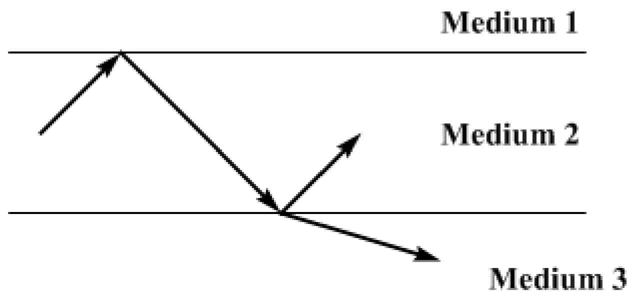
PROBLEM 1 - 10 points

[5 points] (a) Three media are placed on top of one another. A ray of light starting in medium 2 experiences total internal reflection at the top interface but some of the light refracts into medium 3 when the ray reaches the bottom interface. If the two interfaces are parallel, rank the media by their index of refraction, from largest to smallest.



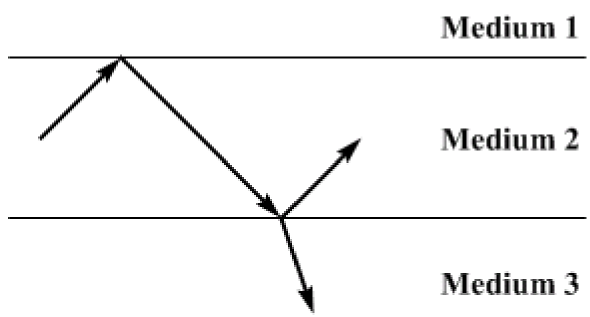
[ ] $n_1>n_2>n_3$ [ ] $n_1>n_3>n_2$ [ ] $n_2>n_1>n_3$ [**X**] $n_2>n_3>n_1$ [ ] $n_3>n_1>n_2$ [ ] $n_3>n_2>n_1$

[ ] There is not enough information given above to decide.

[ ] None of the above

Briefly justify your answer: **For total internal reflection to occur at the interface between medium 2 and medium 1, medium 2 must have a larger index of refraction than medium 1, and the angle of incidence exceeds the critical angle. We have the same angle of incidence at the interface between medium 2 and medium 3, where the beam in medium 3 bends away from the normal. Medium 3 must have a smaller index of refraction than medium 2 (so, medium 2 has the largest index of refraction). We have the same angle of incidence at both interfaces, but at the lower interface the angle of incidence is less than the critical angle. Thus, the index of refraction of medium 3 must be closer to that of medium 2 than is the index of refraction of medium 1.**

[5 points] (b) Medium 3 is now changed, and the rays follow the paths shown at right. Once again rank the media by their index of refraction, from largest to smallest.



[ ] $n_1>n_2>n_3$ [ ] $n_1>n_3>n_2$ [ ] $n_2>n_1>n_3$ [ ] $n_2>n_3>n_1$ [ ] $n_3>n_1>n_2$ [**X**] $n_3>n_2>n_1$

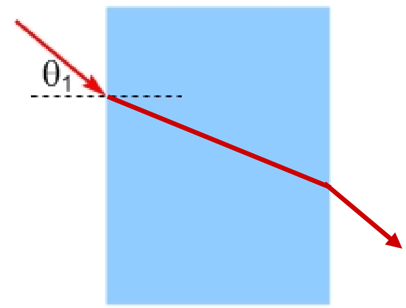
[ ] There is not enough information given above to decide.

[ ] None of the above

Briefly justify your answer: **In this case, the light bends toward the normal as it passes from medium 2 to medium 3, so medium 3 has the largest index of refraction.**

PROBLEM 2 – 10 points

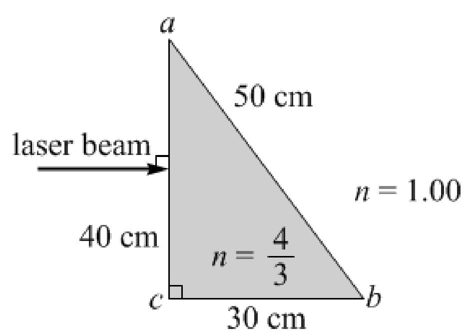
[1 point] (a) A beam of light traveling in air enters a rectangular glass block with refractive index *n*. Assuming the light exits the block along the side opposite to the side it entered, what path does the light follow when it emerges from the block?



[ ] The exact path it was following when it entered the block.
[**X**] A path parallel to the original path, but displaced from it.
[ ] A path perpendicular to the original path.
[ ] A path that makes an angle $\sin^{-1}\left(\frac{\sin\theta_1}{n}\right)$ with the original path.
[ ] None of the above.

[2 points] (b) Briefly justify your answer: **The ray bends towards the normal at the first interface, and then exits the block along the parallel sides. Snell's law is applied exactly the same way at both interfaces, so the angle at which the ray emerges from the block is θ_1. Thus, the ray follows a path parallel to the original path, but displaces from the original path because of the change in direction inside the block.**

A laser beam is incident along the normal to the side *ac* of a right-angled prism. The prism is in the shape of a 3-4-5 triangle, with sides measuring 30 cm (*bc*), 40 cm (*ac*), and 50 cm (*ab*). The prism, which has an index of refraction of 4/3, is surrounded by air (n = 1.00).



[1 point] (c) At what angle, measured from the normal, does the laser beam emerge from the side *ab* of the prism when the beam first encounters that glass-air interface?

[ ] $\sin^{-1}(3/5) = 37^\circ$ [ ] $\sin^{-1}(3/4) = 49^\circ$ [**X**] $\sin^{-1}(4/5) = 53^\circ$ [ ] 60°
[ ] It doesn't – it experiences total internal reflection

[2 points] (d) Briefly justify your answer: **The laser beam, traveling along the normal to side ac, does not change direction when it enters the prism. The angle of incidence at the side ab is then the same as the angle inside the triangle at vertex a, which has a sine of 3/5. Applying Snell's law, we get (4/3)×(3/5) = 1.00sin(θ), which gives θ = $\sin^{-1}$(4/5).**

[1 point] (e) Now, you can adjust the index of refraction of the prism. Given the geometry above, what is the critical index of refraction of the prism? Below this value, the laser beam emerges from the prism into air when it first encounters side *ab* of the prism, while above this value the beam experiences total internal reflection.

[ ] 1 [ ] 5/4 [ ] 4/3 [ ] 1.5 [**X**] 5/3 [ ] 2
[ ] There is no critical index of refraction – the beam never experiences total internal reflection

[3 points] (f) Briefly justify your answer: **When the light is at the critical angle, inside the glass, the angle at which it emerges into the air is 90°. Setting up Snell's law for this situation gives:**
$n \times (3/5) = 1.00\sin(90°) = 1.00$. Solving for n gives 5/3.

PROBLEM 3 – 20 points

An object is placed a certain distance from a lens. The image created by the lens is exactly half as large as the object. If the two focal points of the lens are 20 cm from the lens, where is the object? Where is the image?

[3 points] (a) There are two solutions to this problem. Describe in words one of the solutions, including such information as what kind of lens is being used, what side of the lens the image is on, and what the image characteristics are. Don't draw it yet – we will draw it in part (c).

One solution is when the lens is converging, and the object is farther from the lens than twice the focal length. This gives a smaller, real, and inverted image, on the opposite side of the lens than the object.

[3 points] (b) For the solution you describe in (a), use equations to calculate the object distance and the image distance. Be careful with the signs.

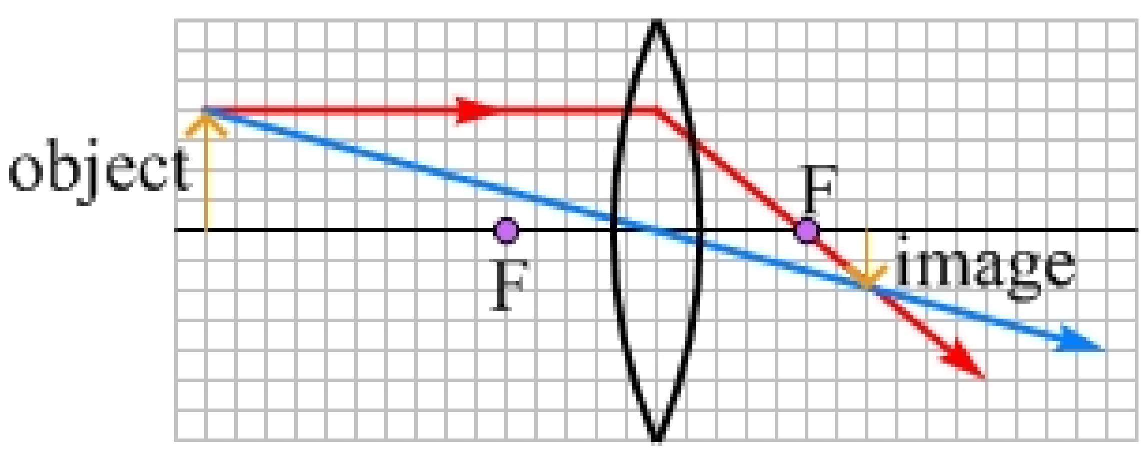
We have two unknowns, but we can work with two equations, the magnification equation and the thin-lens equation.

Magnification: $m = -\frac{d_i}{d_o} = -\left(\frac{1}{2}\right) = -\frac{1}{2}$ **which gives** $d_o = 2d_i$

Thin-lens equation, with *f* = +20 cm: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{2d_i} + \frac{1}{d_i} = \frac{3}{2d_i}$**.**

This gives $d_i = \frac{3}{2} f = 30 \text{ cm}$**, and** $d_o = 2d_i = 60 \text{ cm}$

[3 points] (c) For the solution you describe in (a), sketch a ray diagram on the axis below. Hint: it's a good idea to first draw the lens. The squares on the axis are 4 cm × 4 cm.



[3 points] (d) Describe in words the second solution, including such information as what kind of lens is being used, what side of the lens the image is on, and what the image characteristics are. Don't draw it yet – we'll do that in (f).

The second solution is when the lens is diverging. A diverging lens produces a smaller, virtual, and upright image, on the same side of the lens as the object.

[4 points] (e) For the solution you describe in (d), use equations to calculate the object distance and the image distance. Be careful with the signs.

Once again, we have two unknowns, but we can work with two equations, the magnification equation and the thin-lens equation.

Magnification: $m = -\frac{d_i}{d_o} = +\frac{1}{2}$ **which gives** $d_o = -2d_i$

Thin-lens equation, with *f* = –20 cm: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = -\frac{1}{2d_i} + \frac{1}{d_i} = \frac{1}{2d_i}$.

This gives $d_i = \frac{1}{2} f = -10 \text{ cm}$**, and** $d_o = -2d_i = 20 \text{ cm}$

[3 points] (f) For the solution you describe in (d), sketch a ray diagram on the axis below. Hint: it's a good idea to first draw the lens. The squares on the axis are 2 cm × 2 cm.