Answer to Essential Question 1.3: The Pythagorean theorem is a special case of the Cosine Law that applies to right-angled triangles. With an angle of 90˚ opposite the hypotenuse, the last term in the Cosine Law disappears because cos(90˚) = 0, leaving $c^2 = a^2 + b^2$.

1-4 Vectors

It is always important to distinguish between a quantity that has only a magnitude, which we call a **scalar**, and a quantity that has both a magnitude and a direction, which we call a **vector**. When we work with scalars and vectors we handle minus signs quite differently. For instance, temperature is a scalar, and a temperature of +30˚C feels quite different to you than a temperature of –30˚C. On the other hand, velocity is a vector quantity. Driving at +30 m/s north feels much the same as driving at –30 m/s north (or, equivalently, +30 m/s south), assuming you're going forward in both cases, at least! In the two cases, the speed at which you're traveling is the same, it's just the direction that changes. So, ***a minus sign for a vector tells us something about the direction of the vector; it does not affect the magnitude (the size) of the vector***.

When we write out a vector we draw an arrow on top to represent the fact that it is a vector, for example $\vec{A}$. *A*, drawn without the arrow, represents the magnitude of the vector.

EXPLORATION 1.4 – Vector components

Consider the vectors $\vec{A}$ and $\vec{B}$ represented by the arrows in Figure 1.4 below. The vector $\vec{A}$ lines up exactly with one of the points on the grid. The vector $\vec{B}$ has a magnitude of 4.00 m and is directed at an angle of 63.8˚ below the positive *x*-axis. It is often useful (if we're adding the vectors together, for instance) to find the **components** of the vectors. In this Exploration, we'll use a two-dimensional coordinate system with the positive *x*-direction to the right and the positive *y*-direction up. Finding the *x* and *y* components of a vector involves determining how much of the vector is directed right or left, and how much is directed up or down, respectively.

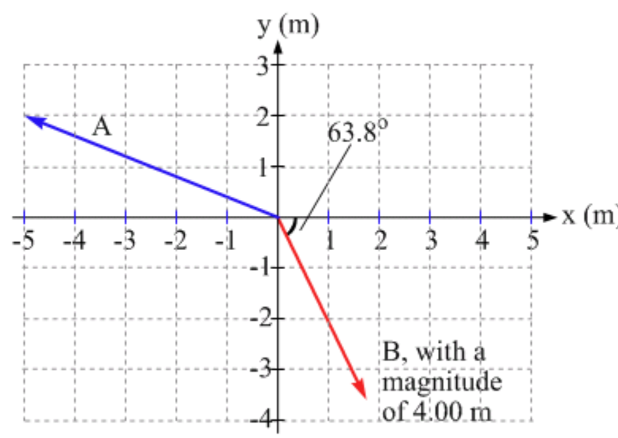


Figure 1.4: The vectors $\vec{A}$ and $\vec{B}$.

Step 1 - *Find the components of the vector* $\vec{A}$. The *x* and *y* components of $\vec{A}$ ($\vec{A}_x$ and $\vec{A}_y$, respectively) can be determined directly from Figure 1.4. Conveniently, the tip of $\vec{A}$ is located at an intersection of grid lines. In this case, we go exactly 5 m to the left and exactly 2 m up, so we can express the *x* and *y* components as:

$\vec{A}_x = +5 \text{ m}$ to the left, or $\vec{A}_x = -5 \text{ m}$ to the right.

$\vec{A}_y = +2 \text{ m}$ up.

This makes it look like we know the components of $\vec{A}$ to an accuracy of only one significant figure. The components are known far more precisely than that, because $\vec{A}$ lines up exactly with the grid lines. The components of $\vec{A}$ are shown in Figure 1.5.

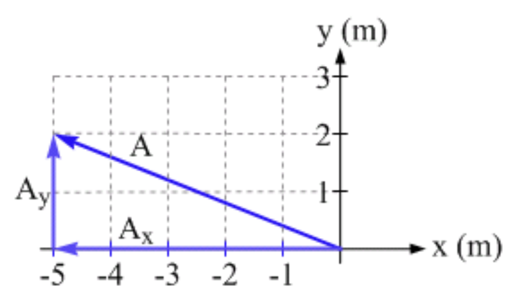


Figure 1.5: Components of the vector $\vec{A}$.

Step 2 – ***Express the vector $\vec{A}$ in unit-vector notation.*** Any vector is the vector sum of its components. For example, $\vec{A} = \vec{A}_x + \vec{A}_y$. This is shown graphically in Figure 1.5. It is rather long-winded to say $\vec{A} = -5$ m to the right + 2 m up. We can express the vector in a more compact form by using **unit vectors**. A unit vector is a vector with a magnitude of 1 unit. We will draw a unit vector with a carat (^) on top, rather than an arrow, such as $\hat{x}$. This notation looks a bit like a hat, so we say $\hat{x}$ as "x hat". Here we make use of the following unit vectors:

$\hat{x}$ = a vector with a magnitude of 1 unit pointing in the positive x-direction

$\hat{y}$ = a vector with a magnitude of 1 unit pointing in the positive y-direction

We can now express the vector $\vec{A}$ in the compact notation: $\vec{A} = (-5 \text{ m})\ \hat{x} + (2 \text{ m})\ \hat{y}$.

Step 3 - ***Find the components of the vector $\vec{B}$.*** We will handle the components of $\vec{B}$ differently from the method we used for $\vec{A}$, because $\vec{B}$ does not conveniently line up with the grid lines like $\vec{A}$ does. Although we could measure the components of $\vec{B}$ carefully off the diagram, we will instead use the trigonometry associated with right-angled triangles to calculate these components because we know the magnitude and direction of the vector.

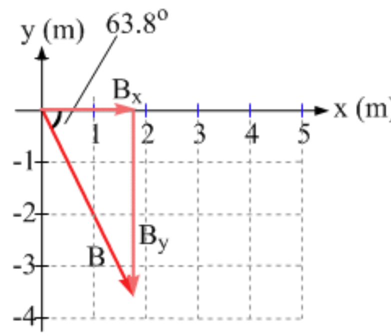


Figure 1.6: Components of the vector $\vec{B}$.

As shown in Figure 1.6, we draw a right-angled triangle with the vector as the hypotenuse, and with the other two sides parallel to the coordinate axes (horizontal and vertical, in this case). The x-component can be found from the relationship:

$$\cos\theta = \frac{B_x}{B}. \qquad \text{So } B_x = B\cos\theta = (4.00 \text{ m})\cos(63.8^\circ) = 1.77 \text{ m}.$$

We can use trigonometry to determine the magnitude of the component and then check the diagram to get the appropriate sign. From Figure 1.6, we see that the x-component of $\vec{B}$ points to the right, so it is in the positive x-direction. We can then express the x-component of $\vec{B}$ as:

$$\vec{B}_x = (+1.77 \text{ m})\ \hat{x}.$$

The y-component can be found in a similar way:

$$\sin\theta = \frac{B_y}{B}. \qquad \text{So, } B_y = B\sin\theta = 4.00\sin(63.8^\circ) = 3.59 \text{ m}.$$

The y-component of $\vec{B}$ points down, so it is in the negative y-direction. Thus:

$$\vec{B}_y = -(3.59 \text{ m})\ \hat{y}.$$

The vector $\vec{B}$ can now be expressed in unit-vector notation as:

$$\vec{B} = \vec{B}_x + \vec{B}_y = (1.77 \text{ m})\hat{x} - (3.59 \text{ m})\hat{y}.$$

Key ideas for vectors: It can be useful to express a vector in terms of its components. One convenient way to do this is to make use of unit vectors; a unit vector is a vector with a magnitude of 1 unit. **Related End-of-Chapter Exercises: 6, 18.**

Essential Question 1.4: Temperature is a good example of a scalar, while velocity is a good example of a vector. List two more examples of scalars, and two more examples of vectors.

Answer to Essential Question 1.4: Other examples of scalars include mass, distance, and speed. Examples of vectors, which have directions associated with them, include displacement, force, and acceleration.

1-5 Adding Vectors

EXAMPLE 1.5 – Adding vectors

Let's define a vector $\vec{C}$ as being the sum of the two vectors $\vec{A}$ and $\vec{B}$ from Exploration 1.4. A vector that results from the addition of two or more vectors is called a **resultant vector**.

(a) Draw the vectors $\vec{A}$ and $\vec{B}$ tip-to-tail to show geometrically the resultant vector $\vec{C}$.

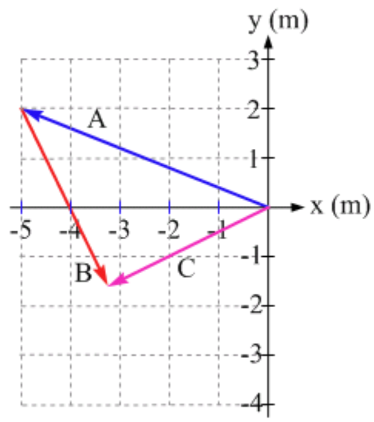
(b) Use the components of vectors $\vec{A}$ and $\vec{B}$ to find the components of $\vec{C}$.

(c) Express $\vec{C}$ in unit-vector notation.

(d) Express $\vec{C}$ in terms of its magnitude and direction.

SOLUTION

(a) To add the vectors geometrically we can move the tail of $\vec{B}$ to the tip of $\vec{A}$, or the tail of $\vec{A}$ to the tip of $\vec{B}$. The order makes no difference. If we had more vectors, we could continue the process, drawing them tip-to-tail in sequence. The resultant vector always goes from the tail of the first vector to the tip of the last vector, as is shown in Figure 1.7.



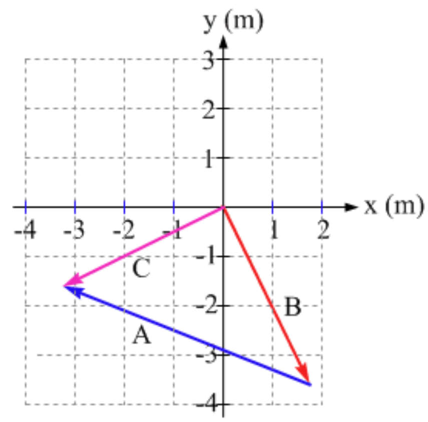


Figure 1.7: Adding vectors geometrically, tip-to-tail. In (a), the tail of vector $\vec{B}$ is placed at the tip of $\vec{A}$; in (b), the tail of vector $\vec{A}$ is placed at the tip of $\vec{B}$. The same resultant vector $\vec{C}$ is produced - the order does not matter.

(b) Now let's add the vectors using their components. We already know the x and y components of $\vec{A}$ and $\vec{B}$ (see Exploration 1.4), so we can use those to find the components of the resultant vector $\vec{C}$. Table 1.2 demonstrates the process. Note that the components of $\vec{A}$ are shown here to two decimal places, even though we know them with more precision. Because we'll be adding the components of $\vec{A}$ to the components of $\vec{B}$, which we know to two decimal places, our final answers should also be expressed with two decimal places.

Vector	*x*-component	*y*-component
$\vec{A}$	$\vec{A}_x = -(5.00 \text{ m})\,\hat{x}$	$\vec{A}_y = +(2.00 \text{ m})\,\hat{y}$
$\vec{B}$	$\vec{B}_x = (1.77 \text{ m})\,\hat{x}$	$\vec{B}_y = -(3.59 \text{ m})\,\hat{y}$
$\vec{C} = \vec{A} + \vec{B}$	$\vec{C}_x = \vec{A}_x + \vec{B}_x$ $\vec{C}_x = -(5.00 \text{ m})\,\hat{x} + (1.77 \text{ m})\,\hat{x}$ $\vec{C}_x = -(3.23 \text{ m})\,\hat{x}$	$\vec{C}_y = \vec{A}_y + \vec{B}_y$ $\vec{C}_y = +(2.00 \text{ m})\,\hat{y} - (3.59 \text{ m})\hat{y}$ $\vec{C}_y = -(1.59 \text{ m})\hat{y}$

Table 1.2: Adding the vectors $\vec{A}$ and $\vec{B}$ using components. The process is shown pictorially in Figure 1.8.

Note that we are solving this two-dimensional vector-addition problem by using a technique that is very common in physics – splitting a two-dimensional problem into two separate one-dimensional problems. It is very easy to add vectors in one dimension, because the vectors can be added like scalars with signs. To find $\vec{C}_x$, for instance, we simply add the *x*-components of $\vec{A}$ and $\vec{B}$ together. To find $\vec{C}_y$, we carry out a similar process, adding the *y*-components of $\vec{A}$ and $\vec{B}$. After finding the individual components of $\vec{C}$, we then combine them, as in parts (c) and (d) below, to specify the vector $\vec{C}$.

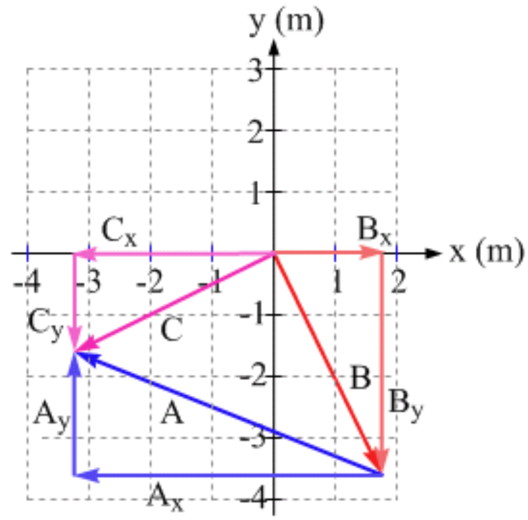


Figure 1.8: This figure illustrates the process of splitting the vectors into components when adding. Each component of the resultant vector, $\vec{C}$, is the vector sum of the corresponding components of the vectors $\vec{A}$ and $\vec{B}$.

(c) Using the bottom line in Table 1.2, the vector $\vec{C}$ can be expressed in unit-vector notation as:
$\vec{C} = \vec{C}_x + \vec{C}_y = -(3.23 \text{ m})\,\hat{x} - (1.59 \text{ m})\,\hat{y}$.

(d) If we know the components of a vector we can draw a right-angled triangle (see Figure 1.9) in which we know the lengths of two sides. Applying the Pythagorean theorem gives the length of the hypotenuse, which is the magnitude of the vector $\vec{C}$.

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{3.23^2 + 1.59^2} = \sqrt{12.961} = 3.60 \text{ m}$$

To find the angle between $\vec{C}$ and $\vec{C}_x$ we can use the relationship:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{C_y}{C_x}.$$

This gives $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{1.59}{3.23}\right) = 26.2^\circ$.

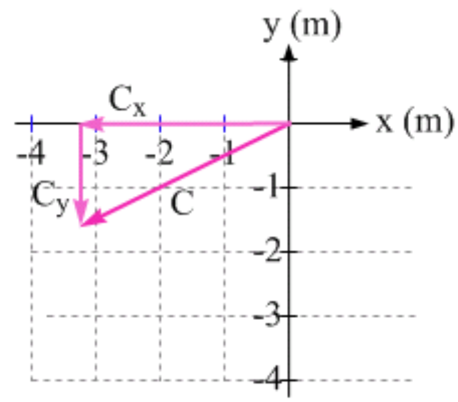


Figure 1.9: The components of the vector $\vec{C}$.

We have dropped the signs from the components, but, in stating the vector $\vec{C}$ correctly in magnitude-direction form, we can check the diagram to make sure we're accounting for which way $\vec{C}$ points: $\vec{C} =$ 3.60 m at an angle of 26.2° below the negative *x*-axis. The phrase "below the negative *x*-axis" accounts for the fact that the vector $\vec{C}$ has negative *x* and *y* components.

Related End-of-Chapter Exercises: 24 – 30.

Essential Question 1.5: Consider again the vectors $\vec{A}$ and $\vec{B}$ from Exploration 1.4 and Example 1.5. If the vector $\vec{D}$ is equal to $\vec{A} - \vec{B}$, express $\vec{D}$ in terms of its components.

Answer to Essential Question 1.5: $\vec{D} = -(6.77 \text{ m})\,\hat{x} + (5.59 \text{ m})\,\hat{y}$.

1-6 Coordinate Systems

Now that we have looked at an example of the component method of vector addition, in Example 1.5, we can summarize the steps to follow.

A General Method for Adding Vectors Using Components

1. Draw a diagram of the situation, placing the vectors tip-to-tail to show how they add geometrically.
2. Show the coordinate system on the diagram, in particular showing the positive direction(s).
3. Make a table showing the *x* and *y* components of each vector you are adding together.
4. In the last line of this table, find the components of the resultant vector by adding up the components of the individual vectors.

Coordinate systems

A coordinate system typically consists of an *x*-axis and a *y*-axis that, when combined, show an origin and the positive directions, as in Figure 1.10. A coordinate system can have just one axis, which would be appropriate for handling a situation involving motion along one line, and it can also have more than two axes if that is appropriate. An important part of dealing with vectors is to think about the coordinate system or systems that is/are appropriate for dealing with a particular situation. Let's explore this idea further.

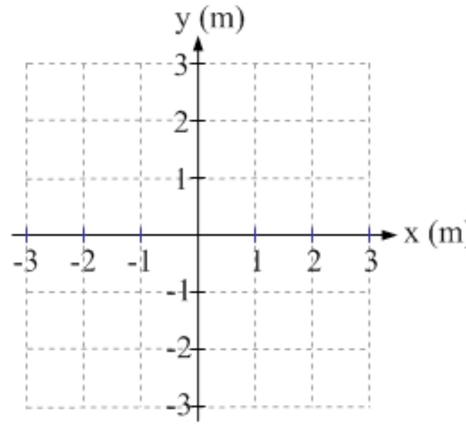


Figure 1.10: A typical *x*-*y* coordinate system.

EXPLORATION 1.6 – Buried treasure

While stranded on a desert island you find a note sealed inside a bottle that is half-buried near a big tree. Unfolding the note, you read: "Start 1 pace north of the big tree. Walk 10 paces northeast, 5 paces southeast, 6 paces southwest, 7 paces northwest, 4 paces southwest, and 2 paces southeast. Then dig." Realizing that your paces might differ in length from the paces of whoever left the note, rather than actually pacing out the distances you begin by drawing an *x*-*y* coordinate system in the sand, with positive *x* directed east and positive *y* directed north. After struggling to split the six vectors into components, however, you wonder whether there is a better way to solve the problem.

Step 1 - ***Is there only one correct coordinate system, or can you choose from a number of different coordinate systems to calculate a single resultant vector that represents the vector sum of the six vectors specified in the note?*** Any coordinate system will work, but there may be one coordinate system that makes the problem relatively easy, while others involve significantly more work to arrive at the answer. It's always a good idea to spend some time thinking about which coordinate system would make the problem easiest.

In fact, you should also think about whether the component method is even the easiest method to use to solve the problem. Adding vectors geometrically would also be a relatively easy way of solving this problem. Thinking about adding them geometrically (it might help to look at the six displacements, as sketched in Figure 1.11), in fact, leads us straight to the most appropriate coordinate system.

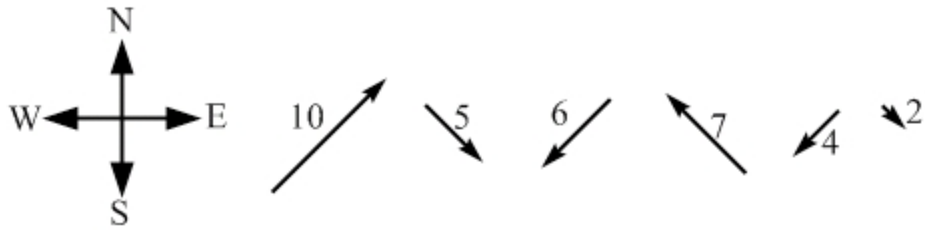


Figure 1.11: A sketch of the six displacements specified on the treasure map.

Step 2 - ***What would be the simplest coordinate system to use to find the resultant vector?*** One thing to notice is that the directions given are northeast, southeast, southwest, or northwest. An appropriate coordinate system is one that is aligned with these directions. For instance, we could point the positive *x*-direction northeast, and the positive *y*-direction northwest. In that case, out of the six different displacements, three are entirely in the *x*-direction and the other three are entirely in the *y*-direction. This makes the problem straightforward to solve. Figure 1.12 shows the vectors grouped by whether they are parallel to the *x*-axis or parallel to the *y*-axis.

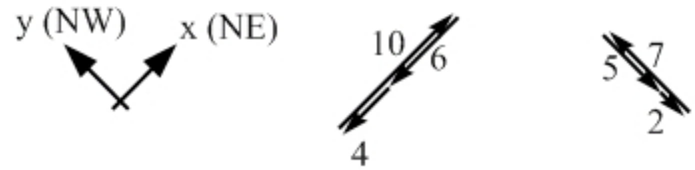


Figure 1.12: Choosing a coordinate system that fits the problem can make the problem easier to solve. In this case we have three vectors aligned with the *x*-axis and three vectors aligned with the *y*-axis.

Step 3 - ***Where should you dig?*** To determine where to dig, focus first on the displacements that are either in the $+x$ direction (10 paces northeast) or the $-x$ direction (6 paces southwest, and 4 paces southwest). Since the total of 10 paces southwest exactly cancels the 10 paces northeast, there is no net displacement along the *x*-axis.

Now turn to the *y*-axis, where we have 7 paces northwest (the $+y$ direction) and a total of $5 + 2 = 7$ paces southeast (the $-y$ direction). Once again these exactly cancel. Because the two components are zero, the resultant displacement vector has a magnitude of zero. You should dig at the starting point, 1 pace north of the tree (assuming you can figure out which way north is!). Digging at that spot, you find a box with a few car batteries, a 12-volt lantern, a solar cell, several wires, and a physics textbook. Reading through the book you figure out how to wire the solar cell to the batteries so the batteries are charged up while the sun shines, and you then figure out how to wire the batteries to the lantern to create a bright light you can use to signal passing planes. Using this system, you are rescued just a few days later, although you make sure to bury everything again carefully near the tree, and place the map back in the bottle, to help the next person who gets stranded there.

Key ideas for coordinate systems: Thinking carefully about the coordinate system to use can save a lot of work. Any coordinate system will work, but, in some cases, choosing the most appropriate coordinate system can make a problem considerably easier to solve.
Related End-of-Chapter Exercises: 5, 31, 41, 42.

Essential Question 1.6: In Exploration 1.6, the six displacements of 10 paces, 5 paces, 6 paces, 7 paces, 4 paces, and 2 paces happen to completely cancel one another because of their particular directions. If you could adjust the directions of each of the six vectors to whatever direction you wanted, what is the maximum distance they could take you away from the starting point?

Answer to Essential Question 1.6: If you lined up all six vectors in the same direction, you would end up 34 paces away from the starting point. When the vectors point in the same direction (and only in this case) you can add their magnitudes. 10 + 5 + 6 + 7 + 4 + 2 = 34 paces.

1-7 The Quadratic Formula

EXAMPLE 1.7 – Solving a quadratic equation

Sometime, such as in some projectile-motion situations, we will have to solve a quadratic equation, such as $2.0x^2 = 7.0 + 5.0x$. Try solving this yourself before looking at the solution.

SOLUTION

The usual first step is to write this in the form $ax^2 + bx + c = 0$, with all the terms on the left side. In our case we get: $2.0x^2 - 5.0x - 7.0 = 0$.

We could graph this on a computer or a calculator to find the values of *x* (if there are any) that satisfy the equation; we could try factoring it out to find solutions; or we can use the quadratic formula to find the solution(s). Let's try the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$ (Equation 1.4: **The quadratic formula**)

In our example, with *a* = 2.0, *b* = –5.0, and *c* = –7.0, the two solutions work out to:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{+5.0 + \sqrt{25 + 56}}{4.0} = \frac{+5.0 + 9.0}{4.0} = +3.5$$, with appropriate units.

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{+5.0 - \sqrt{25 + 56}}{4.0} = \frac{+5.0 - 9.0}{4.0} = -1.0$$, with appropriate units.

These values agree with the graph of the function shown in Figure 1.13. The graph crosses the *x*-axis at two points, at *x* = –1 and also at *x* = +3.5.

Related End-of-Chapter Exercises: 36, 46.

Essential Question 1.7: Could you have a quadratic equation in the form $ax^2 + bx + c = 0$ that had no solutions for *x* (at least, no real solutions)? If so, what would happen when you tried to solve for *x* using the quadratic formula? What would the graph look like?

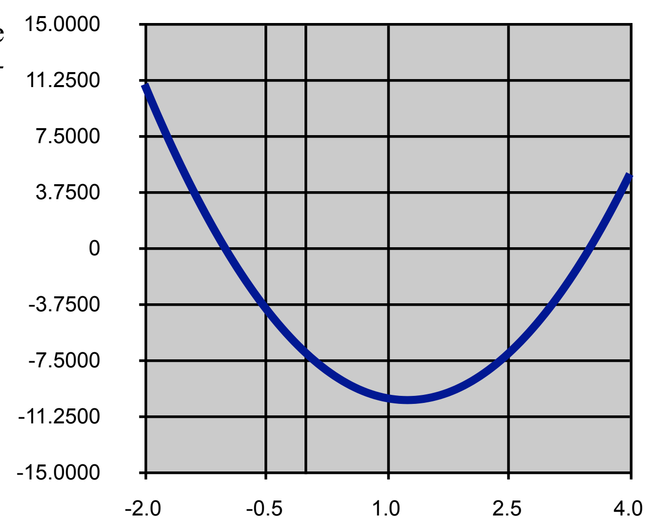


Figure 1.13: A graph of the quadratic equation $2.0x^2 - 5.0x - 7.0 = 0$, for Example 1.7.

Answer to Essential Question 1.7: Yes, you could have an equation with no real solutions. In that case when you applied the quadratic formula you would get a negative under the square root, while the graph would still be parabolic but would not cross (or touch) the *x*-axis.

Chapter Summary

Essential Idea

Physics is the study of how things work, and in analyzing physical situations we will try to apply a logical, systematic approach. Some of the basic tools we will use include:

Units

Our primary set of units is the système international (SI), based on meters, kilograms, and seconds, and four other base units. SI is widely accepted in science worldwide, and convenient because conversions are based on powers of ten. Converting between units is straightforward if you know the appropriate conversion factor(s).

Significant Figures

Three useful guidelines to follow when rounding off include:

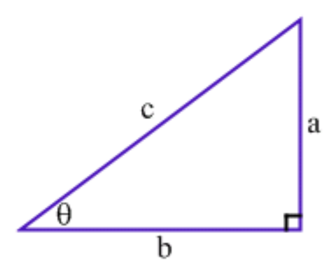
1. Round off only at the end of a calculation when you state the final answer.

2. When you multiply or divide, round your final answer to the smallest number of significant figures in the values going into the calculation.

3. When adding or subtracting, round your final answer to the smallest number of decimal places in the values going into the calculation.

Trigonometry

In a right-angled triangle we use the following relationships:



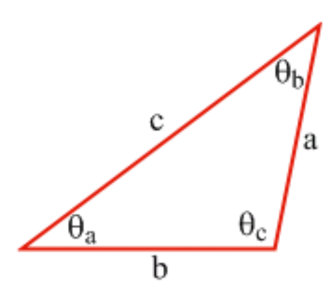
$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{a}{c}; \quad \cos\theta = \frac{adjacent}{hypotenuse} = \frac{b}{c}; \quad \tan\theta = \frac{opposite}{adjacent} = \frac{a}{b}.$$

We relate the three sides using: $c^2 = a^2 + b^2$. (Eq. 1.1: **The Pythagorean Theorem**)

Many triangles do not have a 90˚ angle. For a general triangle, such as that in Figure 1.3, if we know the length of two sides and one angle, or the length of one side and two angles, we can use the Sine Law and the Cosine Law to find the other sides and angles.

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}.$$ (Equation 1.2: **Sine Law**)

$$c^2 = a^2 + b^2 - 2ab\cos\theta_c.$$ (Equation 1.3: **Cosine Law**)



Vectors

A vector is a quantity with both a magnitude and a direction. Vectors can be added geometrically (drawn tip-to-tail), or by using components.

A unit vector is a vector with a length of one unit. A unit vector is denoted by having a carat on top, which looks like a hat, like $\hat{x}$ (pronounced “x hat”).

A vector can be stated in unit-vector notation or in magnitude-direction notation.

A Method for Adding Vectors Using Components

1. Draw a diagram of the situation, placing the vectors tip-to-tail to show how they add geometrically.
2. Show the coordinate system on the diagram, in particular showing the positive direction(s).
3. Make a table showing the x and y components of each vector you are adding together.
4. In the last line of this table, find the components of the resultant vector by adding up the components of the individual vectors.

Algebra and Dimensional Analysis

Dimensional analysis can help check the validity of an equation. Units must be the same for values that are added or subtracted, as well as the same on both sides of an equation.

A quadratic equation in the form $ax^2 + bx + c = 0$ can be solved by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ (Equation 1.4: **The quadratic formula**)

End-of-Chapter Exercises

Exercises 1 – 10 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. You can convert back and forth between miles and kilometers using the approximation that 1 mile is approximately 1.6 km. (a) Which is a greater distance, 1 mile or 1 km? (b) How many miles are in 32 km? (c) How many kilometers are in 50 miles?

2. (a) How many significant figures are in the number 0.040 kg? (b) How many grams are in 0.040 kg?

3. You have two numbers, 248.0 cm and 8 cm. Rounding off correctly, according to the rules of significant figures, what is the (a) sum, and (b) product of these two numbers?