

## 7-1 The Law of Conservation of Energy

In Chapter 6, we looked at the work-energy relationship:  $W_{net} = \Delta K$ . If we break the net work up into two pieces,  $W_{con}$ , the work done by conservative forces (such as gravity), and  $W_{nc}$ , the work done by non-conservative forces (such as tension or friction), then we can write the equation as:  $W_{con} + W_{nc} = \Delta K$ . Recall that the work done by a conservative force does not depend on the path taken between points. The work done by non-conservative forces is path dependent, however.

As we did in Chapter 6 with the work done by gravity, the work done by any conservative force can be expressed in terms of potential energy, using  $W_{con} = -\Delta U$ . We can now write the work-energy equation as

$$-\Delta U + W_{nc} = \Delta K. \quad (\text{Equation A}).$$

Let's use  $i$  to denote the initial state and  $f$  to denote the final state. The change in a quantity is its final value minus its initial value, so we can use  $\Delta K = K_f - K_i$  and  $\Delta U = U_f - U_i$ . Substituting these expressions into Equation A gives  $-(U_f - U_i) + W_{nc} = K_f - K_i$ . With a bit of re-arranging to make everything positive, we get Equation 7.1, below.

Equation 7.1 expresses a basic statement of the **Law of Conservation of Energy**: "Energy can neither be created nor destroyed, it can only be changed from one form to another."

$$K_i + U_i + W_{nc} = K_f + U_f \quad (\text{Equation 7.1: Conservation of energy})$$

The law of conservation of energy is so important that we will use it in Chapters 8, 9, and 10, as well as in many chapters after that. With equation 7.1, we have the only equation we need to solve virtually any energy problem. Let's discuss its five different components.

$K_i$  and  $K_f$  are the initial and final values of the kinetic energy, respectively.

$U_i$  and  $U_f$  are the initial and final values of the potential energy, respectively.

$W_{nc}$  is the work done by non-conservative forces (such as by the force of friction).

The conservation of energy equation is very flexible. So far, we have discussed one form of kinetic energy, the translational kinetic energy given by  $K = (1/2)mv^2$ . When we get to Chapter 11, we will be able to build rotational kinetic energy into energy conservation without needing to modify equation 7.1. Similarly, no change in the equation will be necessary when we define a general form of gravitational potential energy in Chapter 8, and define spring potential energy in Chapter 12. It will only be necessary to expand our definitions of potential and kinetic energy.

**Mechanical energy** is the sum of the potential and the kinetic energies. If no net work is done by non-conservative forces (if  $W_{nc} = 0$ ), then mechanical energy is conserved. This is the **principle of the conservation of mechanical energy**.

### EXAMPLE 7.1 – A frictional best-seller

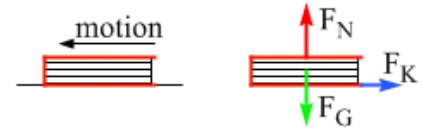
A popular book, with a mass of 1.2 kg, is pushed across a table. The book has an initial speed of 2.0 m/s, and it comes to rest after sliding through a distance of 0.80 m.

- What is the work done by friction in this situation?
- What is the average force of friction acting on the book as it slides?

### SOLUTION

(a) As usual, we should draw a diagram of the situation and a free-body diagram. These diagrams are shown in Figure 7.1. Three forces act on the book as it slides. The normal force is directed up, at  $90^\circ$  to the displacement, so the normal force does no work. The effect of the force of gravity is accounted for via the potential energy terms in equation 7.1, but the gravitational potential energy does not change, because the book does not move up or down. The only force that affects the energy is the force of friction. The work done by friction is accounted for by the  $W_{nc}$  term in the conservation of energy equation.

**Figure 7.1:** A diagram of the sliding book and a free-body diagram showing the forces acting on it as it slides.



So, the five-term conservation of energy equation,  $K_i + U_i + W_{nc} = K_f + U_f$ , can be reduced to two terms, because  $U_i = U_f$  and the final kinetic energy  $K_f = 0$ . We are left with:

$$K_i + W_{nc} = 0 \quad \text{so, } W_{nc} = -K_i = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1.2 \text{ kg})(2.0 \text{ m/s})^2 = -2.4 \text{ J}.$$

The work done by the non-conservative force, which is kinetic friction in this case, is negative because the force of friction is opposite in direction to the displacement.

(b) To find the force of kinetic friction,  $F_K$ , use the definition of work. In this case, we get:  $W_{nc} = F_K \Delta r \cos \theta = -F_K \Delta r$ .

$$\text{Solving for the force of friction gives } F_K = -\frac{W_{nc}}{\Delta r} = -\frac{-2.4 \text{ J}}{0.80 \text{ m}} = 3.0 \text{ N}.$$

**Related End-of-Chapter Exercises: 37, 40, 42.**

### A general method for solving a problem involving energy conservation

This general method can be applied to a wide variety of situations.

- Draw a diagram of the situation. Usually, we use energy to relate a system at one point, or instant in time, to the system at a different point, or a different instant.
- Write out equation 7.1,  $K_i + U_i + W_{nc} = K_f + U_f$ .
- Choose a level to be the zero for gravitational potential energy. Defining the zero level so that either  $U_i$  or  $U_f$  (or both) is zero is often best.
- Identify the terms in the equation that are zero.
- Take the remaining terms and solve.

**Essential Question 7.1:** Did we have to solve Example 7.1 using energy ideas, or could we have applied force and Newton's second law instead?

**Answer to Essential Question 7.1:** In the case of the book sliding on the table, we can apply either an energy analysis or a force analysis. Let's now compare these different methods.

## 7-2 Comparing the Energy and Force Approaches

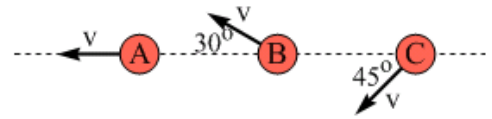
In Example 7.1, there is no real advantage in using an energy analysis over a force analysis. In some cases, however, the energy approach is much easier than the force approach.

### EXPLORATION 7.2A – Which ball has the higher speed?

Three identical balls are launched with equal speeds  $v$  from a height  $h$  above level ground. Ball A is launched horizontally, while the initial velocity of ball B is at  $30^\circ$  above the horizontal, and the initial velocity of ball C is at  $45^\circ$  below the horizontal. Rank the three balls, based on their speeds when they reach the ground, from largest to smallest. Neglect air resistance.

**Step 1 – Sketch a diagram of the situation.** See Figure 7.2.

**Figure 7.2:** A diagram showing the directions of the initial velocities of the three balls in Exploration 7.2A.



**Step 2 – Briefly describe how to solve this problem using methods applied in earlier chapters.**

Consider the projectile-motion analysis we applied in chapter 4. For each ball, we would break the initial velocity into components, determine the  $y$ -component of the final velocity using one of the constant-acceleration equations, and then find the magnitude of the final velocity by using the Pythagorean theorem. We would have to go through the process three times, once for each ball.

**Step 3 – Instead, solve the problem using an energy approach.** Our starting point for energy is always the conservation of energy equation,  $K_i + U_i + W_{nc} = K_f + U_f$ . There is no air resistance, so  $W_{nc} = 0$ . If we define the zero for gravitational potential energy as the ground level, then  $U_f = 0$ , and  $U_i = mgh$ , where  $m$  is the mass of a ball (each ball has the same mass). Substituting this expression into the conservation of energy equation gives:  $K_i + mgh = K_f$ .

Both terms on the left are the same for all three balls. The balls have the same initial kinetic energy and they experience the same change in potential energy. Thus, all three balls have identical final kinetic energies. Because  $K_f = (1/2)mv_f^2$ , and the balls have equal masses, the final speeds are equal. Based on one energy analysis that works for all three balls, instead of three separate projectile-motion analyses, the ranking of the balls based on final speed is  $A=B=C$ .

**Step 4 – Would the answer be different if the balls had unequal masses?** Starting from  $K_i + mgh = K_f$ , and using the definition of kinetic energy, we can show that mass is irrelevant:

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2.$$

Factors of mass cancel, giving:  $v_f = \sqrt{2gh + v_i^2}$ .

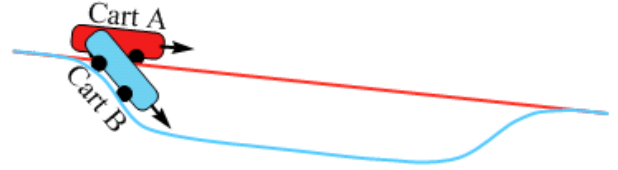
So, the balls have the same final speed even if their masses are different.

**Key idea for solving problems:** We now have two powerful ways of analyzing physical situations. We can either apply force ideas, or apply energy ideas. In certain situations the energy approach is simpler than the force approach. **Related End-of-Chapter Exercises: 6, 38.**

### EXPLORATION 7.2B – Which cart wins the race?

As shown in Figure 7.3, two identical carts have a race on separate tracks. Cart A's track follows a straight path sloping down, while cart B's track dips down below A's just after the start and rises up to meet A's again just before the finish line. If the carts are released at the same time, which cart reaches the end of the track first? Make a prediction, and justify your answer.

**Figure 7.3:** A race between two identical (aside from color!) carts. Cart A's track has a constant slope, while cart B's track dips down below A's before rising up to meet A's again at the end.



After considering the three balls of Exploration 7.2A, it is tempting to predict that the race will end in a tie. An energy analysis, for instance, follows the same logic as that in Exploration 7.2A, except that the analysis in this case is simplified by the initial kinetic energy being zero. Once again, energy tells us that the carts should arrive at the finish line with the same speed. Having the same speed does not mean that they arrive at the same time, however.

Another popular answer is that A wins the race because B travels farther. In actuality, for most tracks, cart B wins the race. Cart A gradually picks up speed as it loses potential energy. In contrast, cart B immediately drops below A, transforming potential energy into kinetic energy, and reaching a speed larger than that of cart A. The carts then travel along parallel paths, with B always moving faster than A. Even while B slows as it climbs the hill near the end, it is traveling faster than A. The larger distance B travels is more than made up for by B's larger average speed.

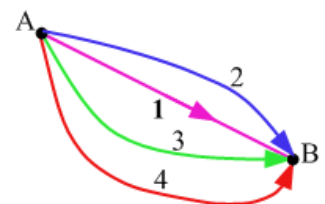
**Key idea about energy and time:** Energy can be a powerful concept, but energy generally gives us no direct information about time. **Related End-of-Chapter Exercises: 4, 35.**

Let's compare and contrast the energy approach with the force approach. Energy can be a very effective tool, because in many cases we only have to consider the initial and final states and we don't have to worry about how the system gets from one state to the other. On the other hand, energy tells us nothing about the time it takes to get from one state to another. In Exploration 7.2B, for instance, the three balls reach the ground at different times, but we would have to use forces, and the constant-acceleration equations, to find those times. Energy is also a scalar, so it tells us nothing about direction. Energy is perfect, however, for connecting speed and position.

If we want to learn about time, or about the direction of a vector, analyzing forces is a better approach. So far, though, we are limited to applying force concepts to situations in which the net force is constant, when we can apply the constant-acceleration equations. We will go beyond this in Chapter 8 but, at the level of physics we are concerned with in this book, we will always be limited in how far we can go with forces. A good example of the limitations of force is shown in Figure 7.4, where an object slides from point A to point B along various paths. If the object comes down path 1, the straight line connecting A and B, we can use forces or energy to analyze the motion, even if friction acts as the object slides. If the object slides along a path other than path 1, then we can't get far with forces. The difficulty is that the force approach is *path dependent* – the forces applied to the object depend on the path, and the forces change if the object moves along a different path.

**Figure 7.4:** Various paths for an object to slide along in traveling from A to B.

**Essential Question 7.2:** Could we use energy to analyze the motion along paths 2, 3, or 4 in Figure 7.4?



**Answer to Essential Question 7.2:** If there is no friction, the energy analysis is path independent, so we treat all paths the same. If friction does act on the object, however, even energy is hard to apply, because then the work done by friction depends on the path.

## 7-7 Combining Energy and Momentum

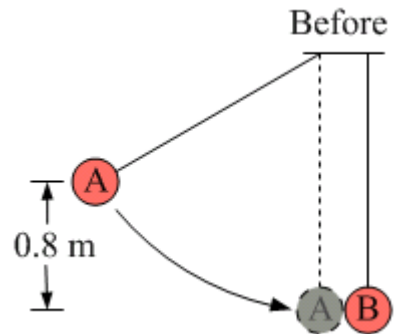
To analyze some situations, we apply both energy conservation and momentum conservation in the same problem. The trick is to know when to apply energy conservation (and when not to!) and when to apply momentum conservation. Consider the following Exploration.

### EXPLORATION 7.7 – Bringing the concepts together

Two balls hang from strings of the same length. Ball A, with a mass of 4.0 kg, is swung back to a point 0.80 m above its equilibrium position. Ball A is released from rest and swings down and hits ball B. After the collision, ball A rebounds to a height of 0.20 m above its equilibrium position and ball B swings up to a height of 0.050 m. Let's use  $g = 10 \text{ m/s}^2$  to simplify the calculations.

**Step 1 – Sketch a diagram of the situation.** This is shown in Figure 7.14.

**Figure 7.14:** A diagram of the two balls on strings. Ball A is swung back until it is 0.80 m higher than its equilibrium point and released from rest.



**Step 2 – Our goal is to find the mass of ball B. Can we find the mass by setting the initial gravitational potential energy of ball A equal to the sum of the final potential energy of ball A and the final potential energy of ball B? Explain why or why not.** The answer to the question is no. We can use energy conservation to help solve the problem, but setting the mechanical energy before the collision equal to the mechanical energy after the collision is assuming too much. The balls make contact in the collision, so it is likely that some of the mechanical energy is transformed to thermal energy, for instance.

**Step 3 – Apply energy conservation to find the speed of ball A just before the collision.** The gravitational potential energy of ball A is transformed into kinetic energy just before the collision. We will neglect the work done by air resistance, so we can apply energy conservation before the collision. Let's start with the conservation of energy equation:  $K_i + U_i + W_{nc} = K_f + U_f$ .

For ball A's swing before the collision, we know that the initial kinetic energy is zero. We are assuming that non-conservative forces do no work. We can also define the zero level for gravitational potential energy to be the lowest point in the swing, just before A hits B, so  $U_f = 0$ . The five-term equation reduces to:

$$U_i = K_f ;$$

$$mgh = \frac{1}{2}mv_f^2 ;$$

$$\text{So, } v_f = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.8} = \sqrt{16} = 4.0 \text{ m/s} .$$

**Step 4 – Apply conservation of energy again to find the speed of ball A just after the collision.** We could try applying conservation of momentum here, but there are too many unknowns. Instead, we can follow the conservation of energy method we used above. Note that we will not

state that the kinetic energy immediately before the collision is equal to the kinetic energy after the collision, because that is not true. We can apply energy conservation, however, if we confine ourselves to the mechanical energy before the collision (as in step 3) or to the mechanical energy after the collision (this step). If we focus on the upswing, we have the kinetic energy of ball A, immediately after the collision, being transformed into gravitational potential energy. The conservation of energy equation reduces to:

$$K_i = U_f ;$$

$$\frac{1}{2}mv_i^2 = mgh ;$$

$$\text{So, } v_i = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = \sqrt{4.0} = 2.0 \text{ m/s} .$$

Let's be clear on what we have calculated in parts 3 and 4, because the notation can be confusing. We are analyzing the motion in three separate parts. The first part of ball A's motion is the downswing, which we analyzed in step 3. The third part is the upswing, which we analyzed in step 4. The second part is the collision, which we still have to analyze. The velocity of ball A immediately before the collision, at the end of the downswing, is  $\vec{v}_{Ai} = 4.0 \text{ m/s}$  to the right, while A's velocity just after the collision, at the start of the upswing, is  $\vec{v}_{Af} = 2.0 \text{ m/s}$  to the left. These are the values we will use in the conservation of momentum equation in step 5.

**Step 5 – First, apply energy conservation to find the speed of ball B after the collision. Then, apply momentum conservation to find the mass of ball B.** We still have to find the velocity of ball B, after the collision, before we use the conservation of momentum equation to find ball B's mass. We can find B's speed immediately after the collision by following the same process we used for ball A in step 3. We get:

$$K_i = U_f$$

$$\frac{1}{2}mv_i^2 = mgh$$

$$\text{So, } v_{Bf} = \sqrt{2gh_B} = \sqrt{2 \times (10 \text{ m/s}^2) \times 0.050 \text{ m}} = \sqrt{1.0 \text{ m}^2/\text{s}^2} = 1.0 \text{ m/s} .$$

The velocity of ball B immediately after the collision is  $\vec{v}_{Bf} = 1.0 \text{ m/s}$  to the right.

Now, we can write out a conservation of momentum equation to solve for the mass of ball B. It is critical to account for the fact that momentum is a vector. In this case, we account for the vector nature of momentum by using a minus sign for the velocity of ball A after the collision to reflect that it is moving to the left, when we chose right to be the positive direction. This gives:  $m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$ , where  $\vec{v}_{Bi} = 0$ . Solving for the mass of ball B gives:

$$m_B = \frac{m_A \vec{v}_{Ai} - m_A \vec{v}_{Af}}{\vec{v}_{Bf}} = \frac{(4.0 \text{ kg}) \times (+4.0 \text{ m/s}) - (4.0 \text{ kg}) \times (-2.0 \text{ m/s})}{+1.0 \text{ m/s}} = 24 \text{ kg} .$$

**Key idea:** In some situations, we can apply conservation of energy and conservation of momentum ideas together. In general, we apply conservation of momentum to connect the situation before the collision to the situation after the collision. We use energy conservation to learn something about the situation before the collision and/or the situation afterwards.

**Related End-of-Chapter Exercises: 30 – 32.**