PROBLEM 1 – 15 points

According to Bob, an observer on Earth, a rocket carrying Martha from Earth directly to the planet Zorg travels at a speed of 0.80 c and takes 30 years to reach Zorg. Zorg is at rest relative to the Earth.

[2 points] (a) What is the distance between Earth and Zorg, according to Bob? Express this distance in light-years.

Bob observes Martha traveling this distance: $d_{Bob} = vt = 0.80c \times 30 \text{ years} = 24 \text{ lightyears}$

[4 points] (b) How long does Martha, the observer on the rocket, think the trip takes?

Martha measures the proper time, in this case. Thus, we can use the time dilation equation: $\Delta t_{proper} = \Delta t \times \sqrt{1-\frac{v^2}{c^2}} = 30 \text{ years} \times \sqrt{1-\frac{0.8c^2}{c^2}} = 30 \text{ years} \times 0.6 = 18 \text{ years}$

[2 points] (c) What is the distance between Earth and Zorg, according to Martha? Express this distance in light-years.

Martha sees Earth and Zorg passing her, traveling at 0.8 c, and separated by 18 years. Thus, the distance is $d_{Martha} = vt = 0.80c \times 18 \text{ years} = 14.4 \text{ lightyears}$

[4 points] (d) Although Bob and Martha disagree about distances and times, they do agree on the value of what is called the space-time interval between the events of Martha passing Earth and Martha reaching Zorg. In fact, every observer agrees on the value of this interval, defined as $\sqrt{(c\Delta t)^2 - (\Delta x)^2}$. Complete the table below to show what the value of the space-time interval is for this situation. Note that, since Martha is present at both events, the spatial distance she measures between the events is zero.

Observer	$c\Delta t$ **(light-years)**	Δx **(light-years)**	$\sqrt{(c\Delta t)^2 - (\Delta x)^2}$ **(light-years)**
Bob	**30**	**24**	**18**
Martha	**18**	**0**	**18**

[3 points] (e) A third observer, Ralph, thinks that the two events of Martha passing Earth and Martha reaching Zorg occur 25 years apart. In Ralph's reference frame, what is the spatial distance Δx between these two events? Ralph's relative velocity relative to Earth does not change.

The space-time interval works for Ralph, too, so: $(c\Delta t)^2 - (\Delta x)^2 = (18 \text{ lightyears})^2$

$(\Delta x)^2 = (c\Delta t)^2 - (18 \text{ lightyears})^2 = (25 \text{ lightyears})^2 - (18 \text{ lightyears})^2 = 301 \text{ lightyears}^2$

Taking the square root gives $\Delta x = 17.3 \text{ lightyears}$**, according to Ralph**

PROBLEM 2 – 10 points

Planet X is 60 lightyears away from Earth. Assume that there is no relative motion between Planet X and Earth (i.e., from the Earth, it appears that Planet X is at rest and vice versa). Note that the time dilation and length contraction equations are sufficient to do this problem, but, in case you would like it, the spacetime interval equation is:

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2 = (\text{spacetime interval})^2$$

[2 points] (a) How long does light take to reach Planet X from Earth? Answer in years.

It takes 60 years, according to an observer on Earth (or on planet X).

You travel on a rocket ship, moving at a constant speed of 0.6 c, directly from Earth to Planet X.

[3 points] (b) According to an observer on Earth, how many years does it take for you to make the journey to Planet X? That is, what is the time interval between the time that you leave Earth and when you arrive at Planet X, according to the Earth bound observer? Answer in years.

According to the observer on Earth: $t = \frac{d}{v} = \frac{60 \text{ lightyears}}{0.6c} = 100 \text{ years}$

[2 points] (c) When you complete your journey to Planet X, according to you, how much older are you than when you left Earth? Answer in years.

Using the spacetime interval, where the spatial separation for the two events (leaving Earth, and arriving at Planet X) is zero, because they both take place outside your rocket ship window, we get:

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$(100 \text{ lightyears})^2 - (60 \text{ lightyears})^2 = (c\Delta t')^2 - (0)^2$$

This gives a time interval of 80 years, which is how much older you are than when you left Earth.

[3 points] (d) According to you, how far apart are Planet X and Earth? Express your answer in units of lightyears.

You see Planet X and Earth moving at 0.6*c*, separated in time by 80 years. This gives a distance between them of (according to you):

$$d = vt = 0.60c \times 80 \text{ years} = 48 \text{ lightyears}$$