Demonstration: Projectile Motion

One ball is released from rest at a height *h*. A second ball is simultaneously fired with a horizontal velocity at the same height. Which ball lands on the ground first?

1. The first ball
2. The second ball
3. Both hit the ground at the same time

Neglect air resistance.

The independence of *x* and *y*

This demonstration shows a powerful idea, that the vertical motion happens completely independently of the horizontal motion.

Equations for 2-D motion

Add an x or y subscript to the usual equations of 1-D motion.

$$v_x = v_{ix} + a_x t \qquad v_y = v_{iy} + a_y t$$

$$x = x_i + v_{ix} t + \frac{1}{2} a_x t^2 \qquad y = y_i + v_{iy} t + \frac{1}{2} a_y t^2$$

$$v_x{}^2 = v_{ix}{}^2 + 2a_x \left(x - x_i \right) \qquad v_y{}^2 = v_{iy}{}^2 + 2a_y \left(y - y_i \right)$$

Worksheet for today

Let’s see the independence idea in action.

Graphing the vertical motion

Here’s a trick – use the average velocity for a 1-s interval.

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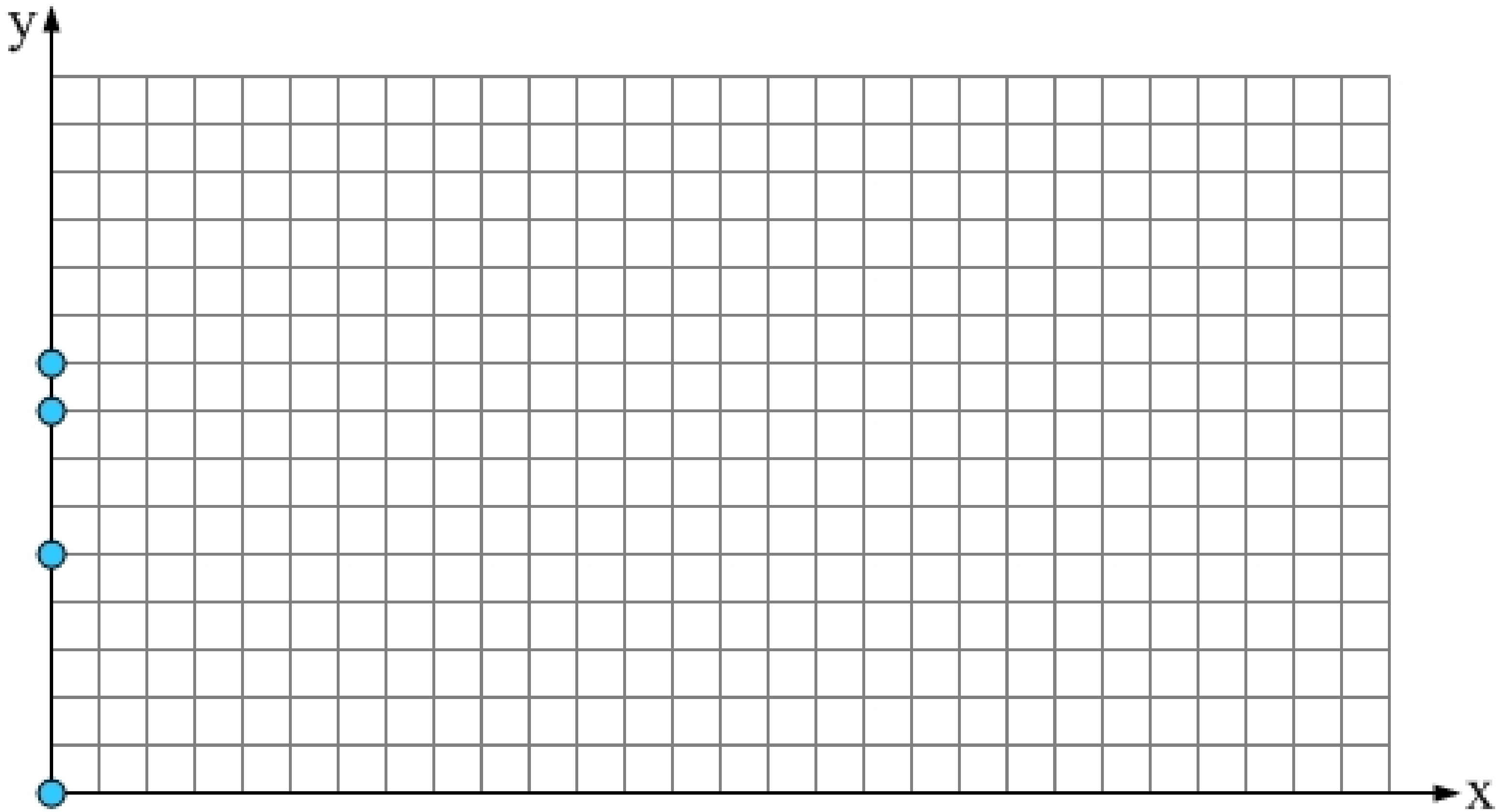
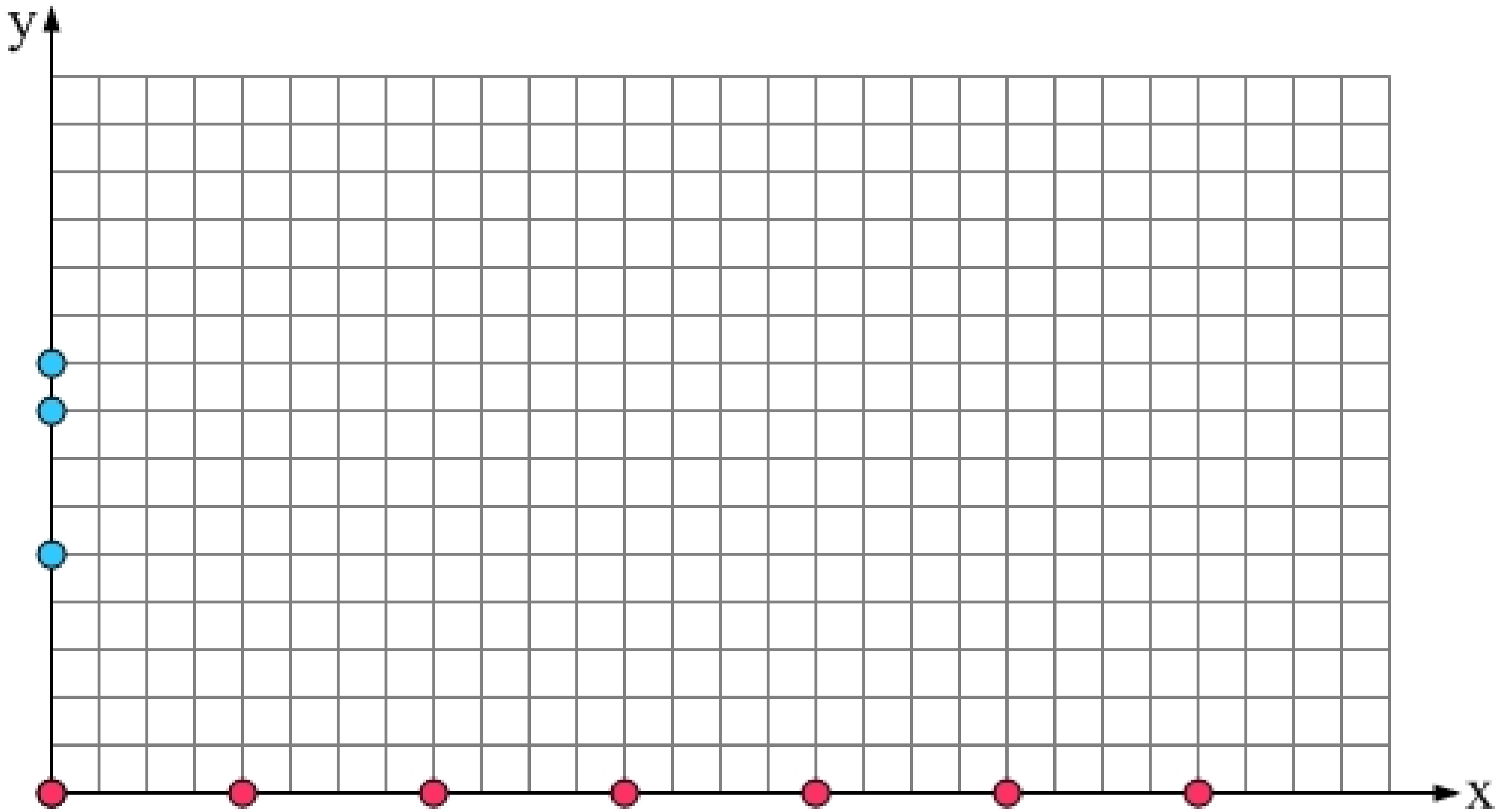


Graphing the horizontal motion

This one should be a lot easier to do.

Graphing the horizontal motion

The dots are equally spaced, with 1 every 4 boxes.

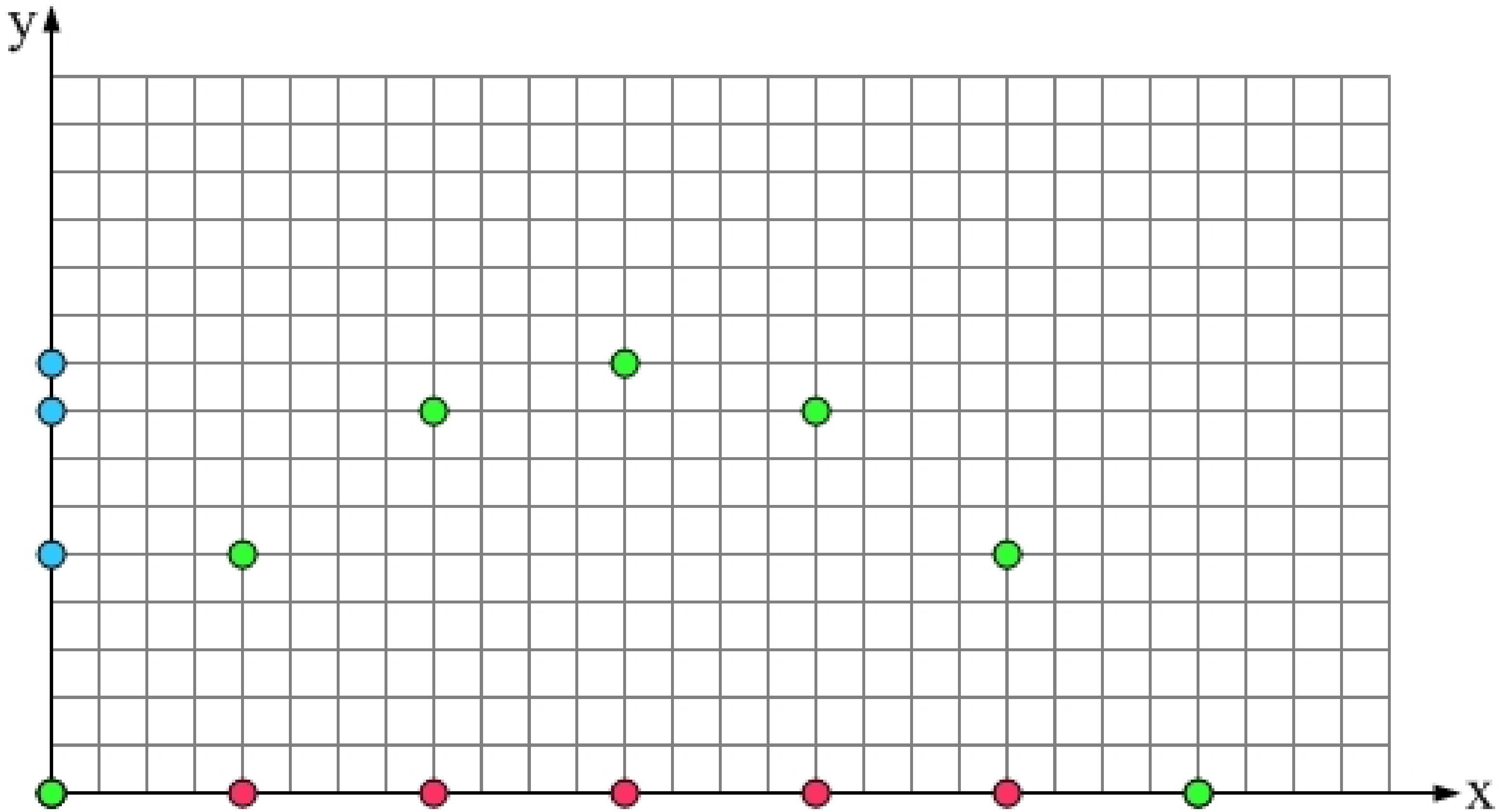


Graphing the projectile motion

Can you make use of anything we did already?

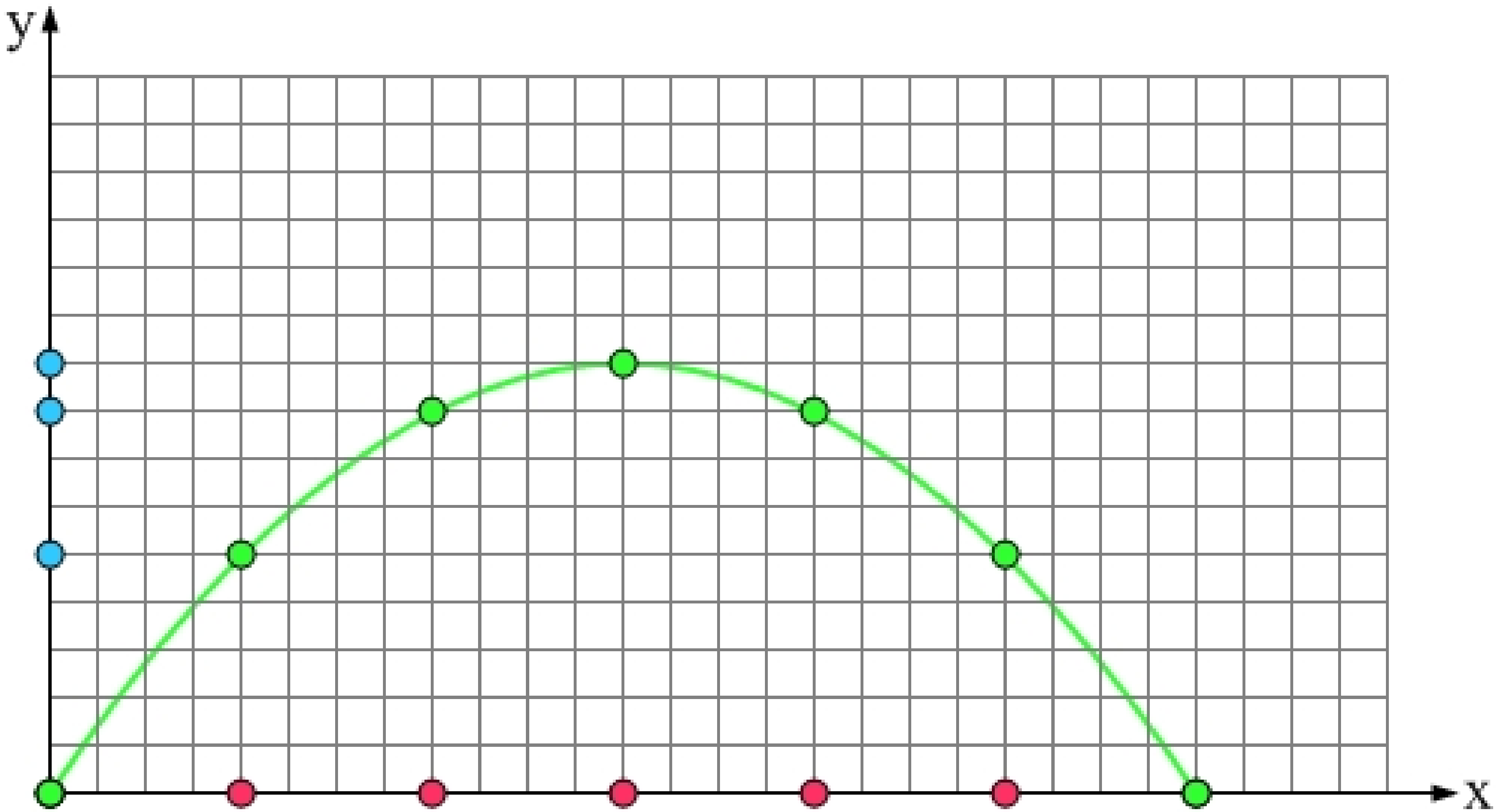
Graphing the projectile motion

Connect the *y*-axis dots and the *x*-axis dots.



Graphing the projectile motion

Connect the dots to get a parabola.



A ballistics cart

While a cart is moving horizontally at constant velocity, it fires a ball straight up into the air. Where does the ball land?

1. Behind the cart
2. Ahead of the cart
3. In the cart

Neglect air resistance.

The ballistics cart

Again, we see that the vertical motion happens completely independently from the horizontal motion.

Projectile motion

Projectile motion is motion under the influence of gravity alone.

A thrown object is a typical example. Follow the motion from the time just **after** the object is released until just **before** it hits the ground.

Air resistance is neglected. The only acceleration is the acceleration due to gravity.

Maximum height

To find the maximum height reached by a projectile, use

$$v_y^2 = v_{iy}^2 + 2a_y\left(y - y_i\right)$$

Let's say up is positive, $y_i = 0$, and $v_y = 0$ at maximum height. Also, $a_y = -g$.

Solving for y_{max}:

$$y_{\max} = \frac{v_y^2 - v_{iy}^2}{2a_y} = \frac{-v_{iy}^2}{-2g} = \frac{v_{iy}^2}{2g}$$

The maximum height depends on what planet you're on, and on the y-component of the initial velocity.

Time to reach maximum height

To find the maximum height reached by a projectile, use

$$v_y = v_{0y} + a_y t$$

Let's say up is positive, and $v_y = 0$ at maximum height. Also, $a_y = -g$.

Solving for $t_{maxheight}$:

$$t_{\text{maxheight}} = \frac{-v_{oy}}{a_y} = \frac{-v_{0y}}{-g} = \frac{v_{0y}}{g}$$

The time to reach maximum height depends on what planet you're on, and on the y-component of the initial velocity.

Time for the whole motion

If the projectile starts and ends at the same height, how does the time for the whole trip compare to the time to reach maximum height?

Time for the whole motion

If the projectile starts and ends at the same height, how does the time for the whole trip compare to the time to reach maximum height?

The down part of the trip is a mirror image of the up part of the trip, so:

$$t_{\text{total}} = 2t_{\text{maxheight}} = \frac{2v_{iy}}{g}$$

Example problem

3.00 seconds after being launched from ground level with an initial speed of 25.0 m/s, an arrow passes just above the top of a tall tree. The base of the tree is 45.0 m from the launch point. Neglect air resistance and assume that the arrow lands at the same level from which it was launched. Use g = 10.0 m/s^2.

(a) At what angle, measured from the horizontal, was the arrow launched? Feel free to find the sine, cosine, or tangent of the angle instead of the angle itself if you find that to be easier.

Step 1

Draw a diagram.

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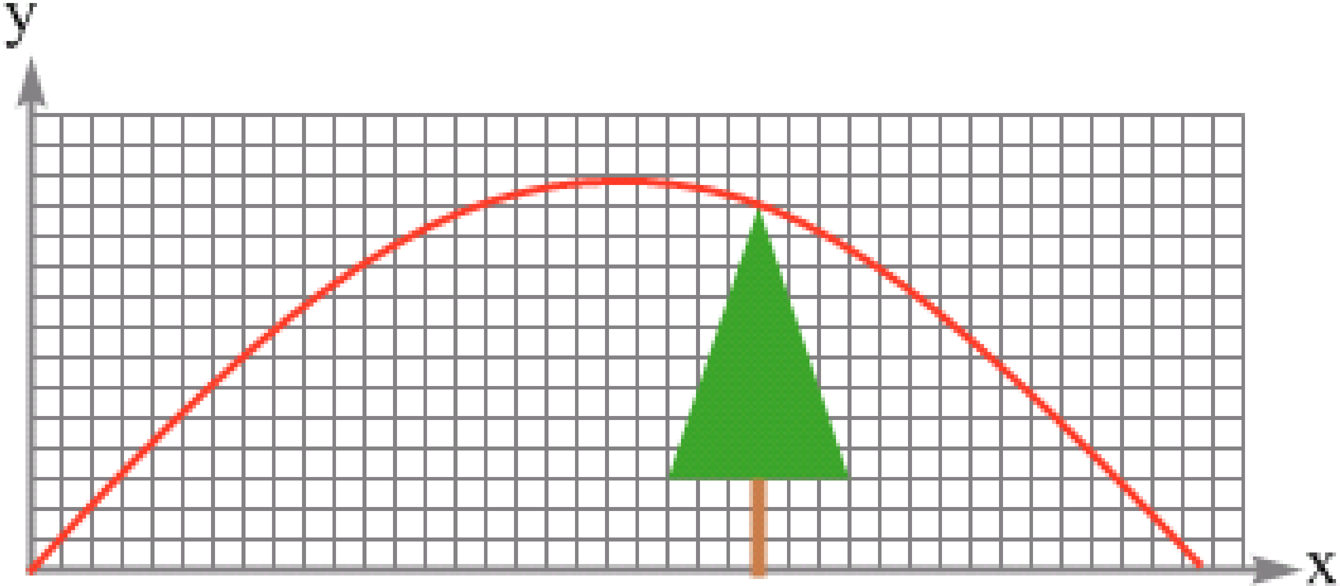
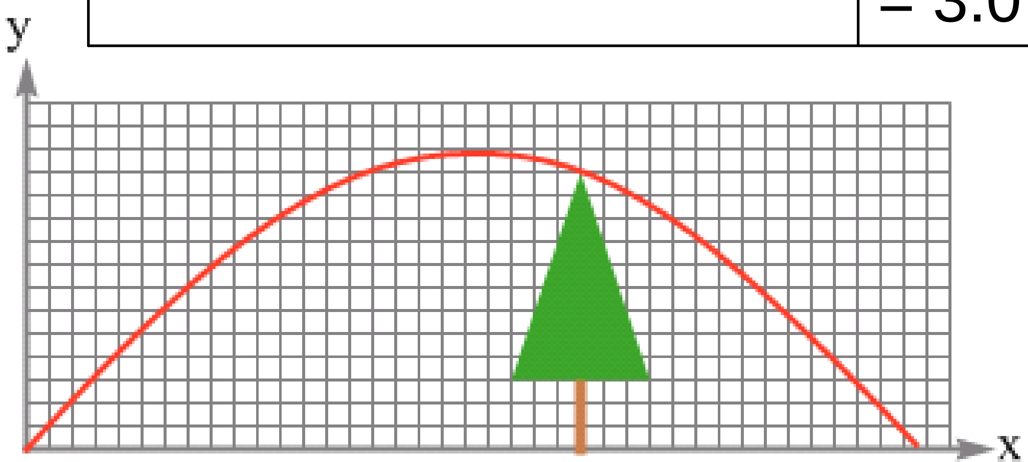


Step 2 – Make a data table

Keep the *x* information separate from the *y* information.

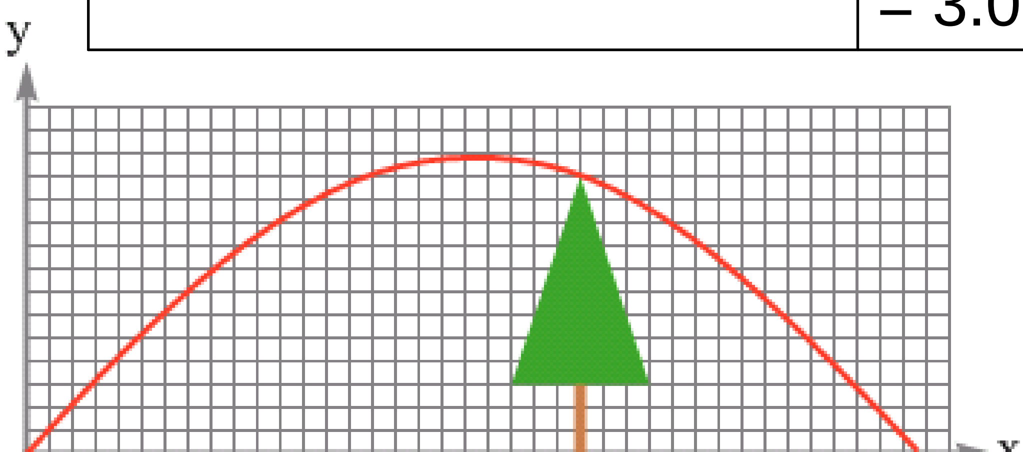
	x-direction	*y*-direction
Positive direction	?	?
Initial position	x_0 = ?	y_0 = ?
Initial velocity	v_{0x} = ?	v_{0y} = ?
Acceleration	a_x = ?	a_y = ?
Displacement and time	x = 45.0 m at t = 3.0 s	y = height of tree at t = 3.00 s



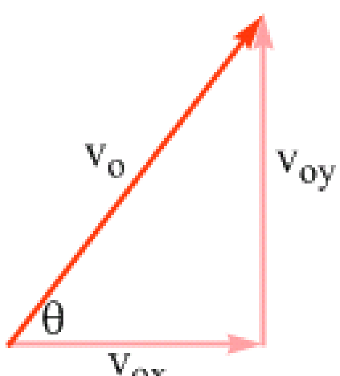
Step 2 – Make a data table

Keep the *x* information separate from the *y* information.

	x-direction	*y*-direction
Positive direction	right	up
Initial position	$x_0 = 0$	$y_0 = 0$
Initial velocity	$v_{0x} = v_0 \cos\theta$	$v_{0y} = v_0 \sin\theta$
Acceleration	$a_x = 0$	$a_y = -g$
Displacement and time	$x = 45.0$ m at $t = 3.00$ s	y = height of tree at $t = 3.00$ s



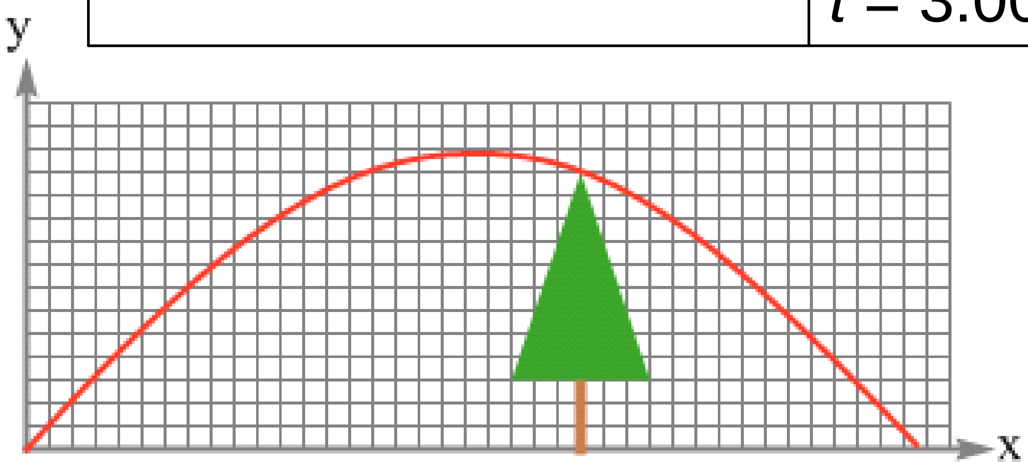
$v_0 = 25.0$ m/s



Part (a) – Find the launch angle.

Should we solve the *x* sub-problem or the *y* sub-problem?

	x-direction	*y*-direction
Positive direction	right	up
Initial position	$x_0 = 0$	$y_0 = 0$
Initial velocity	$v_{0x} = v_0 \cos\theta$	$v_{0y} = v_0 \sin\theta$
Acceleration	$a_x = 0$	$a_y = -g$
Displacement and time	x = 45.0 m at t = 3.00 s	y = height of tree at t = 3.00 s



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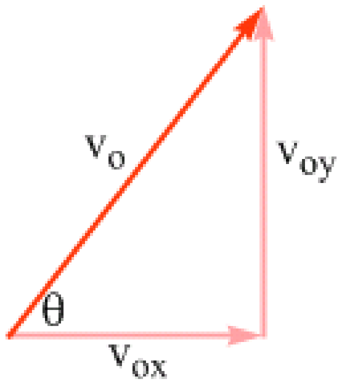

Part (a) – Find the launch angle.

Let's solve the *x* sub-problem.

	x-direction
Positive direction	right
Initial position	$x_0 = 0$
Initial velocity	$v_{0x} = v_0 \cos\theta$
Acceleration	$a_x = 0$
Displacement and time	$x = 45.0$ m at $t = 3.00$ s

$v_0 = 25.0$ m/s

Part (a) – Find the launch angle.

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Positive direction	right
Initial position	$x_0 = 0$
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Acceleration	$a_x = 0$
Displacement and time	$x = 45.0$ m at $t = 3.00$ s

$v_0 = 25.0$ m/s

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

Part (a) – Find the launch angle.

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Positive direction	right
Initial position	$x_0 = 0$
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Displacement and time	$x = 45.0$ m at $t = 3.00$ s

$v_0 = 25.0$ m/s

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$x = v_{0x}t$$

$$v_{0x} = \frac{x}{t} = \frac{45.0 \text{ m}}{3.00 \text{ s}} = 15.0 \text{ m/s}$$

Part (a) – Find the launch angle.

Let's solve the *x* sub-problem.

	x-direction
Positive direction	right
Initial position	$x_0 = 0$
Initial velocity	$v_{0x} = v_0 \cos\theta$
Acceleration	$a_x = 0$
Displacement and time	$x = 45.0$ m at $t = 3.00$ s

$v_0 = 25$ m/s

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$x = v_{0x}t$$

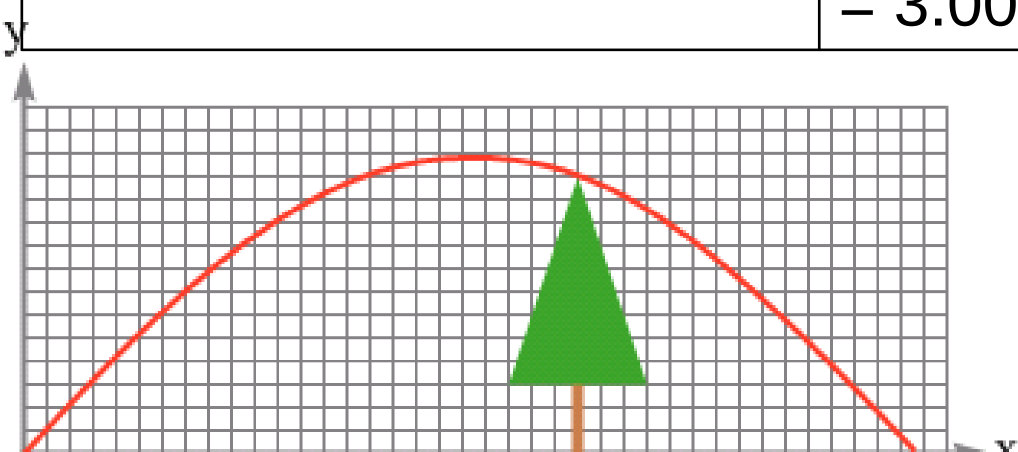
$$v_{0x} = \frac{x}{t} = \frac{45.0\text{ m}}{3.00\text{ s}} = 15.0\text{ m/s}$$

$$\cos\theta = \frac{v_{0x}}{v_0} = \frac{15.0\text{ m/s}}{25.0\text{ m/s}} = \frac{3}{5}$$

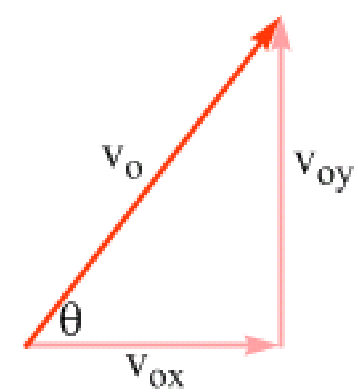
Back to the data table

Let's fill in what we know in the table.

	x-direction	y-direction
Positive direction	right	up
Initial position	$x_0 = 0$	$y_0 = 0$
Initial velocity	$v_{0x} = +15.0$ m/s	$v_{0y} = v_0 \sin\theta$
Acceleration	$a_x = 0$	$a_y = -g$
Displacement and time	$x = 45.0$ m at $t = 3.00$ s	y = height of tree at $t = 3.00$ s



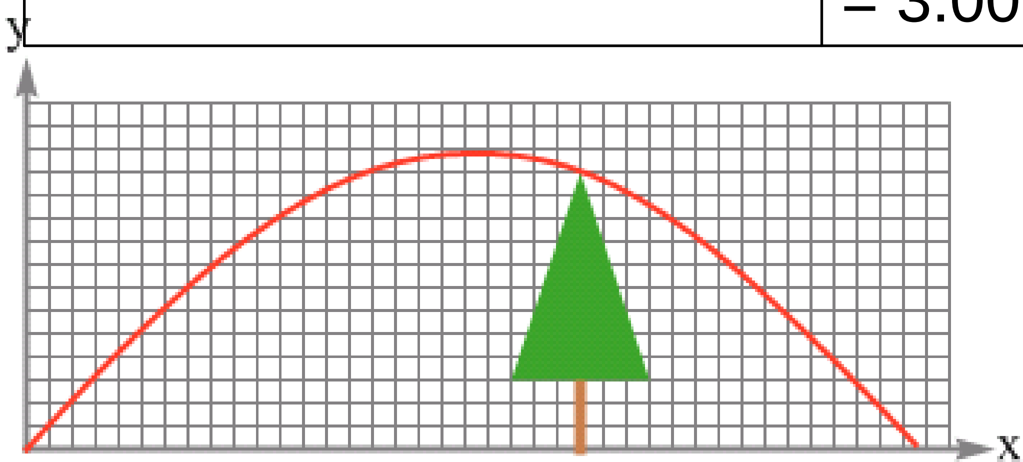
$v_0 = 25.0$ m/s



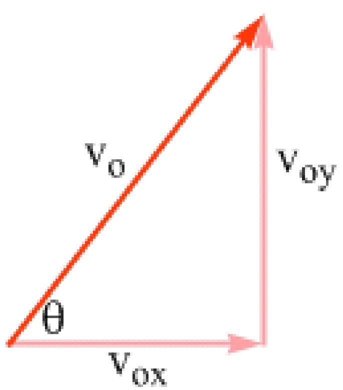
Back to the data table

Let's fill in what we know in the table.

	x-direction	y-direction
Positive direction	right	up
Initial position	$x_0 = 0$	$y_0 = 0$
Initial velocity	$v_{0x} = +15.0$ m/s	$v_{0y} = +20.0$ m/s
Acceleration	$a_x = 0$	$a_y = -g$
Displacement and time	$x = 45.0$ m at $t = 3.00$ s	y = height of tree at $t = 3.00$ s



$v_0 = 25.0$ m/s



How tall is the tree?

The arrow barely clears the tree when it passes over the tree 3.00 s after being launched.

Find the height of the tree.

1. less than 60.0 m
2. 60.0 m
3. more than 60.0 m

Part (b) – the height of the tree

Should we solve the *x* or the *y* sub-problem?

	x-direction	*y*-direction
Positive direction	right	up
Initial position	$x_0 = 0$	$y_0 = 0$
Initial velocity	$v_{0x} = +15.0$ m/s	$v_{0y} = +20.0$ m/s
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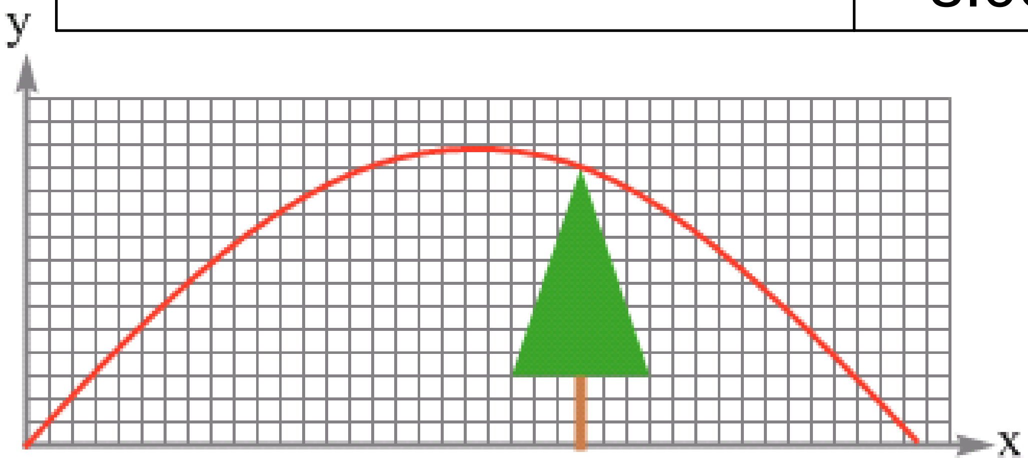
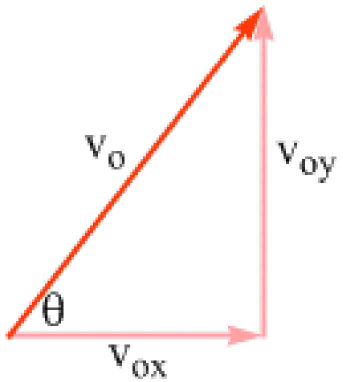


$v_0 = 25.0$ m/s



Part (b) – the height of the tree

This a job for the *y* direction.

	y-direction
Positive direction	up
Initial position	$y_0 = 0$
Initial velocity	$v_{0y} = +20.0$ m/s
Acceleration	$a_y = -g$
Displacement and time	y = height of tree at $t = 3.00$ s

Part (b) – the height of the tree

This a job for the *y* direction.

	y-direction
Positive direction	up
Initial position	$y_0 = 0$
Initial velocity	$v_{0y} = +20.0$ m/s
Acceleration	$a_y = -g$
Displacement and time	y = height of tree at $t = 3.00$ s

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

Part (b) – the height of the tree

This a job for the *y* direction

	y-direction
Positive direction	up
Initial position	$y_0 = 0$
Initial velocity	v_{0y} = +20.0 m/s
Acceleration	$a_y = -g$
Displacement and time	y = height of tree at t = 3.00 s

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$y = 0 + (+20.0\ \text{m/s}) \times (3.00\ \text{s}) + \frac{1}{2}(-10.0\ \text{m/s}^2) \times (3.00)^2$$

Part (b) – the height of the tree

This a job for the *y* direction

	y-direction
Positive direction	up
Initial position	$y_0 = 0$
Initial velocity	v_{0y} = +20.0 m/s
Acceleration	$a_y = -g$
Displacement and time	y = height of tree at t = 3.00 s

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$y = 0 + (+20.0\ \text{m/s}) \times (3.00\ \text{s}) + \frac{1}{2}(-10.0\ \text{m/s}^2) \times (3.00)^2$$

$$y = +60.0\ \text{m} - 45.0\ \text{m} = +15.0\ \text{m}$$

Ballistic cart, going down

The ballistic cart is released from rest on an incline, and it shoots the ball out perpendicular to the incline as it rolls down. Where does the ball land?

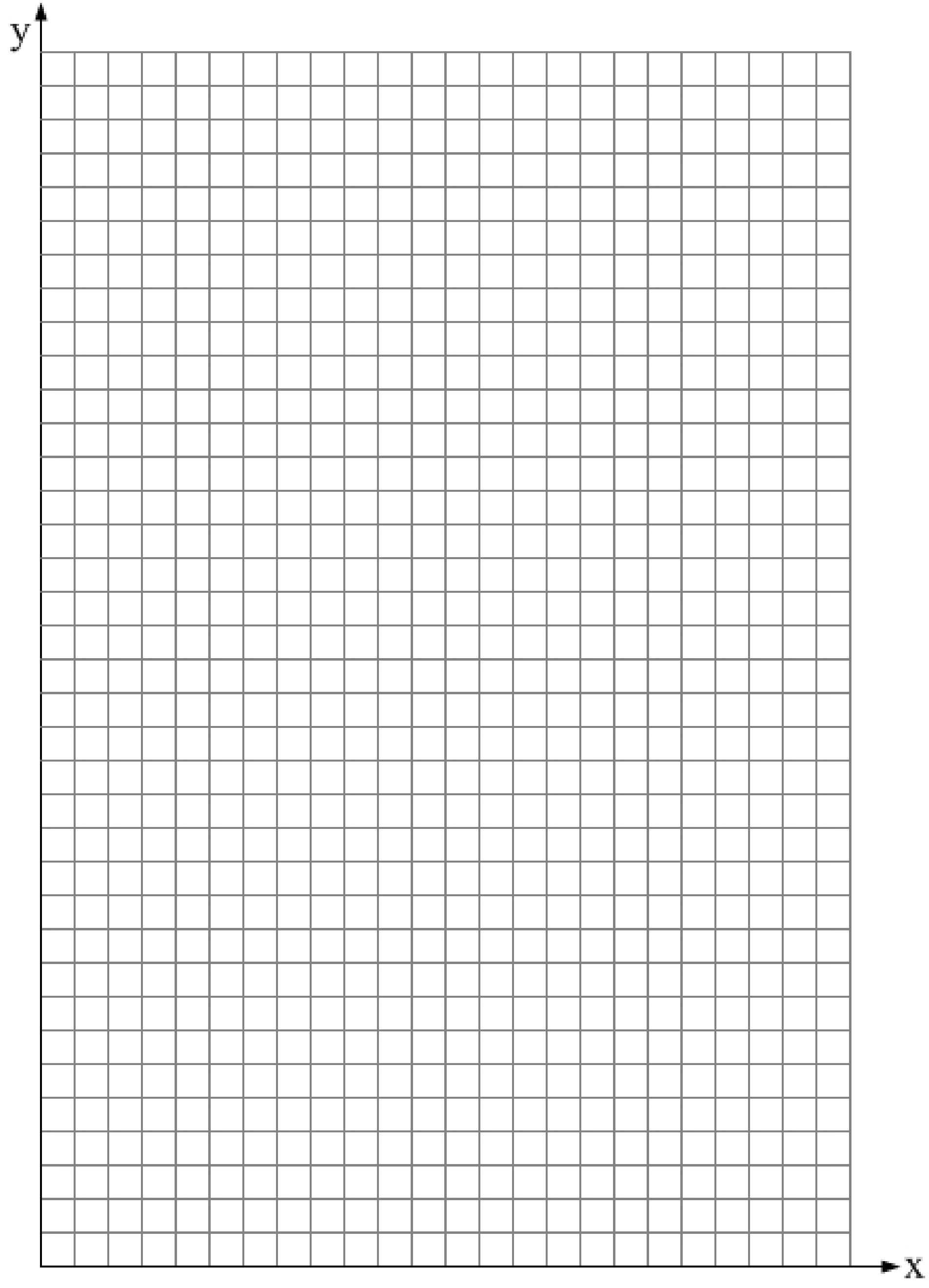
1. Ahead of the cart
2. In the cart
3. Behind the cart

Ballistic cart, going up

The ballistic cart is released from rest on an incline, and it shoots the ball out perpendicular to the incline as it rolls down. Where does the ball land?

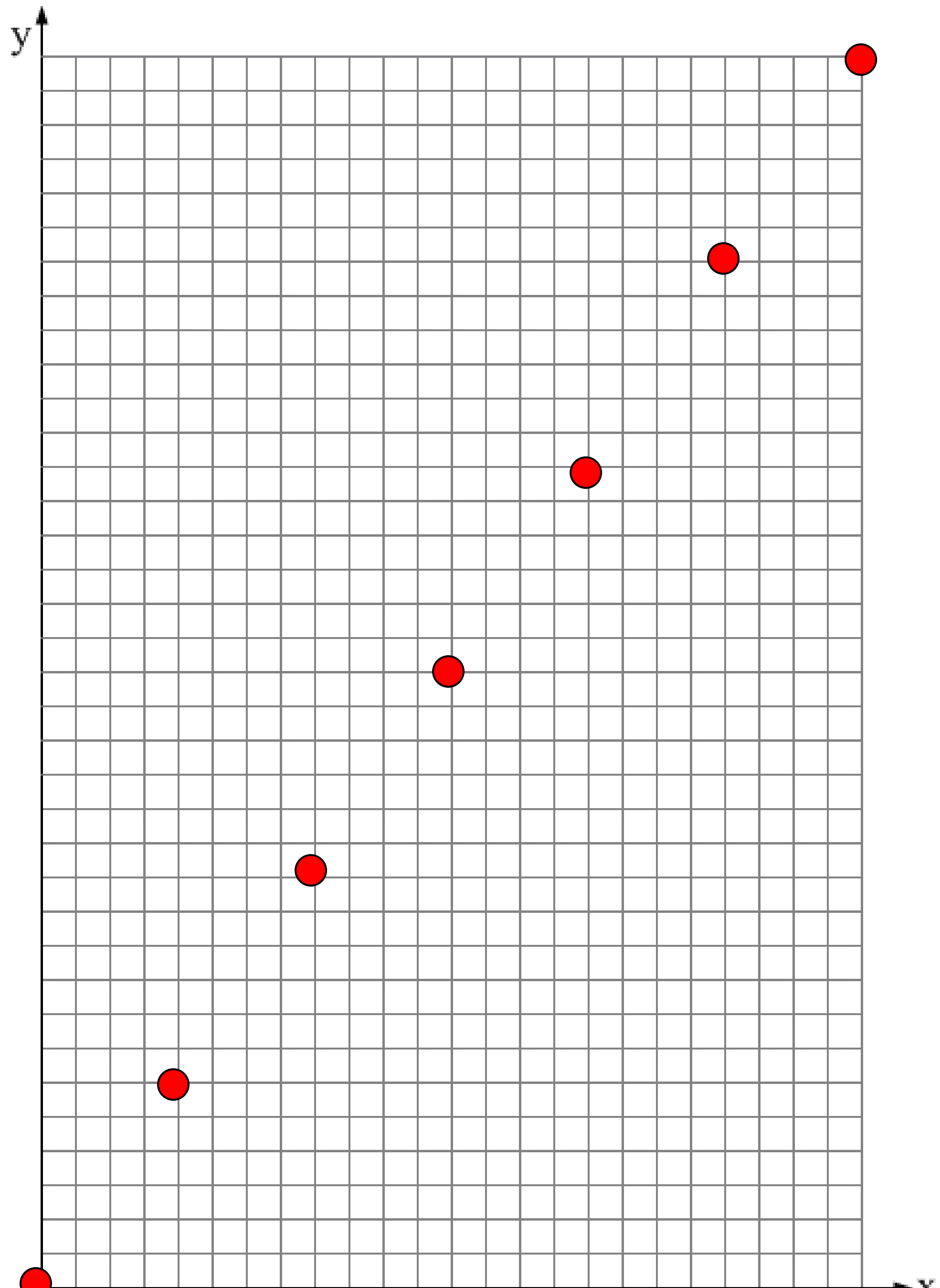
1. Ahead of the cart (uphill)
2. In the cart
3. Behind the cart (downhill)

Whole Vectors



Whole Vectors

$r = v_i\ t$



Whole Vectors

$r = v_i\,t - 0.5\,g\,t^2$