PROBLEM 1 – 15 points

[5 points] (a) A green laser beam (λ = 532 nm in air) is incident on a double slit, creating an interference pattern of bright and dark spots on a screen some distance away. **If you want the spots in the pattern to be closer together** (measuring the distance between spots as the distance between their centers) which of the following changes could you make? **Select all that apply.** Grading scheme: +1 for each correct choice; –1 for each incorrect choice. Negative scores will be given zero.

[ ] Replace the green laser by a red laser. $\sin\theta = \frac{m\lambda}{d}$

[**X**] Replace the green laser by a violet laser. **Smaller wavelength = closer spots**

[**X**] Increase d, the distance between the two slits. **Larger *d* = closer spots**

[ ] Decrease d, the distance between the two slits.

[ ] Increase the distance between the double slit and the screen. $\tan\theta = \frac{y}{L}$

[**X**] Decrease the distance between the double slit and the screen. **y goes down**

[**X**] Immerse the entire apparatus in water. **Decreases the wavelength.**

[**X**] Immerse the entire apparatus in olive oil. **Also decreases the wavelength.**

[ ] Replace the double slit by a diffraction grating, keeping d the same

[ ] Use a beam of electrons instead of a green laser, with the electrons having a de Broglie wavelength of 532 nm.

Now assume that the slits are separated by a distance of d = 5.32 x10^{-5} m. A screen is placed 20 m away from the slits.
Remember that for small angles we can use the approximation $\theta \approx \sin\theta \approx \tan\theta$.

[5 points] (b) Find the spacing between the central maximum and one of the first-order maxima on the screen.

$\sin\theta = \frac{m\lambda}{d}$ and $\tan\theta = \frac{y}{L}$. Setting these equal gives: $y = \frac{m\lambda L}{d}$.

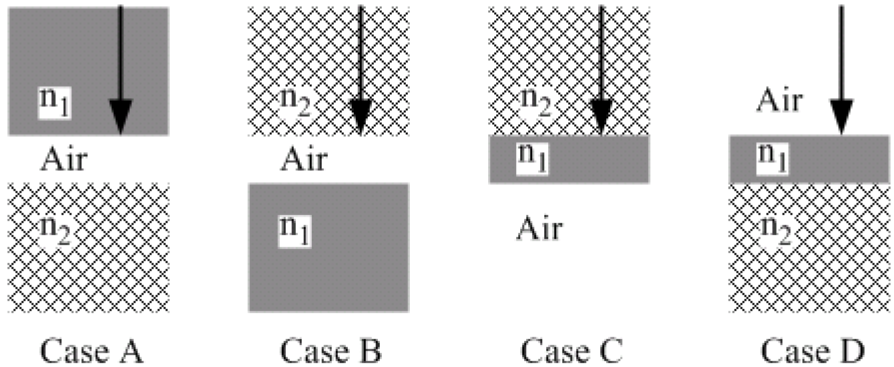
In this case, *m* = 1, and the wavelength divided by the grating spacing is 1/100. This gives a separation of *y* = 20 m / 100 = 20 cm.

[5 points] (c) The entire apparatus is now immersed in a liquid that has an index of refraction n = 1.5. What is the new spacing between the central maximum and one of the first-order maxima on the screen?

The wavelength decreases by a factor of 1.5 (the index of refraction) in this new medium, which reduces *y* by a factor of 1.5. The spacing shrinks to about 13.3 cm.

PROBLEM 2 – 15 points

The diagram shows four situations in which light of wavelength λ is incident perpendicularly on a very thin layer (the middle layer in each case). The indicated indices of refraction are $n_1 = 1.50$ and $n_2 = 2.00$.



[8 points] (a) In each case, consider what happens to the reflected light in the limit where the thickness of the thin layer approaches zero.

(i) In case A, a thin-film thickness approaching zero causes the reflected light to be

[**X**] eliminated by destructive interference [ ] bright by constructive interference

(ii) In case B, a thin-film thickness approaching zero causes the reflected light to be

[**X**] eliminated by destructive interference [ ] bright by constructive interference

(iii) In case C, a thin-film thickness approaching zero causes the reflected light to be

[ ] eliminated by destructive interference [**X**] bright by constructive interference

(iv) In case D, a thin-film thickness approaching zero causes the reflected light to be

[ ] eliminated by destructive interference [**X**] bright by constructive interference

In this situation, the wave that goes down and back through the film travels no extra distance, because the film thickness approaches zero. Any effective path length difference comes from any half-wavelength shifts because of reflection from a higher-n medium. In both cases A and B, the wave reflecting from the top surface of the air film experiences no shift, but the wave reflecting from the bottom surface of the air film is inverted, which is equivalent to a half wavelength shift. The two reflected waves are shifted, in effect, by a half wavelength, and cancel one another by destructive interference. In cases C and D, we have a different situation. In case C, both waves are inverted, so they end up having no shift with respect to one another. In case D, neither wave is inverted, so they also have no shift with respect to one another. In both of these cases, because the two waves have a net shift of zero with respect to one another, they interfere constructively.

[7 points] (b) In **case B**, what is the minimum **non-zero** thickness of the thin-film that would produce **destructive interference** for reflected light if the wavelength of the incident light is 600 nm (measured in air)?

Let's go through the five-step process to figure this out.

Step 1: find the effective shift for the wave reflecting off the top surface of the film. For the wave traveling in medium 2, reflecting from the air, there is no inversion when the reflection occurs, because the wave is reflecting from a lower-*n* medium. Thus, the effective shift is $\Delta_t = 0$**.**

Step 2: find the effective shift for the wave reflecting off the bottom surface of the film. This wave is traveling in air, and reflecting from a higher-*n* medium, which gives a half-wavelength shift. The wave also travels down and back through the film, for an extra path-length distance of 2*t*, if we say the thickness of the air film is *t*. Thus, the effective shift is $\Delta_b = 2t + \frac{\lambda'}{2}$**.**

Step 3: find the net shift (the effective path-length difference) of the two waves by subtracting our result in step 1 from the result in step 2. This gives:

$\Delta = \Delta_b - \Delta_t = 2t + \frac{\lambda'}{2}$**.**

Step 4: bring in the appropriate interference condition. In this case, we want destructive interference, so we set our net shift equal to $m + \frac{1}{2}$ **wavelengths.**

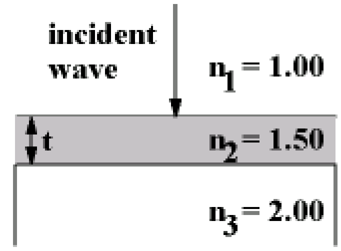
$2t + \frac{\lambda'}{2} = \left(m + \frac{1}{2}\right)\lambda'$**.**

Step 5: Solve the equation, remembering that the wavelength in the equation is the wavelength in the thin film. In this case, the film is air, so we can use the 600 nm value stated above.

Solving the equation, we get $2t = m\lambda'$**. To get the smallest non-zero thickness (** t **), we use the smallest non-zero integer for *m*, which is *m* = 1.**

This gives $t = \frac{\lambda'}{2} = \frac{600 \text{ nm}}{2} = 300 \text{ nm}$**.**

PROBLEM 3 - 15 points



A thin piece of glass with an index of refraction of n = 1.50 is placed on top of a medium that has an index of refraction n = 2.00. A beam of light traveling in air (n = 1.00) shines perpendicularly down on the glass. The beam contains light of only two colors, blue light with a wavelength in air of 450 nm and orange light with a wavelength in air of 600 nm.

[5 points] (a) What is the minimum non-zero thickness of the glass that gives completely constructive interference for the blue light reflecting from the film?

Once again, we can go through the five-step process, similar to that in the previous problem.

Step 1: find the effective shift for the wave reflecting off the top surface of the film. The wave traveling in medium 1, reflecting from medium 2, experiences an inversion when the reflection occurs, because the wave is reflecting from a higher-*n* medium. This is, in effect, a half-wavelength shift. Thus, the effective shift is $\Delta_t = \frac{\lambda'}{2}$**.**

Step 2: find the effective shift for the wave reflecting off the bottom surface of the film. This wave also experiences a half-wavelength shift, because it reflects from a higher-*n* medium. The wave also travels down and back through the film, for an extra path-length distance of 2*t*, if we say the thickness of the air film is *t*. Thus, the effective shift is $\Delta_b = 2t + \frac{\lambda'}{2}$**.**

Step 3: find the net shift (the effective path-length difference) of the two waves by subtracting our result in step 1 from the result in step 2. This gives: $\Delta = \Delta_b - \Delta_t = 2t$**.**

Step 4: bring in the appropriate interference condition. In this case, we want constructive interference, so we set our net shift equal to m **wavelengths:** $2t = m\lambda'$**.**

Step 5: Solve the equation, remembering that the wavelength in the equation is the wavelength in the thin film. In this case, the film has an index of refraction of 1.50, so we divide the wavelength in vacuum by the index of refraction to find the wavelength in the thin film.

To get the smallest non-zero thickness (*t*), we use the smallest non-zero integer for *m*, which is *m* = 1.

This gives $t_{blue} = \frac{\lambda'}{2n_2} = \frac{450 \text{ nm}}{2 \times 1.50} = 150 \text{ nm}$**.**

[5 points] (b) What is the minimum non-zero thickness of the glass that gives completely constructive interference for the orange light reflecting from the film?

The analysis method here is the same as that in part (a), so we get the same equation, with a different value of wavelength.

This gives $t_{orange} = \frac{\lambda'}{2n_2} = \frac{600 \text{ nm}}{2 \times 1.50} = 200 \text{ nm}$**.**

[5 points] (c) What is the minimum non-zero thickness of the glass that gives completely constructive interference for BOTH the blue and orange light simultaneously?

Our equation tells us that the thicknesses that give constructive interference are integer multiples of the minimum non-zero thickness. Thus, the set of film thicknesses that give completely constructive interference for blue light consists of:

t_{blue} **= 150 nm, 300 nm, 450 nm, 600 nm, 750 nm, …**

For orange light, we get integer multiples of 200 nm:

t_{orange} **= 200 nm, 400 nm, 600 nm, 800 nm, 1000 nm, …**

By comparing these two sets, we can see that the smallest thickness that gives completely constructive interference for both blue and orange light simultaneously is 600 nm.