Maximizing range

A particular projectile lands at the same height from which it was launched. Assuming the launch speed is constant, what launch angle maximizes the range? (The range is the horizontal distance between the launch point and the landing point.)

1. 30°
2. An angle less than 45°
3. 45°
4. An angle more than 45°
5. It depends on the mass of the projectile.

Maximizing range

What's our equation for range? Range = horizontal distance traveled.

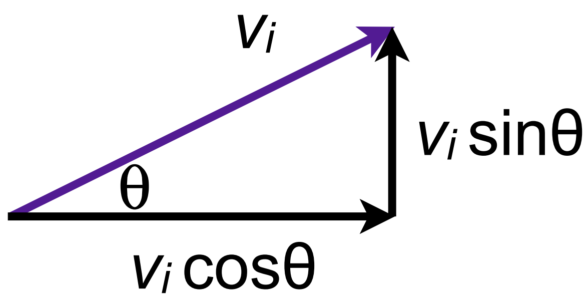
$\Delta x = v_{ix}t$, where *t* is the time of flight.

If we increase the launch angle, what happens to v_{ix}? What happens to *t*?

v_{ix} decreases, while *t* increases, and who knows what happens to the range.

The range equation

This equation applies only when the landing height is the same as the launch height.



$$\Delta x = v_{ix} t = v_{ix}\left(\frac{2v_{iy}}{g}\right)$$

$$\Delta x = \frac{2(v_i \cos\theta)(v_i \sin\theta)}{g} = \frac{v_i^2}{g}\sin(2\theta)$$

The range is maximum at a launch angle of 45°. The equation also tells us that angles of θ and 90°- θ produce the same range.

Maximizing range

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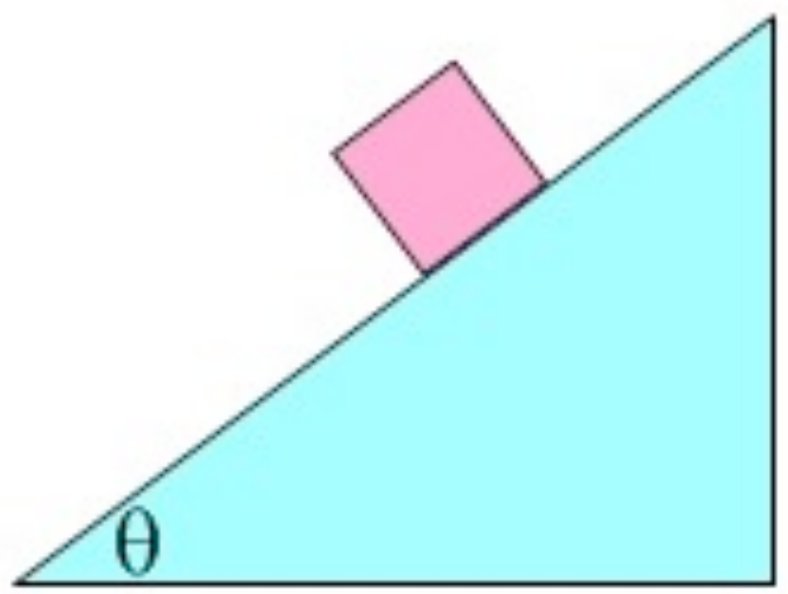
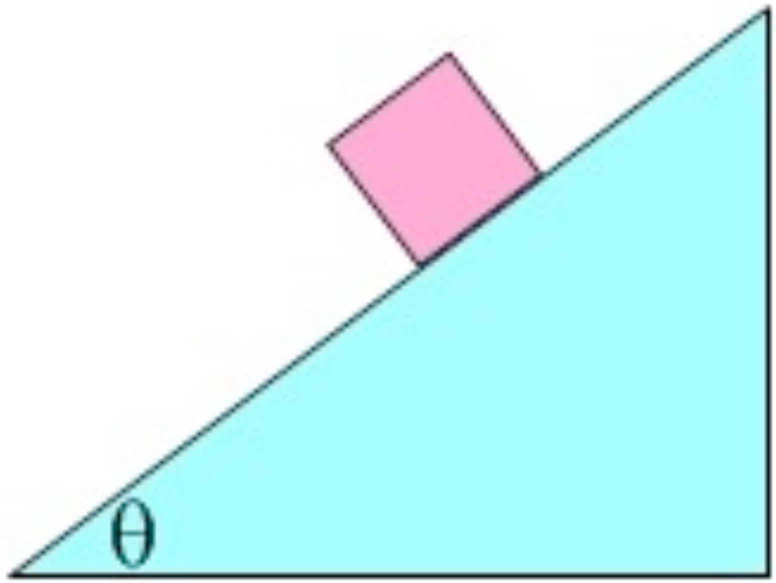
Step 1, draw a diagram

Let's analyze the situation of a block that remains at rest on an incline.



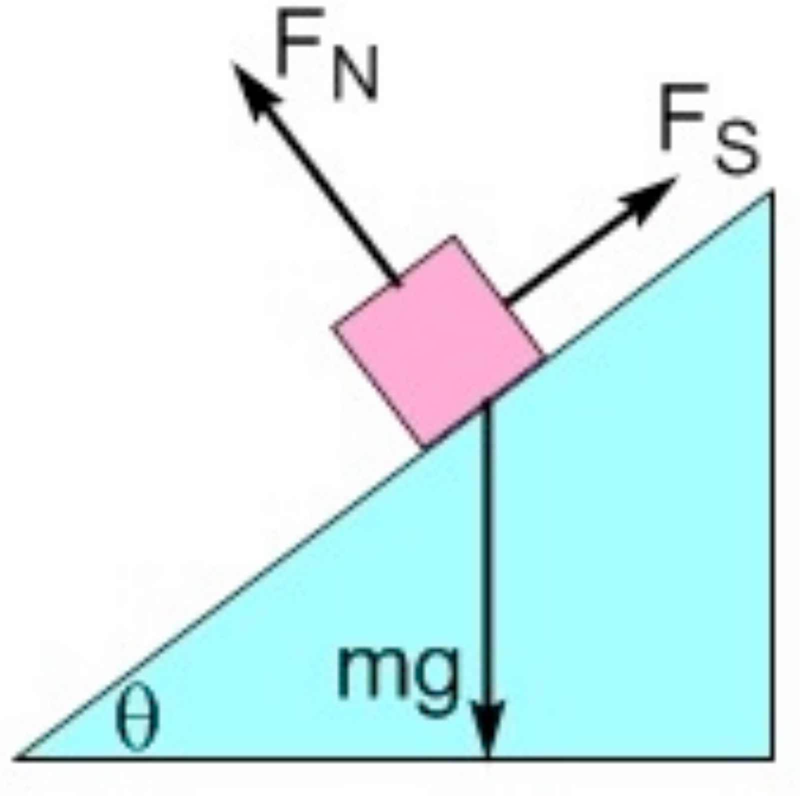
Step 2, draw a free-body diagram

Show the different forces acting on the block.



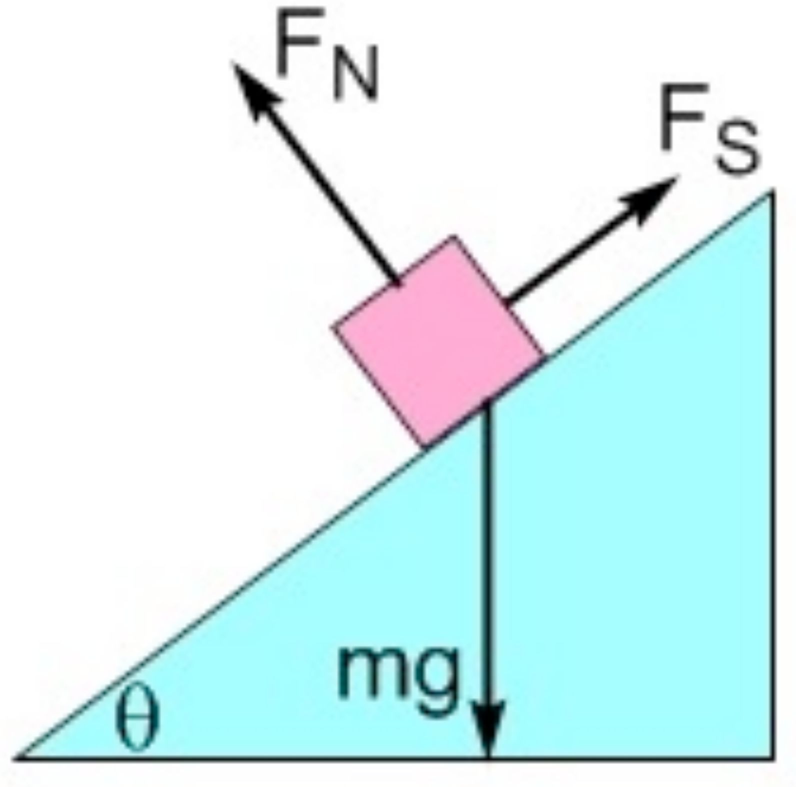
Step 2, draw a free-body diagram

There are two forces, but the contact force applied by the ramp on the box is shown as two separate components, the normal force and the force of static friction.



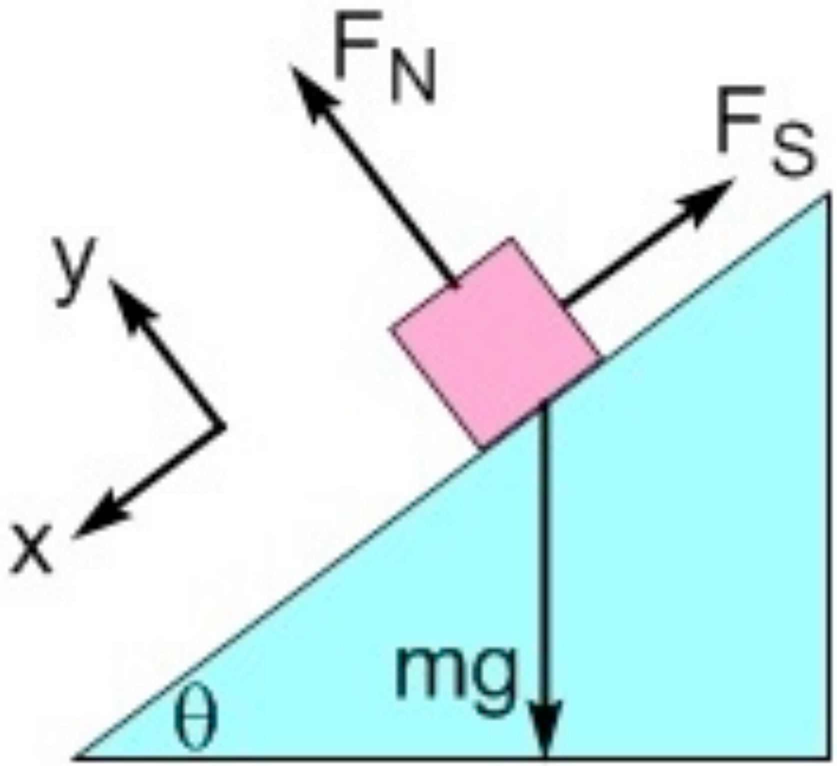
Step 3, choose a coordinate system

What is a good coordinate system in this case?



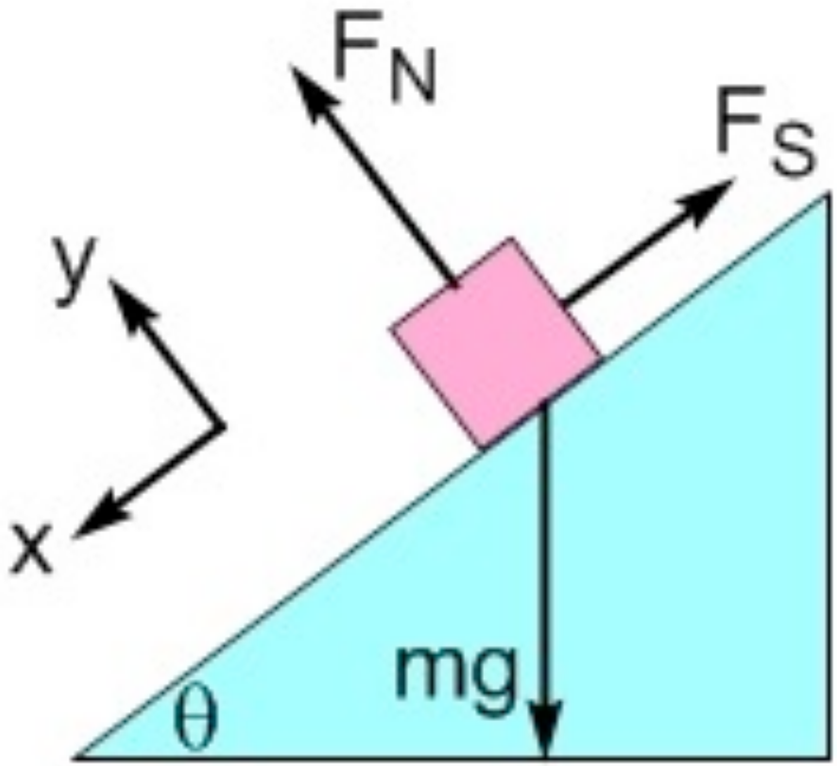
Step 3, choose a coordinate system

Let’s align the coordinate system with the incline.



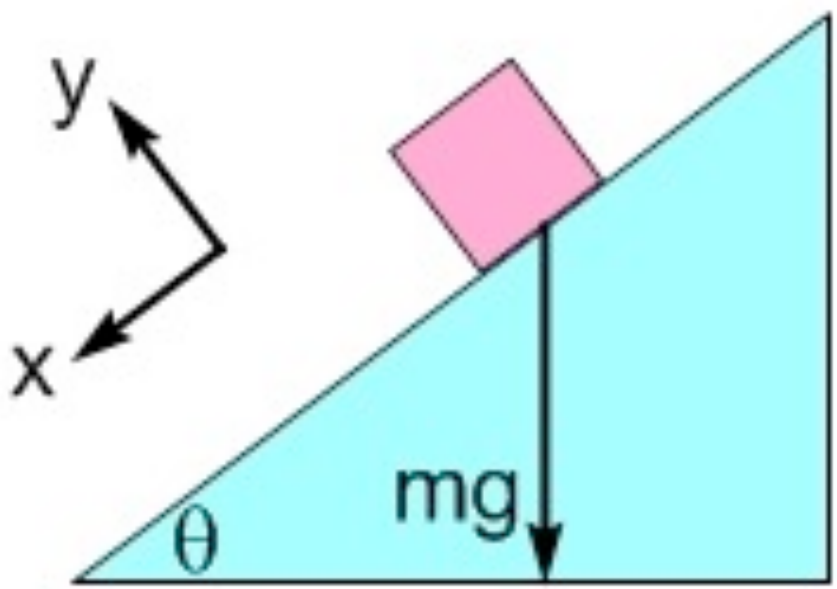
Step 4, break forces into components, parallel to the coordinate axes

Which force(s) do we have to split into components?



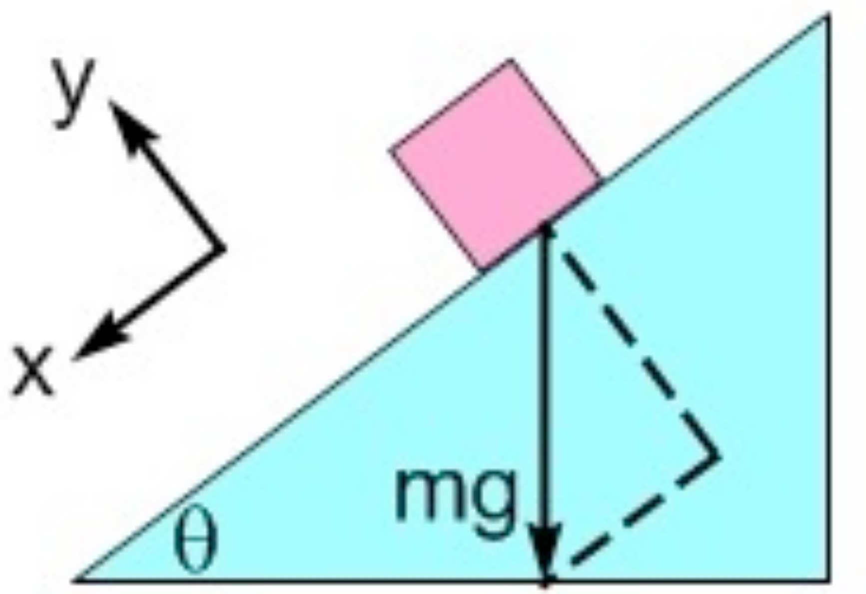
Step 4, break forces into components, parallel to the coordinate axes

We just have to break the force of gravity into components.



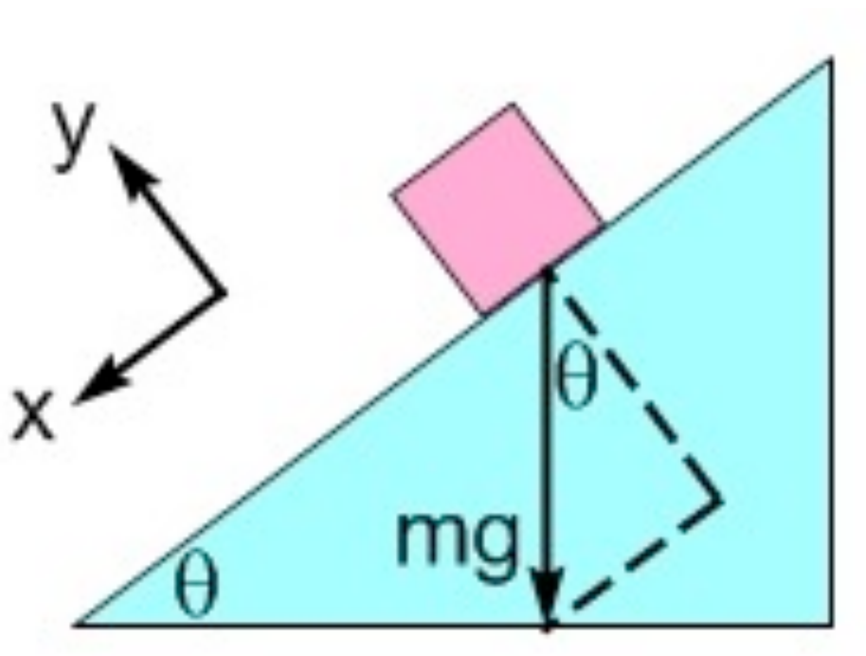
Step 4, break mg into components

Draw a right-angled triangle, with sides parallel to the axes, and the force as the hypotenuse. Where does θ figure into the triangle?



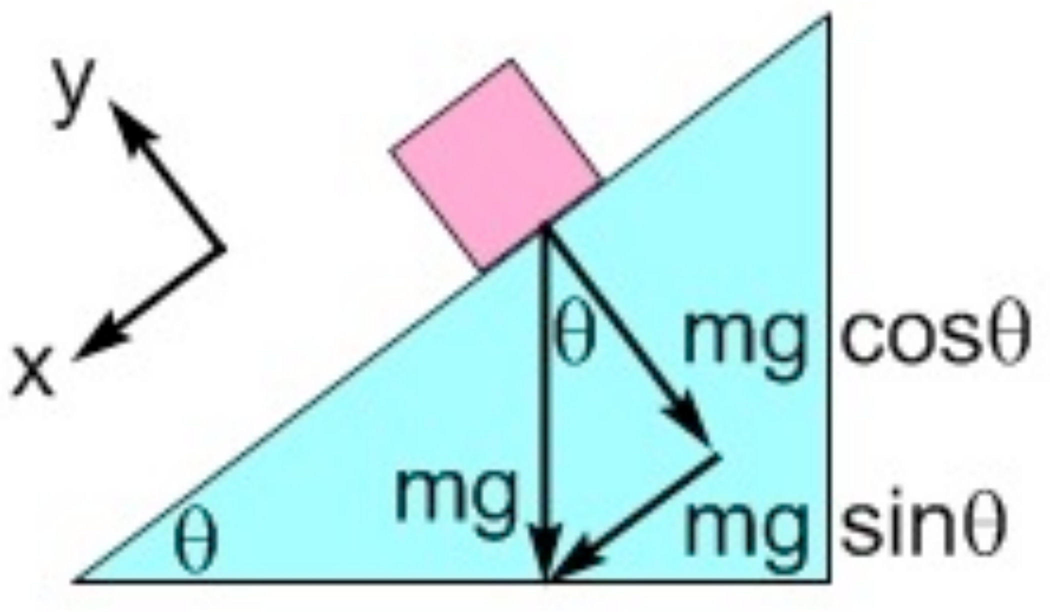
Step 4, break mg into components

It's at the top. Now use sine and cosine.



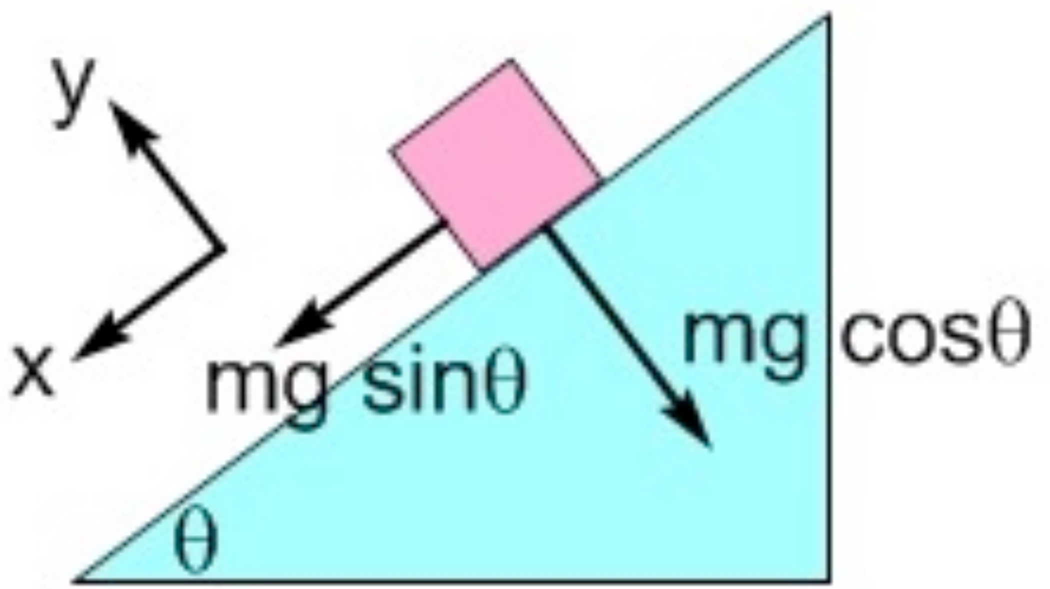
Step 4, break mg into components

Sine goes with the component down the slope, cosine with the component into the slope.

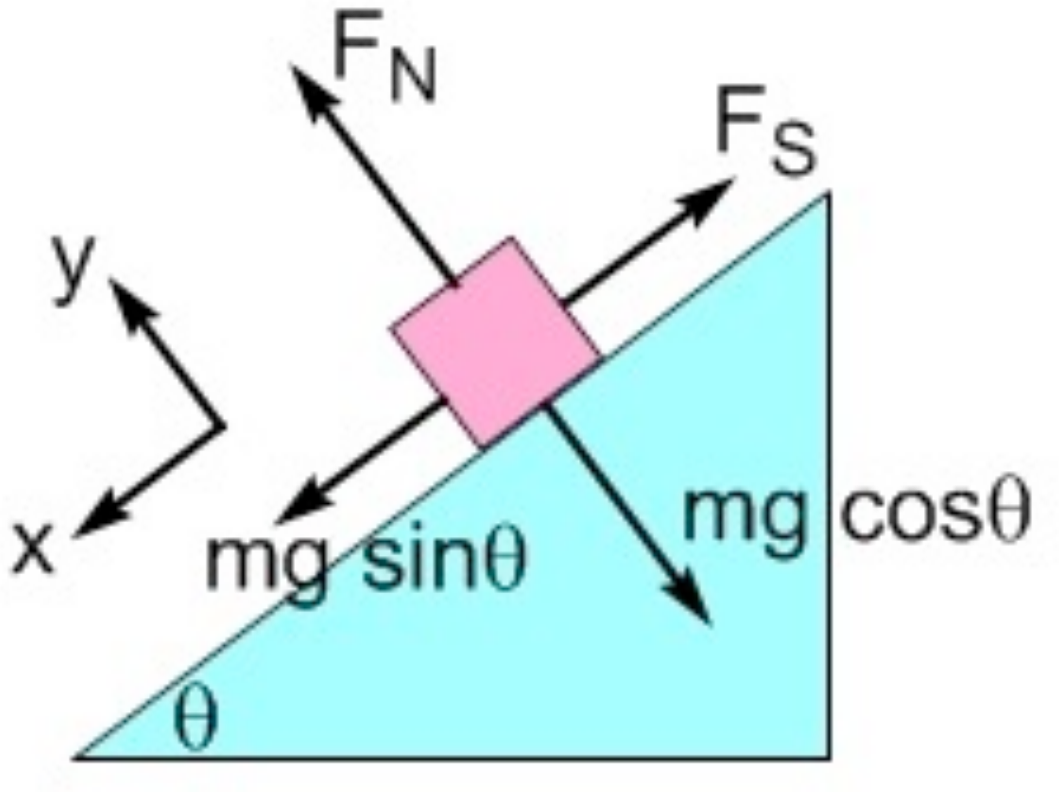


Step 4, break mg into components

The end result – we replaced *mg* by its components.



The full free-body diagram



Step 5, apply Newton's Second Law

We apply Newton's Second Law twice, once for the *x*-direction and once for the *y*-direction.

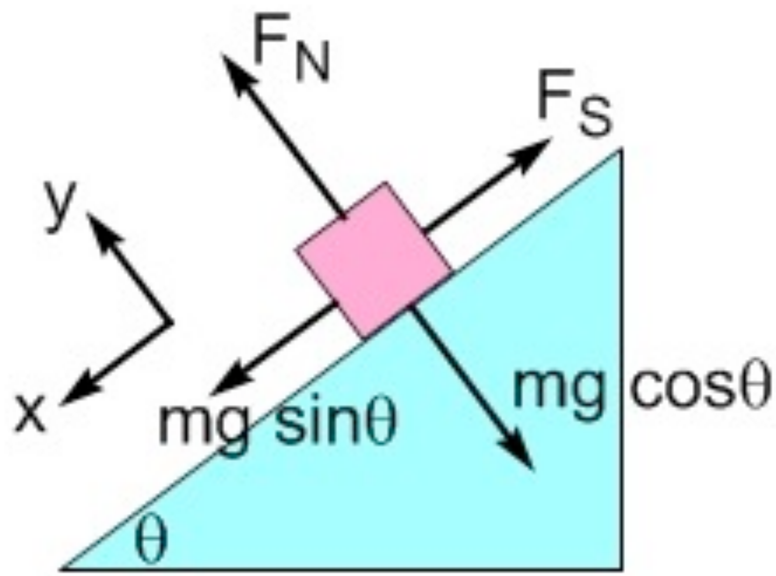
x-direction

$$\sum \vec{F}_x = m\vec{a}_x$$

y-direction

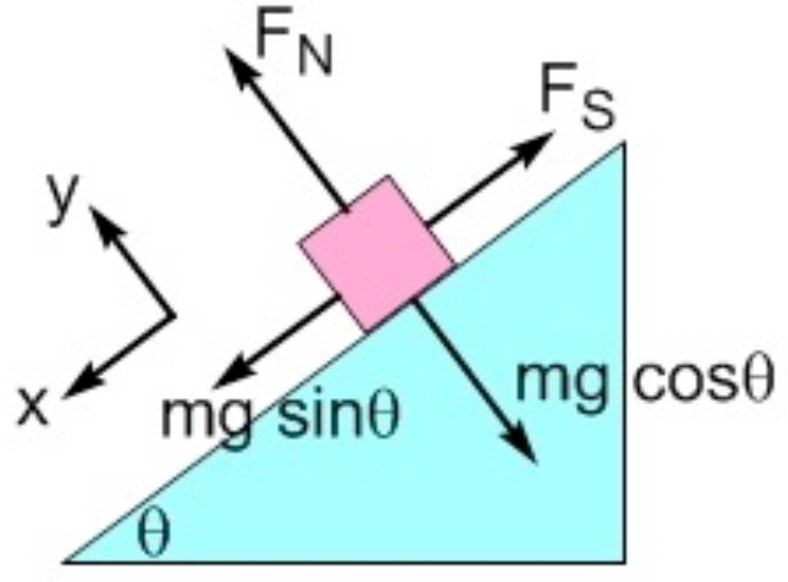
$$\sum \vec{F}_y = m\vec{a}_y$$

Evaluate the left-hand side of each equation by looking at the free-body diagram.



Step 5, apply Newton's Second Law

Special case - we're at the maximum angle - any more, and the block starts sliding.



<u>*x*-direction</u>

$$\sum \vec{F}_x = m\vec{a}_x$$

$$+mg\sin\theta - F_{S,\max} = 0$$

$$mg\sin\theta = F_{S,\max} = \mu_S F_N$$

<u>*y*-direction</u>

$$\sum \vec{F}_y = m\vec{a}_y$$

$$+F_N - mg\cos\theta = 0$$

$$F_N = mg\cos\theta$$

Step 6, combine the equations

$$mg\sin\theta = F_{S,\max} = \mu_S F_N \qquad F_N = mg\cos\theta$$

$$mg\sin\theta = \mu_S\, mg\cos\theta$$

$$\sin\theta = \mu_S \cos\theta$$

$$\mu_S = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

The coefficient of static friction is just the tangent of the angle when the top object starts to slide over the bottom object.

Graphs for a block on an incline

For a block on an incline, plot, as a function of the angle of the incline

- The normal force applied to the block

- The force of static friction that must act on the block to prevent the block from sliding

- The value of the maximum possible force of static friction

Graphs

For a block on an incline, plot, as a function of the angle of the incline

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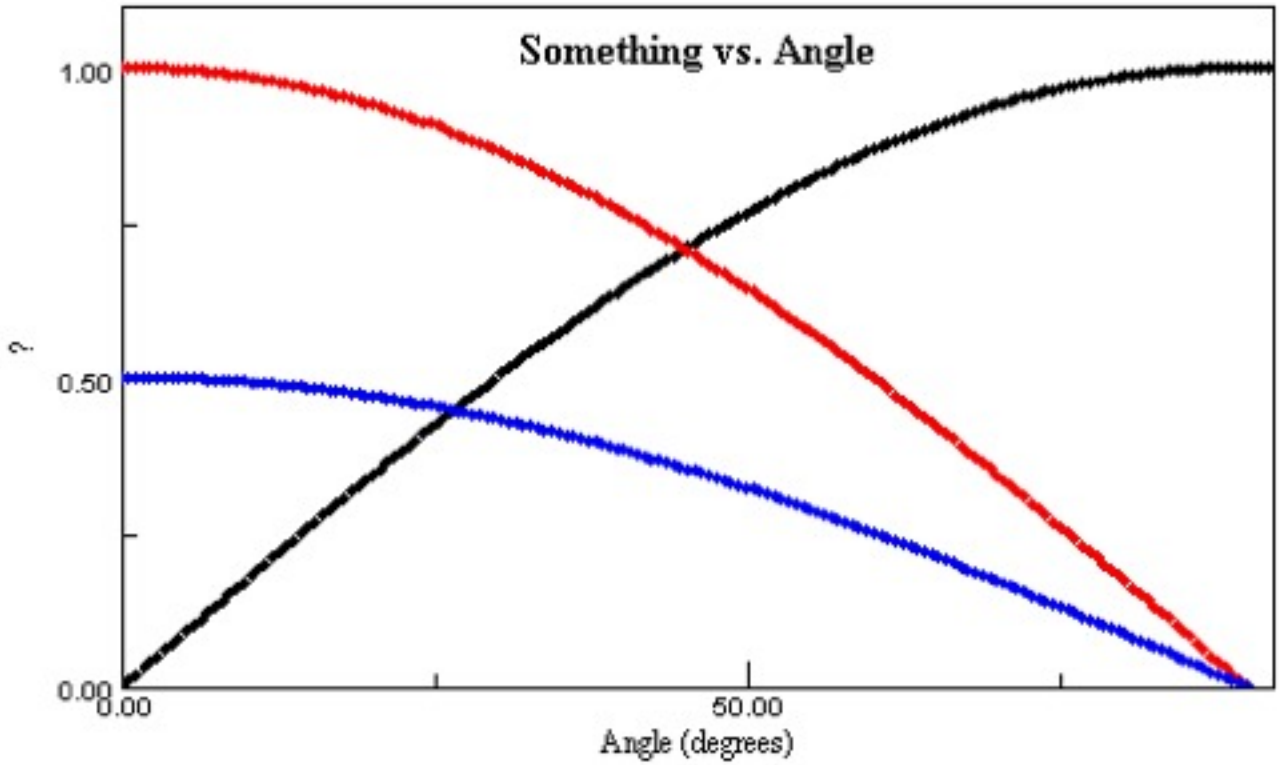

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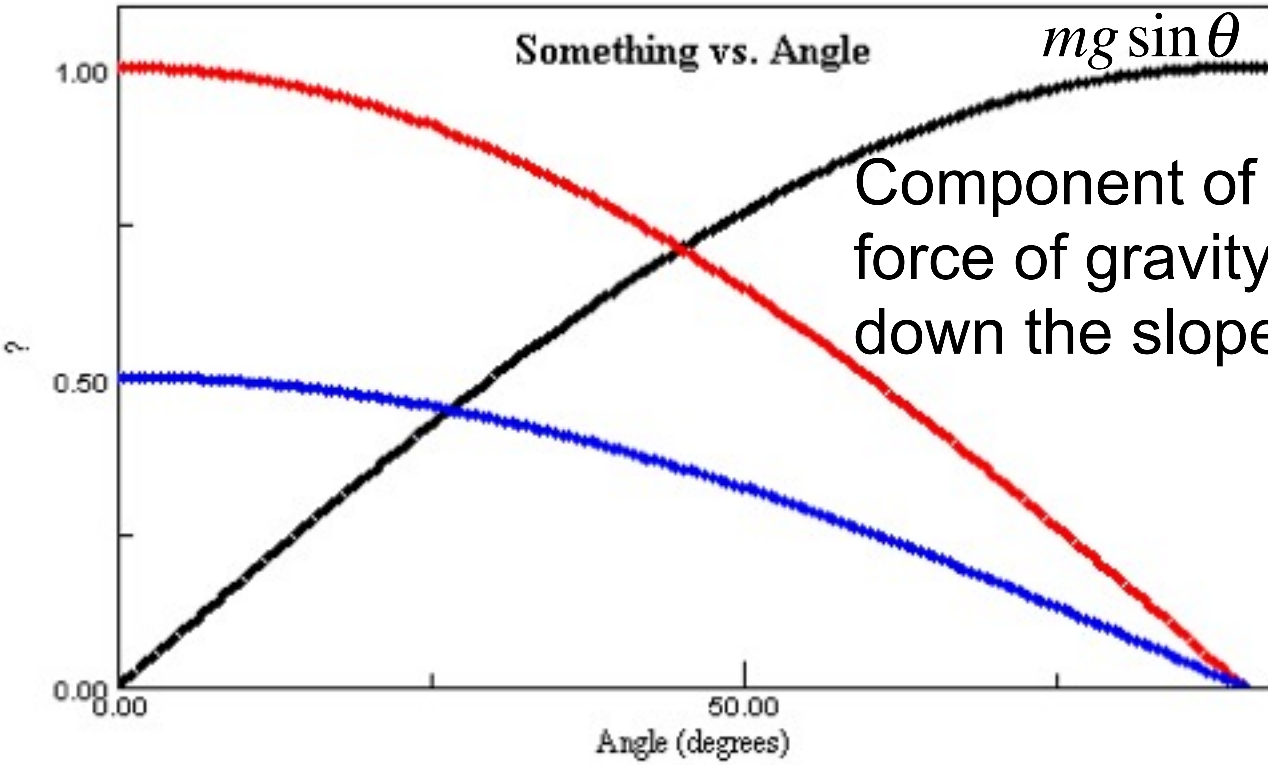


$mg\sin\theta$

Component of the force of gravity acting down the slope

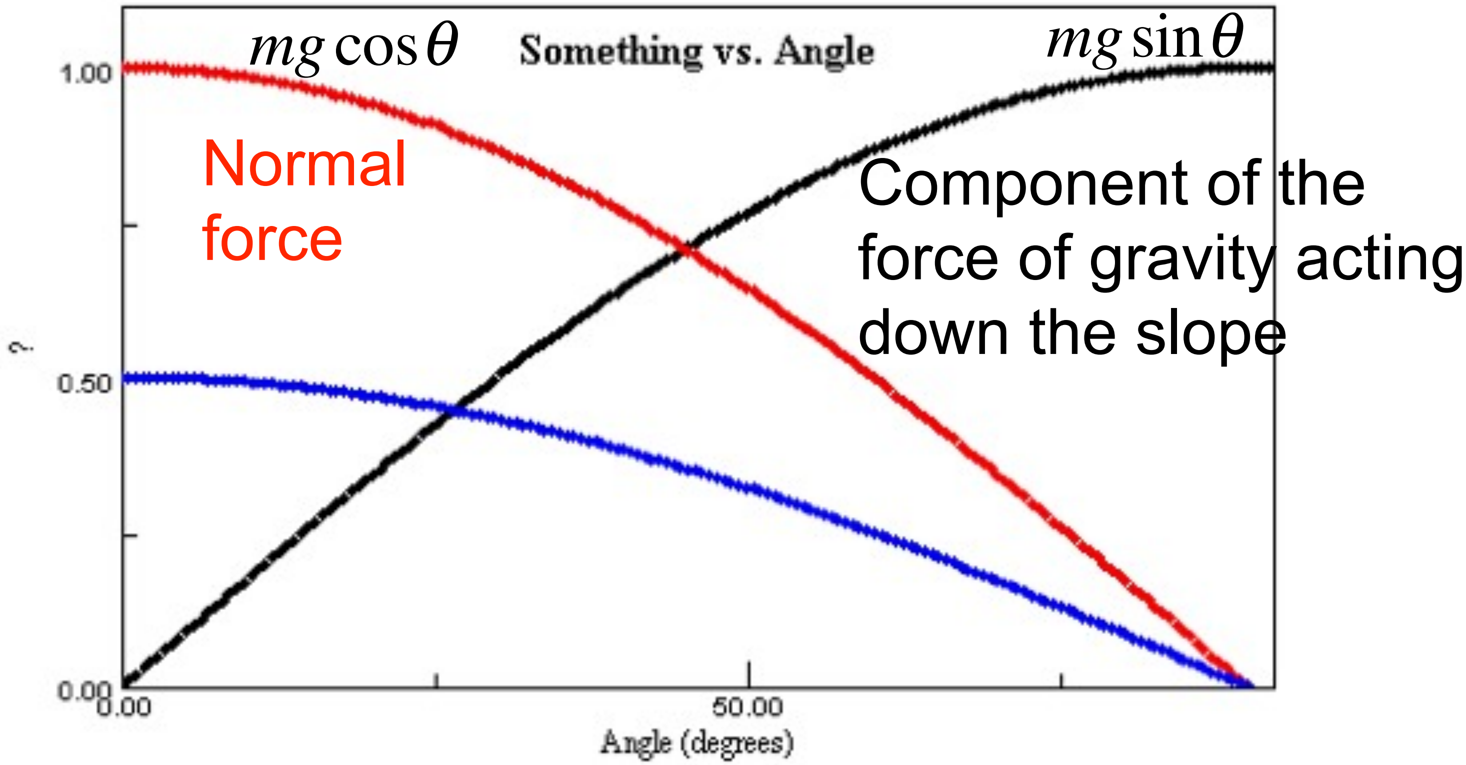
Graphs

For a block on an incline, plot, as a function of the angle of the incline

- The normal force applied to the block

- The force of static friction that must act on the block to prevent the block from sliding

- The value of the maximum possible force of static friction



Graphs

For a block on an incline, plot, as a function of the angle of the incline

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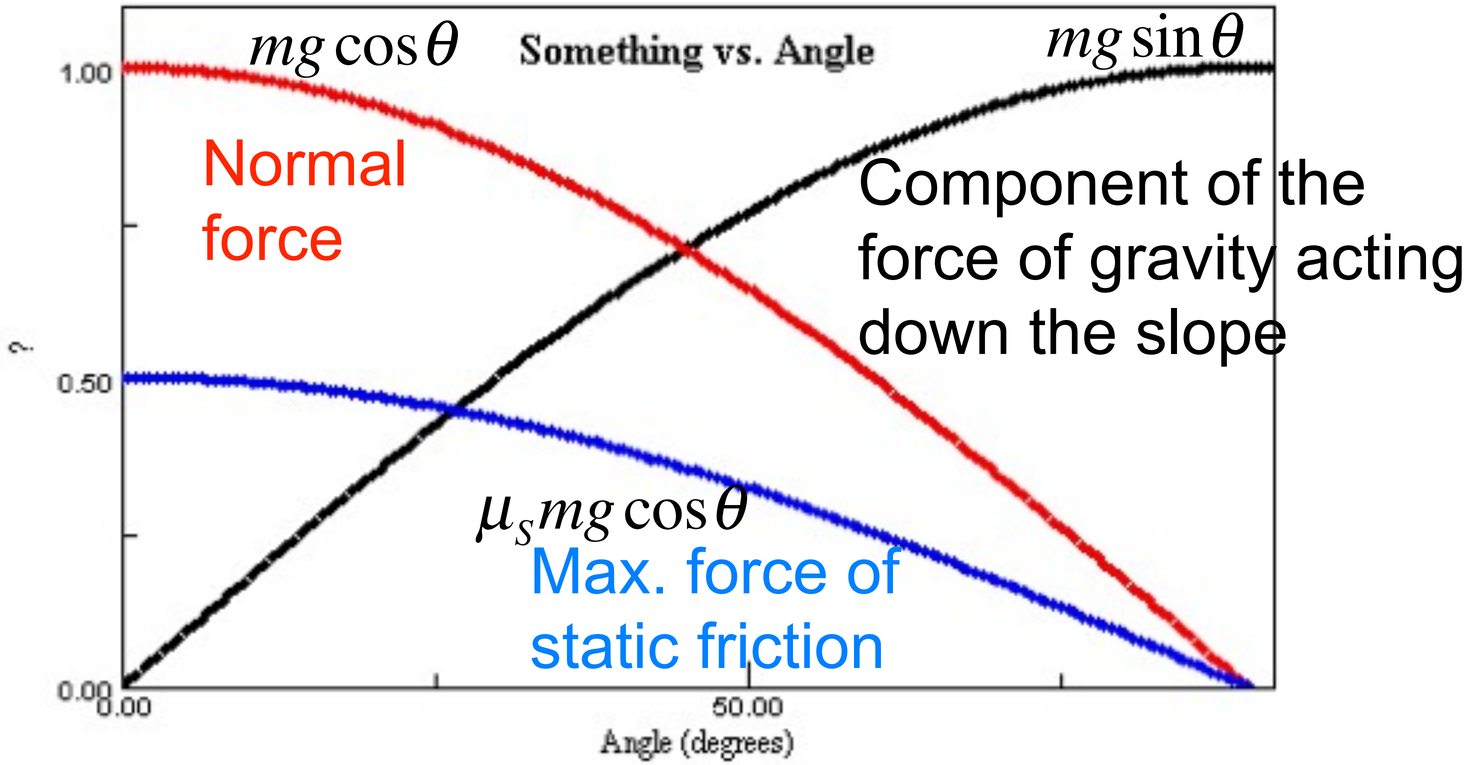

Graphs

For a block on an incline, plot, as a function of the angle of the incline

- The normal force applied to the block

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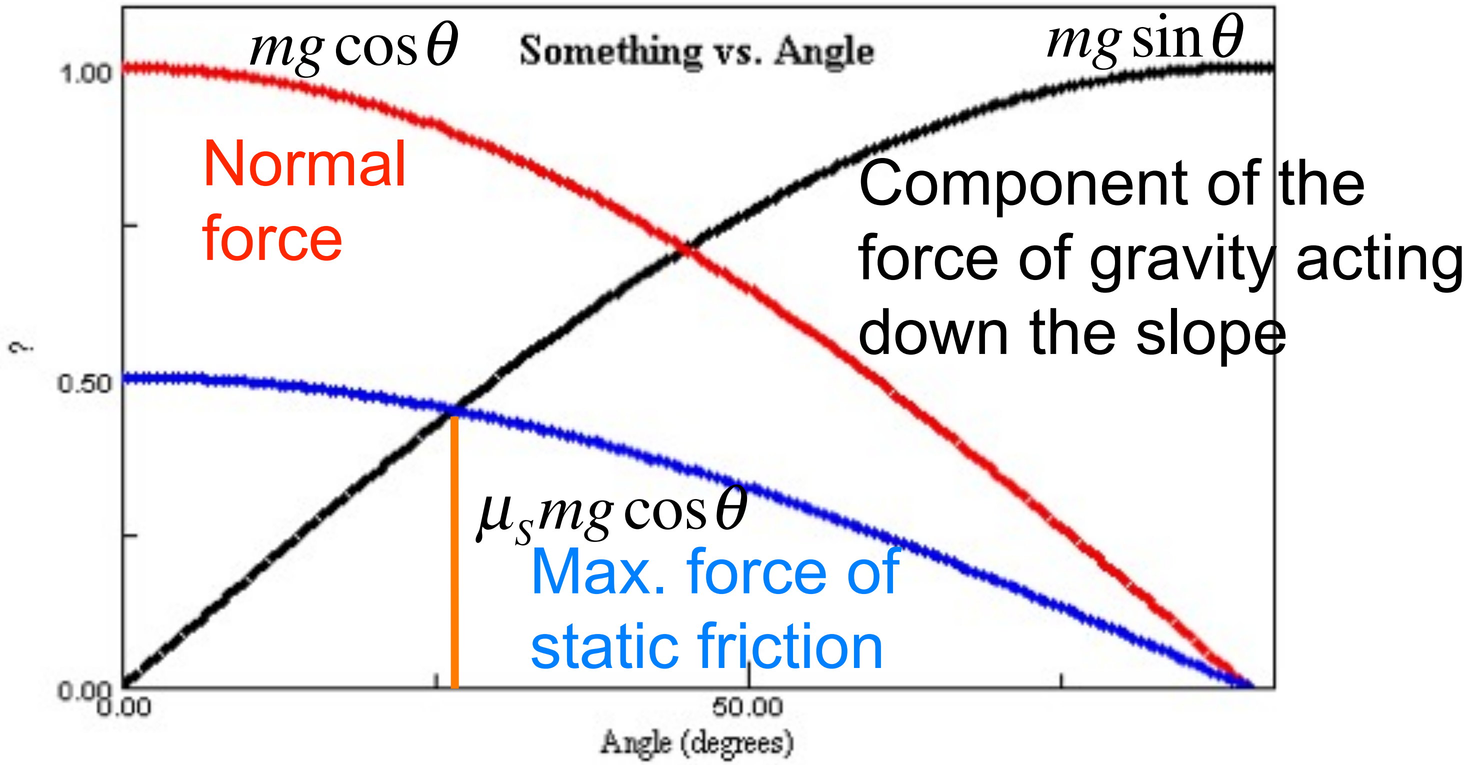


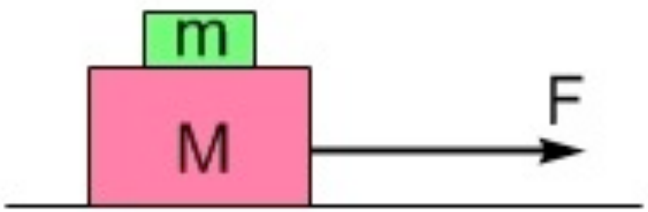
A general method for solving problems involving forces

- Draw a diagram of the situation.
- Draw one or more free-body diagrams showing all the forces acting on the object(s).
- Choose a coordinate system. It is often most convenient to align one of your coordinate axes with the direction of the acceleration.
- Break the forces up into their x and y components.
- Apply Newton's Second Law in (usually) both directions.

Two boxes (see the worksheet)

A small box of mass m = 1 kg sits on top of a large box of mass M = 2 kg on a flat table. The coefficients of friction between all surfaces in contact are:

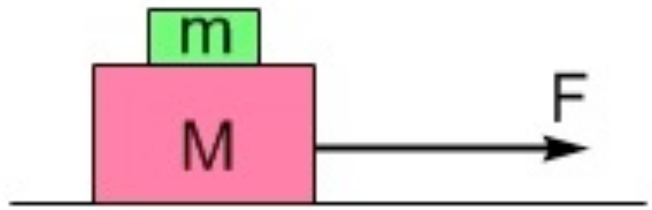
$\mu_K = 0.40$ and $\mu_S = 0.50$



Use g = 10 m/s^2

A string of negligible mass is tied to the large box and a horizontal force F = 21 N is applied to the string so that the two boxes accelerate together to the right, with the small box maintaining its position on top of the large box at all times.

Free-body diagram for the two-box system



Treat the two boxes as one system, and sketch the free-body diagram of the system.

Sketch the free-body diagram of the small box.

Sketch the free-body diagram of the large box.

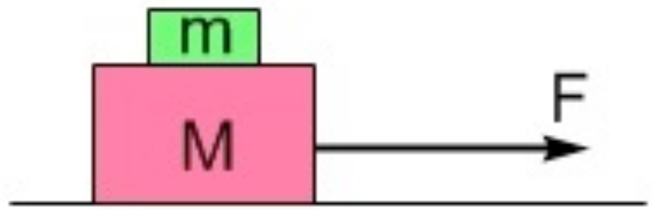
How many forces?

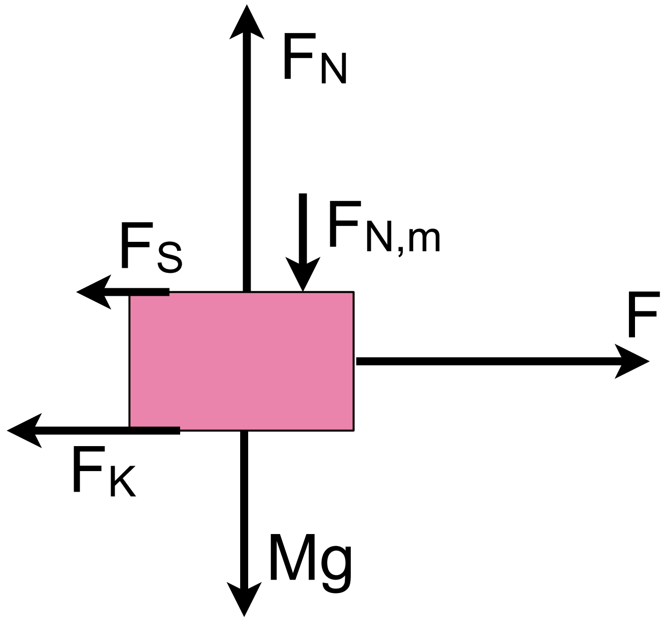
How many forces should be included on the free-body diagram of the large box?

Press the number corresponding to the number of forces you think should be shown on the box.

Free-body diagram for the large box

Sketch the free-body diagram of the large box.





Doing the calculations

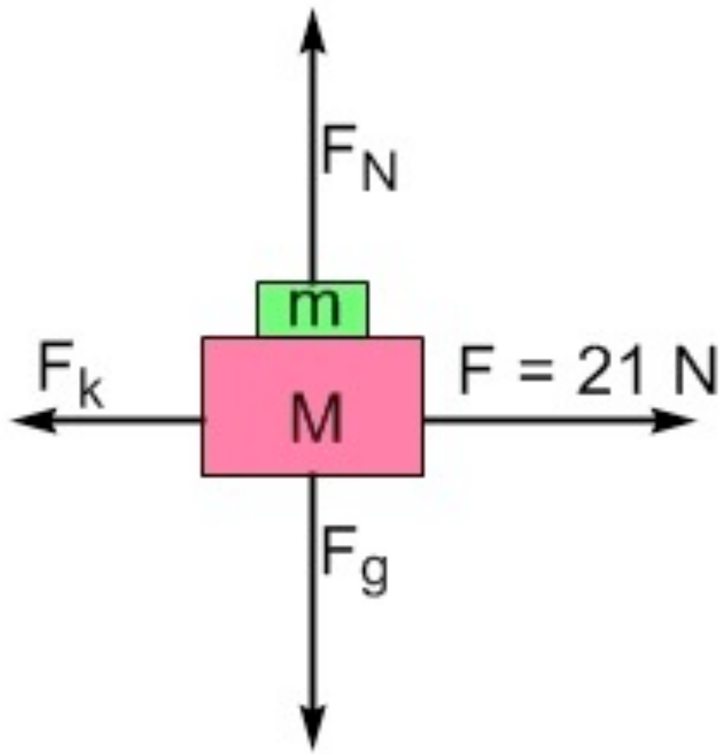
Let's say that M = 2.0 kg and m = 1.0 kg, and we'll use the approximation that g = 10 m/s^2 to simplify the calculations.

If the force applied to the string is F = 21 N, the boxes accelerate together to the right with the small box maintaining its position on top of the large box. What is the acceleration of the system?

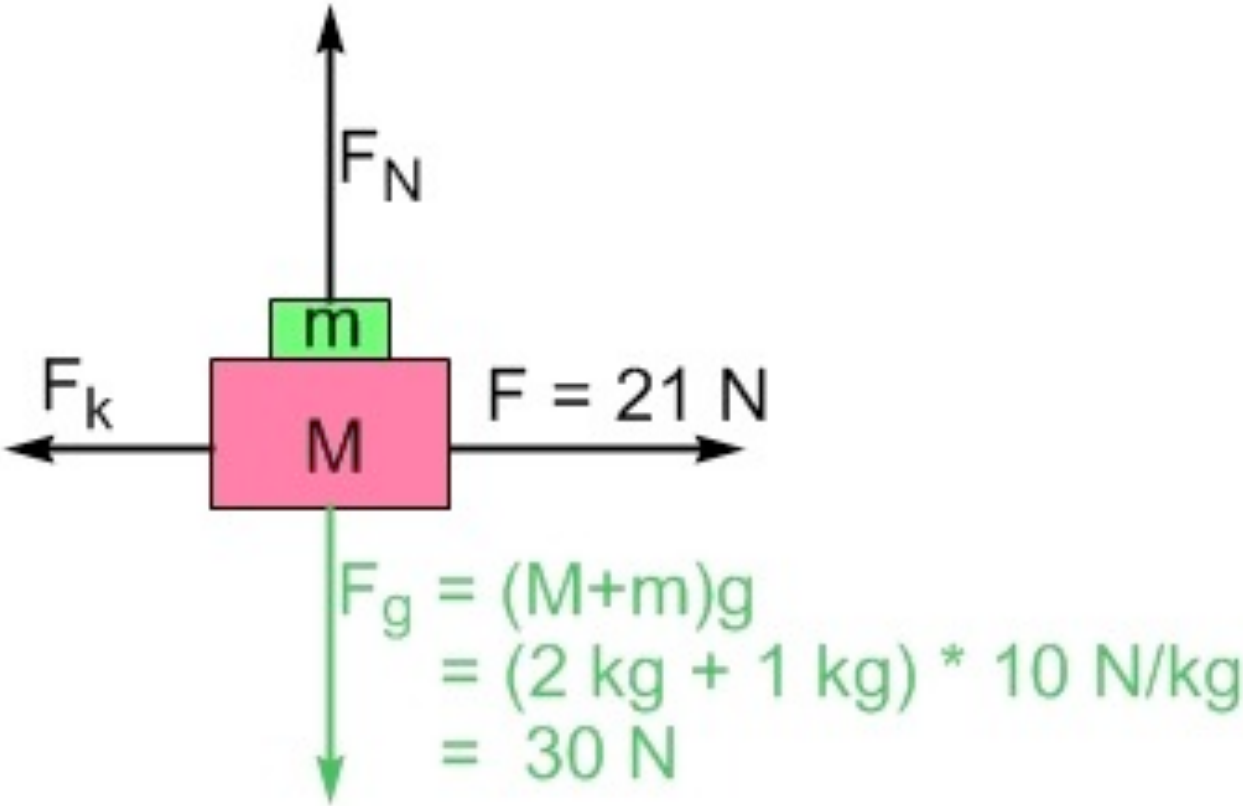
Apply Newton's Second Law

To what?

The two-box system.



Find the force of gravity



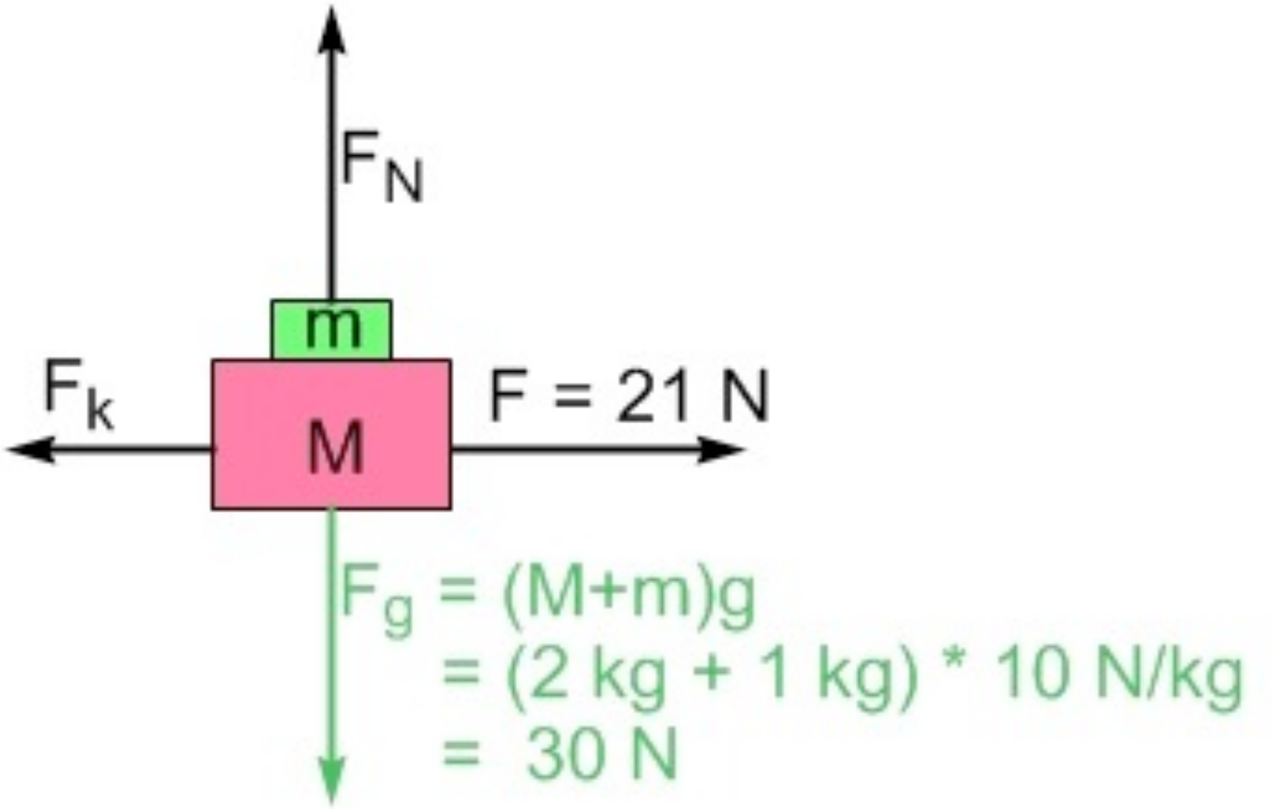
Apply Newton's Second Law

x-direction

$$\sum \vec{F}_x = (m+M)\vec{a}_x$$

y-direction

$$\sum \vec{F}_y = (m+M)\vec{a}_y$$



Apply Newton's Second Law

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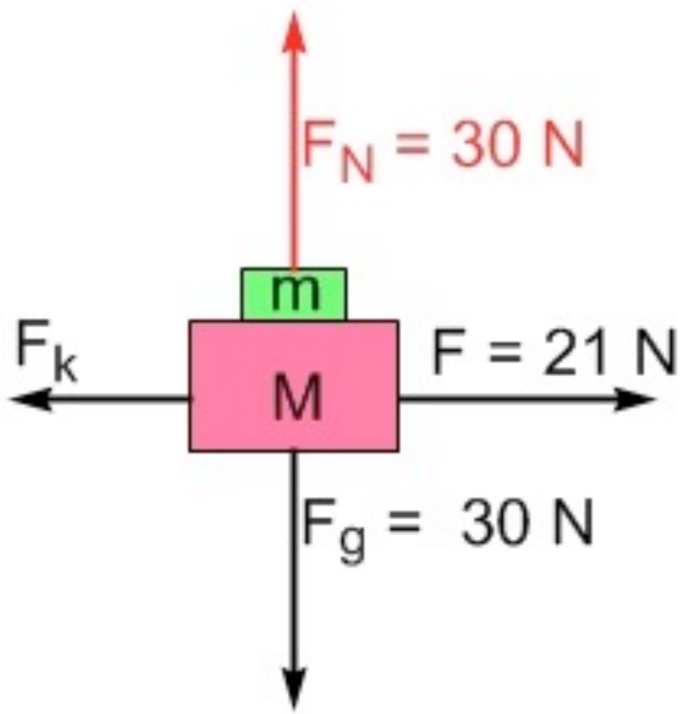
$$\sum \vec{F}_x = (m+M)\vec{a}_x$$



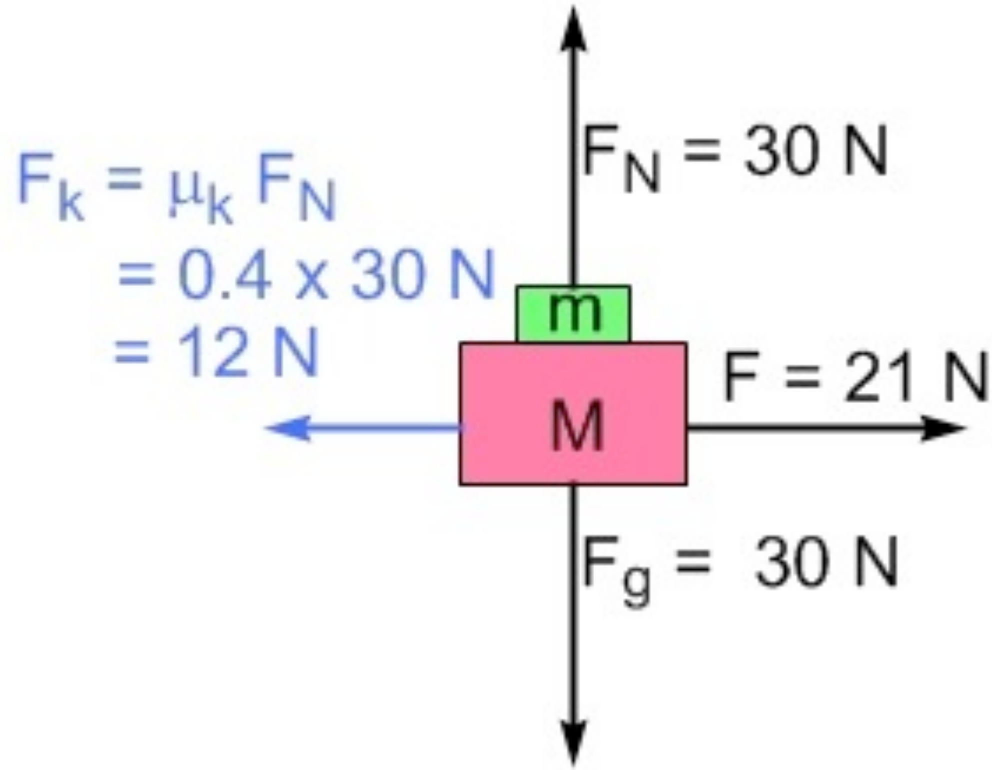
y-direction

$$\sum \vec{F}_y = (m+M)\vec{a}_y$$

$$+F_N - F_g = 0$$

$$F_N = F_g = 30\text{ N}$$

Find the force of kinetic friction



Apply Newton's Second Law

x-direction

$$\sum \vec{F}_x = (m+M)\vec{a}_x$$

$$+F - F_K = (m+M)\vec{a}_x$$

$$\vec{a}_x = \frac{+F - F_K}{m+M}$$

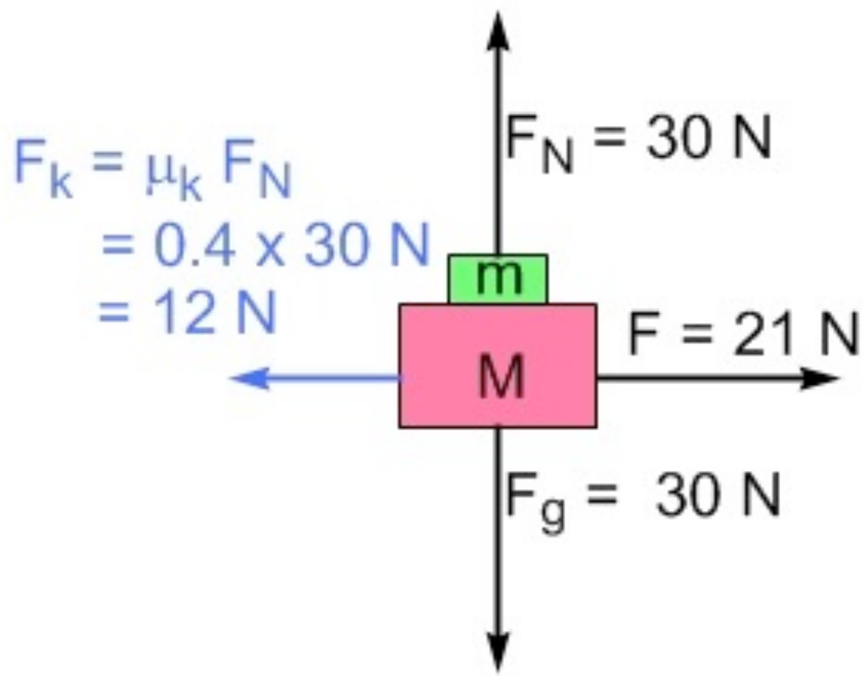
$$\vec{a}_x = \frac{+21\text{ N} - 12\text{ N}}{m+M}$$

$$\vec{a}_x = \frac{+9.0\text{ N}}{3.0\text{ kg}} = +3.0\text{ m/s}^2$$

y-direction

$$\sum \vec{F}_y = (m+M)\vec{a}_y$$

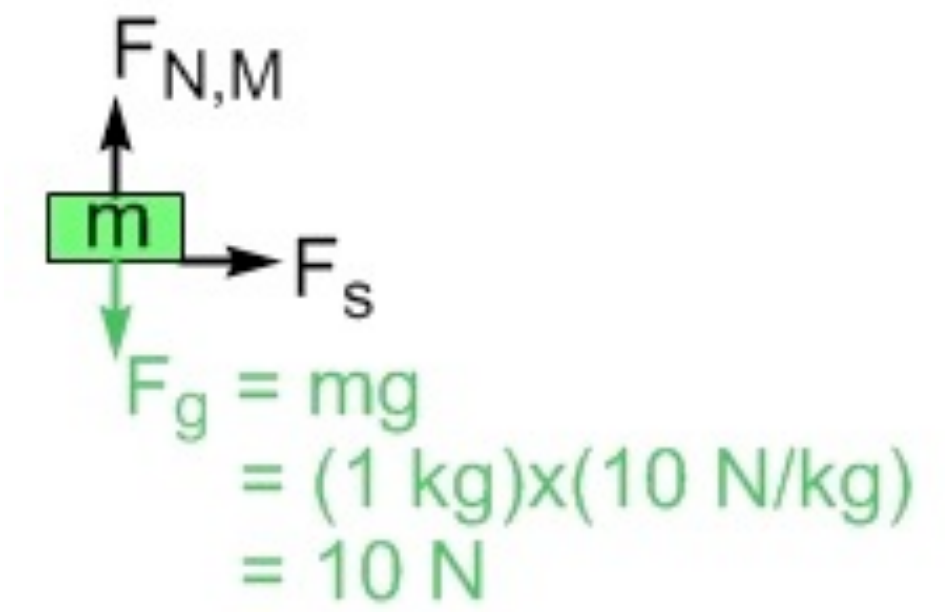
$$F_N = F_g = 30\text{ N}$$



Free-body diagram of the small box

Find the magnitude of the force of friction acting on the small box in this situation.

It may or may not help to know that the normal force applied by the large box on the small box has a magnitude of 10 N.



Free-body diagram of the small box

Find the magnitude of the force of friction acting on the small box in this situation.

It may or may not help to know that the normal force applied by the large box on the small box has a magnitude of 10 N.

$$\sum \vec{F}_y = m\vec{a}_y = 0$$

$$+F_{N,M} - F_g = 0$$

$$F_{N,M} = F_g = 10\ \text{N}$$

The force of friction on the small box

The magnitude of the force of friction acting on the small box is ________ N.

Press the number that is equal to the number of newtons you think the force is.

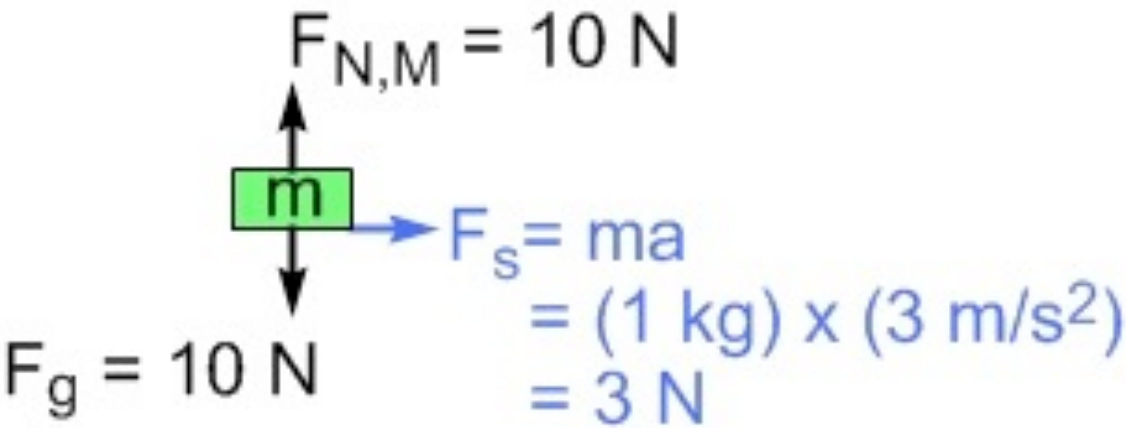
Apply Newton's Second Law

Remember that this force of friction is a static force of friction - there is nothing in the situation stating that we are in a limiting case, so we should not assume that $F_S = F_{S,\max} = \mu_S F_N$.

Instead, apply Newton's second law.

$$\sum \vec{F}_x = m\vec{a}_x$$

$$+F_S = m\vec{a}_x = (1.0 \text{ kg}) \times (3.0 \text{ m/s}^2) = 3.0 \text{ N}$$



Whiteboard