Answer to Essential Question 21.7: How does the flat string pictured in Figure 21.12(b) differ from a regular flat string, from which no pulses would emerge? What is not obvious from the static image shown in Figure 21.12(b) is that the string, where the pulses overlap, is moving. Some sections of the string are moving down, while others are moving up, with the various parts of the string moving with velocities that are just right to re-create the two pulses properly. Thus, the energy needed to re-form the pulses is in the kinetic energy of various parts of the string.

21-8 Beats; and Reflections

When you listen to two sound waves of similar, but different, frequency, you generally hear the sound rising and falling in intensity, typically at the rate of a few cycles per second. This phenomenon is known as **beats**, and it is caused by interference between the two waves. Let's say the waves are initially in phase, with their peaks coinciding. The waves interfere constructively, producing a largeamplitude sound. Because the waves have different frequencies, however, they gradually drift out of phase. Eventually, the peak from one wave lines up with the trough (negative-displacement peak) in the other wave, leading to destructive interference and, when the interference is completely destructive, no sound. The larger the difference between the two frequencies, the faster the waves drift out of phase with one another. The phase difference continues to grow, but this eventually leads to peaks in the two waves lining up again. This cycle is demonstrated in Figure 21.17.

The beat frequency, which is the frequency at which the intensity oscillates, is simply the difference between the frequencies of the two waves.



Figure 21.17: The phenomenon of beats is caused by interference between two waves that have different frequencies. The two individual waves are shown at the top and middle, while the superposition of the two waves is shown at the bottom. The rise and fall in the amplitude of the resultant wave is what we hear as beats.

 $f_{beat} = f_{high} - f_{low}$. (Equation 21.12: the beat frequency)

String musicians can even tune their instrument using beats, by playing two strings at once and adjusting the tension in one string (which adjusts the frequency of the string). When the beats disappear, the frequencies of the two strings are equal.

Reflections

When a wave traveling along a string encounters the end of the string, the wave reflects. Exactly how the wave reflects depends on whether the end of the string is tied down or loose (or even something in between, such as tied to a spring, but we will consider only the two extremes).

On stringed instruments, for instance, the strings are fixed at the ends. The leading (rightmost) edge of an upward going pulse, like that shown in Figure 21.18(a), propagates to the right along the string by each part of the string successively pulling up on the next part of the string. This propagation method works until the pulse reaches the end of the string, which is tied down. The part of the string next to the right end pulls up on the end, but the end does not move. Instead, by Newton's Third Law, the end exerts a downward force on the piece of the string next to it, leading to an inverted pulse traveling back along the string, as shown in part (e) of Figure 21.18.



Figure 21.18: When a wave reflects from a fixed end, it reflects upside down.

Note that the string is completely flat in Figure 21.18 (c), halfway through the reflection of the pulse. This is caused by completely destructive interference taking place between the first half of the pulse, which has been inverted and is moving left, and the second half of the pulse, which is still upright and moving right. Figure 21.19 shows a way to visualize the reflection, as if a pulse directed right on the string is interfering with a mirror-image pulse, which is inverted, directed left on the string. Superposition can only work on the string itself, so we don't have to worry about any areas of overlap of the two pulses that are to the right of the end of the string.



Figure 21.19: Reflection from a fixed end can be visualized as interference between a rightmoving pulse, and an inverted copy of the pulse that is moving left. We are imagining the pulses existing to the right of the end of the string (in the shaded region), even though they cannot do so.

If the end of a string is not tied down, but is free to move, it is known as a **free end**. When a wave reflects from a free end, the end responds to the wave by moving, and the wave reflects without being inverted. Figure 21.20 shows the process for a pulse, which we can visualize as if a pulse directed right on the string is interfering with a mirror-image pulse directed left. Note that, in Figure 21.20(c), the end of the string is displaced by twice the amplitude of the pulse, because of constructive interference between the half of the pulse that has been reflected and is moving to the left, and the other half which is still moving to the right.



Figure 21.20: Reflection from a free end can be visualized as interference between a rightmoving pulse, and an exact copy of the pulse that is moving left.

Reflection of waves: If a wave reflects from a fixed boundary of a medium, the reflected wave is inverted. If, instead, a wave reflects from a free boundary, such as the free end of a string, the reflected wave reflects without being inverted (that is, the reflected wave is upright). **Related End-of-Chapter Exercises: 9, 10, 53 – 55.**

Essential Question 21.8: You hear a beat frequency of 6 Hz when you play two guitar strings simultaneously. If one string has a frequency of 330 Hz, what is the frequency of the other string?