5-1 Kinetic Friction

When two objects are in contact, the friction force, if there is one, is the component of the contact force between the objects that is parallel to the surfaces in contact. (The component of the contact force that is perpendicular to the surfaces is the normal force.) Friction tends to oppose relative motion between objects. When there is relative motion, the friction force is the kinetic force of friction. An example is a book sliding across a table, where kinetic friction slows, and then stops, the book.

If there is relative motion between objects in contact, the force of friction is the kinetic force of friction (F_K). We will use a simple model of friction that assumes the force of kinetic friction is proportional to the normal force. A dimensionless parameter, called the coefficient of kinetic friction, μ_K, represents the strength of that frictional interaction.

$$\mu_K = \frac{F_K}{F_N} \quad \text{so} \quad F_K = \mu_K F_N .$$ (Equation 5.1: **Kinetic friction**)

Some typical values for the coefficient of kinetic friction, as well as for the coefficient of static friction, which we will define in Section 5-2, are given in Table 5.1. The coefficients of friction depend on the materials that the two surfaces are made of, as well as on the details of their interaction. For instance, adding a lubricant between the surfaces tends to reduce the coefficient of friction. There is also some dependence of the coefficients of friction on the temperature.

Interacting materials	**Coefficient of kinetic friction (μ_K)**	**Coefficient of static friction (μ_S)**
Rubber on dry pavement	0.7	0.9
Steel on steel (unlubricated)	0.6	0.7
Rubber on wet pavement	0.5	0.7
Wood on wood	0.3	0.4
Waxed ski on snow	0.05	0.1
Friction in human joints	0.01	0.01

Table 5.1: Approximate coefficients of kinetic friction, and static friction (see Section 5-2), for various interacting materials.

EXPLORATION 5.1 – A sliding book

You slide a book, with an initial speed v_i, across a flat table. The book travels a distance L before coming to rest. What determines the value of L in this situation?

Step 1 – ***Sketch a diagram of the situation and a free-body diagram of the book.*** These diagrams are shown in Figure 5.1.

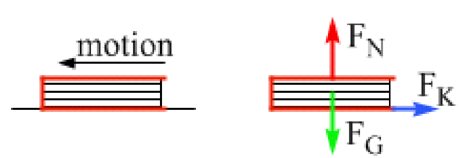

Figure 5.1: A diagram of the sliding book and a free-body diagram showing the forces acting on the book as it slides.

The Earth applies a downward force of gravity on the book, while the table applies a contact force. Generally, we split the contact force into components and show an upward normal force and a horizontal force of kinetic friction, $\vec{F}_K$, acting on the book. $\vec{F}_K$ points to the right on the book, opposing the relative motion between the book (moving left) and the table (at rest).

Step 2 – ***Find an expression for the book's acceleration.*** We are working in two dimensions, and a coordinate system with axes horizontal and vertical is convenient. Let's choose up to be positive for the *y*-axis and, because the motion of the book is to the left, let's choose left to be the positive *x*-direction. We will apply Newton's Second Law twice, once for each direction. There is no acceleration in the *y*-direction, so we have: $\sum \vec{F}_y = m\vec{a}_y = 0$.

Because we're dealing with the *y* sub-problem, we need only the vertical forces on the free-body diagram. Adding the vertical forces as vectors (with appropriate signs) tells us that:

$$+F_N - F_G = 0 \,, \quad \text{so} \quad F_N = F_G = mg \,.$$

Repeat the process for the *x* sub-problem, applying Newton's Second Law, $\sum \vec{F}_x = m\vec{a}_x$. Because the acceleration is entirely in the *x*-direction, we can replace $\vec{a}_x$ by $\vec{a}$. Now, we focus only on the forces in the *x*-direction. All we have is the force of kinetic friction, which is to the right while the positive direction is to the left. Thus: $-F_K = m\vec{a}$.

We can now solve for the acceleration of the book, which is entirely in the *x*-direction:

$$\vec{a} = \frac{-F_K}{m} = \frac{-\mu_K F_N}{m} = \frac{-\mu_K mg}{m} = -\mu_K g \,.$$

Step 3 – ***Determine an expression for length, L, in terms of the other parameters.*** Let's use our expression for the book's acceleration and apply the method for solving a constant-acceleration problem. Table 5.2 summarizes what we know. Define the origin to be the book's starting point and the positive direction to be the direction of motion.

Table 5.2: A summary of what we know about the book's motion to help solve the constant-acceleration problem.

Initial position	$x_i = 0$
Final position	$x = +L$
Initial velocity	$+v_i$
Final velocity	$v = 0$
Acceleration	$a = -\mu_K g$

We can use equation 2.10 to relate the distance traveled to the coefficient of friction:

$$v^2 = v_i^2 + 2a\Delta x \,, \quad \text{so} \quad \Delta x = \frac{v^2 - v_i^2}{2a} \,.$$

In this case, we get $L = \dfrac{0 - v_i^2}{-2\mu_K g} = \dfrac{v_i^2}{2\mu_K g}$.

Let's think about whether the equation we just derived for *L* makes sense. The equation tells us that the book travels farther with a larger initial speed or with smaller values of the coefficient of friction or the acceleration due to gravity, which makes sense. It is interesting to see that there is no dependence on mass – all other parameters being equal, a heavy object travels the same distance as a light object.

Key idea: A useful method for solving a problem with forces in two dimensions is to split the problem into two one-dimensional problems. Then, we solve the two one-dimensional problems individually. **Related End-of-Chapter Exercises 13 – 15, 31.**

Essential Question 5.1: You are standing still and you then start to walk forward. Is there a friction force involved here? If so, is it the kinetic force of friction or the static force of friction?

Answer to Essential Question 5.1: When asked this question, most people are split over whether the friction force is kinetic or static. Think about what happens when you walk. When your shoe (or foot) is in contact with the ground, the shoe does not slip on the ground. Because there is no relative motion between the shoe and the ground, the friction force is static friction.

5-2 Static Friction

If there is no relative motion between objects in contact, then the friction force (if there is one) is the static force of friction (F_S). An example is a box that remains at rest on a ramp. The force of gravity acting down the ramp is opposed by a static force of friction acting up the ramp. A more challenging example is when the box is placed on the floor of a truck. When the truck accelerates and the box moves with the truck (remaining at rest relative to the truck), it is the force of static friction that acts on the box to keep it from sliding around in the truck.

Consider again the question about the friction force between the sole of your shoe and the floor, when you start to walk. In which direction is the force of static friction? Many people think this friction force is directed opposite to the way you are walking, but the force of static friction is actually directed the way you are going. To determine the direction of the force of static friction, think about the motion that would result if there were no friction. To start walking, you push back with your foot on the floor. Without friction, your foot would slide back, moving back relative to the floor, as shown in Figure 5.2. Static friction opposes this motion, the motion that would occur if there was no friction, and thus static friction is directed forward.

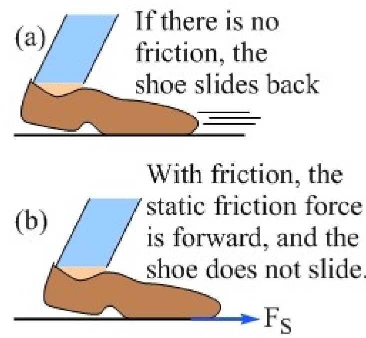

Figure 5.2: On a frictionless floor, your shoe slides backward over the floor when you try to walk forward (a). Static friction opposes this motion, so the static force of friction, applied by the ground on you, is directed forward (b).

The static force of friction opposes the relative motion that would occur if there were no friction. Another interesting feature is that ***the static force of friction adjusts itself to whatever it needs to be to prevent relative motion between the surfaces in contact.*** Within limits, that is. The static force of friction has a maximum value, $F_{S,\max}$, and the coefficient of static friction is defined in terms of this maximum value:

$$\mu_S = \frac{F_{S,\max}}{F_N} \quad \text{so} \quad F_S \le \mu_S F_N \,. \qquad \text{(Equation 5.2: \textbf{Static friction})}$$

Let's now explore a situation that involves the adjustable nature of the force of static friction.

EXPLORATION 5.2 – A box on the floor

A box with a weight of $mg = 40$ N is at rest on a floor. The coefficient of static friction between the box and the floor is $\mu_S = 0.50$, while the coefficient of kinetic friction between the box and the floor is $\mu_K = 0.40$.

Step 1 - ***What is the force of friction acting on the box if you exert no force on the box?*** Let's draw a free-body diagram of the box (see Figure 5.3b) as it sits at rest. Because the box remains at rest, its acceleration is zero and the forces must balance. Applying Newton's Second Law tells us that $F_N = mg = 40\text{ N}$. There is no tendency for the box to move, so there is no force of friction.

Step 2 - ***What is the force of friction acting on the box if you push horizontally on the box with a force of 10 N, as in Figure 5.3a?*** Nothing has changed vertically, so we still have $F_N = mg = 40 \text{ N}$. To determine whether or not the box moves, let's use equation 5.2 to determine the maximum possible force of static friction in this case. We get:

$$F_S \leq \mu_S F_N = 0.50 \times 40 \text{ N} = 20 \text{ N}.$$

The role of static friction is to keep the box at rest. If we exert a horizontal force of 10 N on the box, the force of static friction acting on the box must be 10 N in the opposite direction, to keep the box from moving. The free-body diagram of this situation is shown in Figure 5.3c. 10 N is below the 20 N maximum value, so the box will not move.

Step 3 - ***What is the force of friction acting on the box if you increase your force to 15 N?*** This situation is similar to step 2. Now, the force of static friction adjusts itself to 15 N in the opposite direction of your 15 N force. 15 N is still less than the maximum possible force of static friction (20 N), so the box does not move.

Step 4 - ***What is the force of friction acting on the box if you increase your force to 20 N?*** If your force is 20 N, the force of static friction matches you with 20 N in the opposite direction. We are now at the maximum possible value of the force of static friction. Pushing even a tiny bit harder would make the box move.

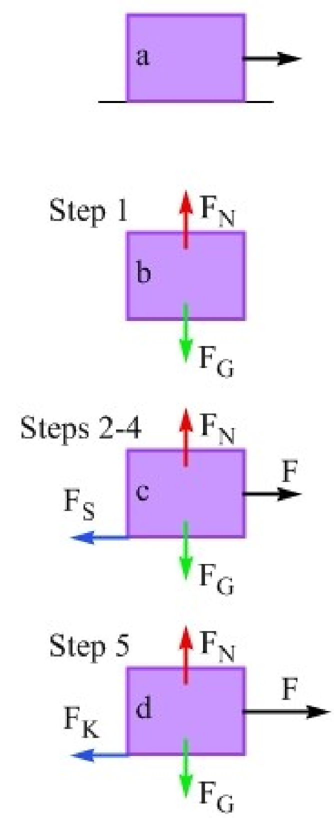

Figure 5.3: (a) The top diagram shows the box, and the force you exert on it. (b) The free-body diagram for step 1, in which you exert no force. (c) The free-body diagram that applies to steps 2 – 4, in which the force you exert is less than or equal to the maximum possible force of static friction. (d) The free-body diagram that applies to step 5, in which your force is large enough to cause the box to move.

Step 5 - ***What is the force of friction acting on the box if you increase your force to 25 N?*** Increasing your force to 25 N, which is larger in magnitude than the maximum possible force of static friction, makes the box move. Because the box moves, the friction is the kinetic force of friction, which is in the direction opposite to your force with a magnitude of $F_K = \mu_K F_N = 0.40 \times 40 \text{ N} = 16 \text{ N}$.

Key ideas for static friction: The static force of friction is whatever is required to prevent relative motion between surfaces in contact. The static force of friction is adjustable only up to a point. If the required force exceeds the maximum value $F_{S,\max} = \mu_S F_N$, then relative motion will occur. **Related End-of-Chapter Exercises: 32, 34.**

A microscopic model of friction

Figure 5.4 shows a magnified view of two surfaces in contact. Surface irregularities interfere with the motion of one surface left or right with respect to the other surface, giving rise to friction.

Figure 5.4: A magnified view of two surfaces in contact. The irregularities in the objects prevent smooth motion of one surface over the other, giving rise to friction.

Essential Question 5.2: What is the magnitude of the net contact force exerted by the floor on the box in step 5 of Exploration 5.2?

Answer to Essential Question 5.2: The two components of the contact force are the upward normal force and the horizontal force of kinetic friction. These two components are at right angles to one another, so we can use the Pythagorean theorem to find the magnitude of the contact force:

$$F_C = \sqrt{F_K^2 + F_N^2} = \sqrt{(16\text{ N})^2 + (40\text{ N})^2} = 43\text{ N}.$$

5-3 Measuring the Coefficient of Friction

Let's connect the force ideas to the one-dimensional motion situations from Chapter 2.

EXPLORATION 5.3 – Measuring the coefficient of static friction

Coefficients of static friction for various pairs of materials are given in Table 5.1. Here's one method for experimentally determining these coefficients for a particular pair of materials. Take an aluminum block of mass *m* and a board made from a particular type of wood (we could also use a block of the wood and a piece of inflexible aluminum). Place the block on the board and slowly raise one end of the board. The angle of the board when the block starts to slide gives the coefficient of static friction. How?

To answer this question, let's extend the general method for solving a problem that involves Newton's laws.

Step 1 – ***Draw a diagram of the situation.*** The diagram is shown in Figure 5.5a.

Step 2 – ***Draw a free-body diagram of the block when it is at rest on the inclined board.*** Two forces act on the block, the downward force of gravity and the upward contact force from the board. We generally split the contact force into components, the normal force perpendicular to the incline, and the force of static friction acting up the slope. This free-body diagram is shown in Figure 5.5b

Step 3 – ***Choose an appropriate coordinate system.*** In this case, if we choose a coordinate system aligned with the board (one axis parallel to the board and the other perpendicular to it), as in Figure 5.5c, we will only have to split the force of gravity into components.

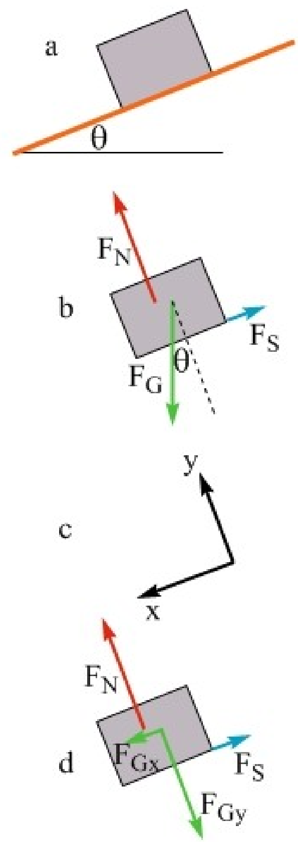

Figure 5.5: (a) A diagram showing the block on the board. (b) The initial free-body diagram of the block. (c) An appropriate coordinate system, aligned with the incline. (d) A second free-body diagram, with the forces aligned parallel to the coordinate axes.

Step 4 – ***Split the force of gravity into components.*** If the angle between the board and the horizontal is θ, the angle between the force of gravity and the *y*-axis is also θ. The component of the force of gravity acting parallel to the slope has a magnitude of $F_{Gx} = F_G \sin\theta = mg\sin\theta$. The perpendicular component is $F_{Gy} = F_G \cos\theta = mg\cos\theta$. These components are shown on the lower free-body diagram, in Figure 5.5d.

Step 5 – ***Apply Newton's second law twice, once for each direction.*** Again, we break a two-dimensional problem into two one-dimensional problems. With no acceleration in the *y*-direction we get: $\sum \vec{F}_y = m\vec{a}_y = 0$. Looking at the lower diagram in Figure 5.5 for the *y*-direction forces:

$$+F_N - mg\cos\theta = 0, \quad \text{which tells us that } F_N = mg\cos\theta .$$

While the box is at rest, there is no acceleration in the x-direction so: $\sum \vec{F}_x = m\vec{a}_x = 0$. Looking at the lower free-body diagram, Figure 5.5d, for the forces in the x-direction:

$$mg\sin\theta - F_S = 0, \quad \text{which tells us that } F_S = mg\sin\theta .$$

These two equations, $F_N = mg\cos\theta$ and $F_S = mg\sin\theta$, tell us a great deal about what happens as the angle of the incline increases. When the board is horizontal, the normal force is equal to mg and the static force of friction is zero. As the angle increases, $\cos\theta$ decreases from 1 and $\sin\theta$ increases from zero. Thus, as the angle increases, the normal force (and the maximum possible force of static friction) decreases, while the force of static friction required to keep the block at rest increases. At some critical angle θ_C, the force of static friction needed to keep the block at rest equals the maximum possible force of static friction. If the angle exceeds this critical angle, the block will slide. Using the definition of the coefficient of static friction:

$$\mu_S = \frac{F_{S,\max}}{F_N} = \frac{mg\sin\theta_C}{mg\cos\theta_C} = \tan\theta_C .$$

Thus, it is easy to determine coefficients of static friction experimentally. Take two objects and place one on top of the other. Gradually tilt the objects until the top one slides off. The tangent of the angle at which sliding occurs is the coefficient of static friction.

Key idea regarding the coefficient of static friction: The coefficient of static friction between two objects is the tangent of the angle beyond which one object slides down the other.
Related End-of-Chapter Exercises: 7, 36.

The steps we used to solve the problem in Exploration 5.3 can be applied generally to most problems involving Newton's laws. Let's summarize the steps here. Then, we will get some more practice applying the method in the next section.

A General Method for Solving a Problem Involving Newton's Laws in Two Dimensions

1. Draw a diagram of the situation.

2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object.

3. For each free-body diagram, choose an appropriate coordinate system. Coordinate systems for different free-body diagrams should be consistent with one another. A good rule of thumb is to align each coordinate system with the direction of the acceleration.

4. Break forces into components that are parallel to the coordinate axes.

5. Apply Newton's second law twice to each free-body diagram, once for each coordinate axis. Put the resulting force equations together and solve.

Related End-of-Chapter Exercises 17, 19.

Essential Question 5.3: Could we modify the procedure described in Exploration 5.3 to measure the coefficient of kinetic friction? If so, how could we modify it?