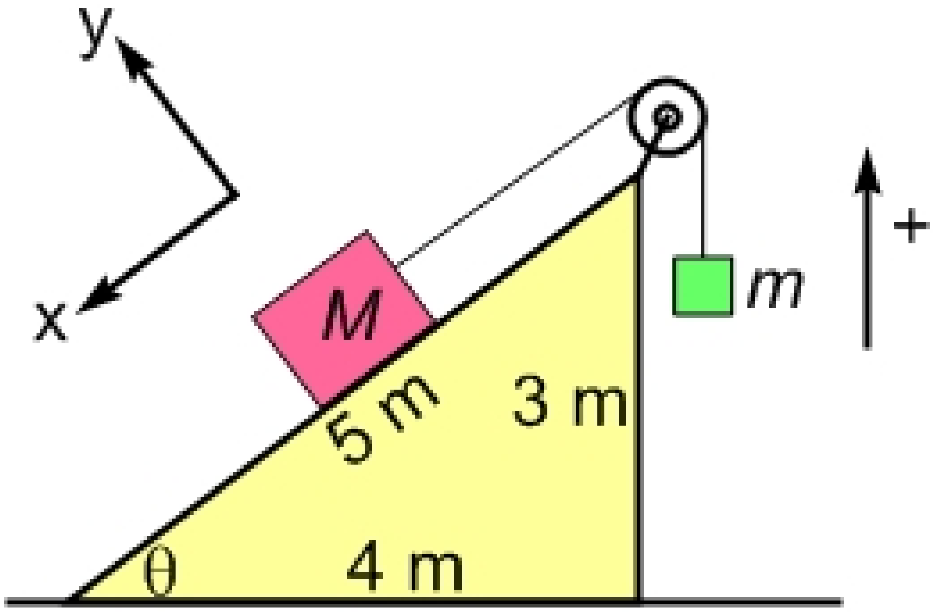
$$+F_T - mg = 0$$

$$F_T = mg$$

Analyzing the large block

This is different.

Now, we need to account for friction. Because we're looking for the maximum *m*, we need the maximum possible value of the force of static friction.



$$\sum \vec{F}_x = M\vec{a}_x$$

$$+Mg\sin\theta + F_{S,\max} - F_T = 0$$

$$F_T = Mg\sin\theta + F_{S,\max}$$

$$F_T = Mg\sin\theta + \mu_S F_N$$

Combining the equations

Small box: $F_T = mg$

Large block: $F_T = Mg\sin\theta + \mu_S F_N$

Normal force: $F_N = Mg\cos\theta$

Combining: $mg = Mg\sin\theta + \mu_S Mg\cos\theta$

$$m = M\sin\theta + \mu_S M\cos\theta$$

$$m = M\left(\sin\theta + \mu_S\cos\theta\right)$$

Final result: $m = M\left(\frac{3}{5} + 0.5\times\frac{4}{5}\right) = M\times 1 = M$ By chance

More things you can find

What is the minimum possible value *m* can be if we want the system to remain at rest?

What is the acceleration of the system if *m* = 1.0 kg?