***Answer to Essential Question 21.8***: Because the beat frequency is 6 Hz, we know that the two frequencies differ by 6 Hz. If one string is 330 Hz, the other string is either 336 Hz (6 Hz higher) or 324 Hz (6 Hz lower).

## *21-9 Standing Waves on Strings*

In sections 21-9 and 21-10, we will discuss physics related to musical instruments, focusing on stringed instruments in this section and wind instruments in section 21-10.

Some stringed instruments (such as the harp) have strings of different lengths, while others (such as the guitar) use strings of the same length. We can apply the same principles to understand either kind of instrument. Consider a single string of a particular length that is fixed at both ends. The string is under some tension, so that when you pluck the string it vibrates and you hear a nice sound from the string, dominated by one particular frequency. How does that work?

When you pluck the string, you send waves of many different frequencies along the string, in both directions. Each time a wave reaches an end, the wave reflects so that is inverted. All of these reflected waves interfere with one another. For most waves, after multiple reflections the superposition leads to destructive interference. For certain special frequencies, for which an integral number of half-wavelengths fit exactly into the length of the string, the reflected waves interfere constructively, producing large-amplitude oscillations on the string at those frequencies.

These special frequencies produce **standing waves** on the string. Identical waves travel left and right on the string, and the superposition of such identical waves leads to a situation where the positions of zero displacement (the **nodes**) remain fixed, as do the positions of maximum displacement (the **anti-nodes**), so the wave appears to stand still. Figure 21.21 shows the left and right-moving waves on the string, and their superposition, which is the actual string profile, for the lowest-frequency standing wave on the string at various times.

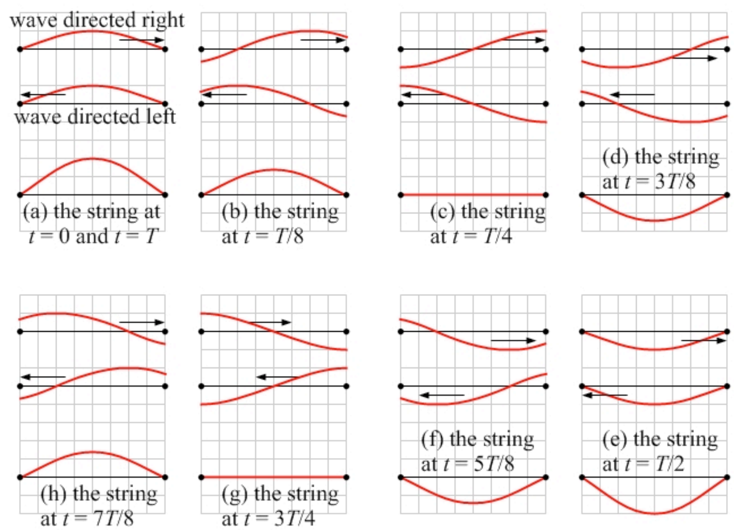


**Figure 21.21**: The string profile for the lowest-frequency standing wave (the **fundamental**) on the string at $t = 0$, and at regular time intervals after that, showing how the identical left and right-moving waves combine to form a standing wave. Go clockwise around the diagram to see what the string looks like as time goes by.

**For a string fixed at both ends:** The standing waves have a node (a point of zero displacement) at each end of the string. The various wavelengths that correspond to the special standing-wave frequencies are related to $L$, the length of the string, by:

$$n\frac{\lambda_n}{2} = L, \qquad \text{so} \qquad \lambda_n = \frac{2L}{n}, \text{ where } n \text{ is an integer.}$$

Using Equation 21.1, $v = f\lambda$ , the particular frequencies that tend to be excited on a stretched string are:

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L},$$ (Eq. 21.13: **Standing-wave frequencies for a string fixed at both ends**)

The lowest-frequency standing wave on the string, corresponding to $n = 1$, is known as the ***fundamental***. The other frequencies, or ***harmonics***, are simply integer multiples of the fundamental. In general, when you pluck a string, the dominant sound is the fundamental, but the harmonics make the sound more pleasing than what a single-frequency note sounds like. Figure 21.22 shows the standing wave patterns for the fundamental and the two lowest harmonics.

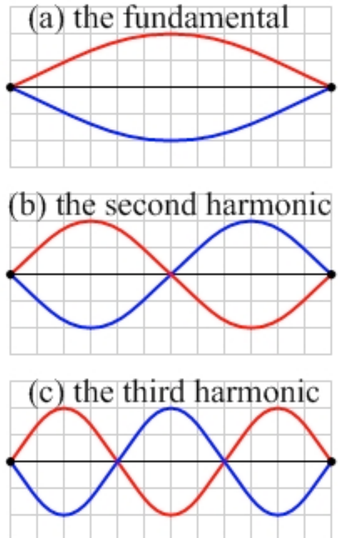


**Figure 21.22**: The standing wave patterns for the fundamental and the second and third harmonics, for a string fixed at both ends.

**EXAMPLE 21.9 – Waves on a guitar string**

A particular guitar string has a length of 72 cm and a mass of 6.0 grams.

(a) What is the wavelength of the fundamental on this string?

(b) If you want to tune that string so its fundamental frequency is 440 Hz (an A note), what should the speed of the wave be?

(c) When the string is tuned to 440 Hz, what is the string's tension?

(d) Somehow, you excite only the third harmonic, which has a frequency three times that of the fundamental. At $t = 0$, the profile of the string is shown in Figure 21.23, with the middle of the string at its maximum displacement from equilibrium. What is the oscillation period, $T$?

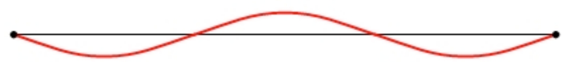

**Figure 21.23**: The string profile at $t = 0$ when the third harmonic has been excited on the string, with the middle of the string at its maximum displacement from equilibrium.

**SOLUTION**

(a) For the fundamental, exactly half a wavelength fits in the length of the string. Thus, the wavelength is twice the length of the string: $\lambda = 144$ cm = 1.44 m.

(b) Knowing the frequency and the wavelength, we can determine the wave speed: $v = f\lambda = (440 \text{ Hz})(1.44 \text{ m}) = 634$ m/s.

(c) Knowing the speed, we can use Equation 21.5, to find the tension in the string.

$$v = \sqrt{\frac{F_T}{(m/L)}}, \quad \text{so} \quad F_T = v^2 \times \frac{m}{L}.$$

In this case, we get: $F_T = v^2 \times \frac{m}{L} = (633.6 \text{ m/s})^2 \times \frac{0.006 \text{ kg}}{0.72 \text{ m}} = 3350 \text{ N}$.

(d) The fundamental frequency is 440 Hz, so the third harmonic has a frequency of 1320 Hz, three times that of the fundamental. The period is the inverse of the frequency, so:

$$T = \frac{1}{f} = \frac{1}{1320 \text{ Hz}} = 760 \text{ µs}.$$

**Related End-of-Chapter Exercises: 28, 29, 36, 59.**

***Essential Question 21.9:*** Return to the situation discussed in Example 21-9. Figure 21.23 shows the string profile at $t = 0$. Show the string profile at times of $t = T/4$, $T/2$, $3T/4$, and $T$.