Answer to Essential Question 21.2: (a) When $t = 0$, the equation gives, at $x = 2.0$ m,
$y = (8.0\text{ mm})\cos\left[(4.0\pi\text{ rad})\right] = +8.0\text{ mm}$, because the cosine of an even multiple of pi is +1.0.
Note that if you work this out on a calculator, your calculator needs to be in radians mode.
(b) If $t = 2.5$ s, the equation gives:
$y = (8.0\text{ mm})\cos\left[(7.5\pi\text{ rad}) + (4.0\pi\text{ rad})\right] = (8.0\text{ mm})\cos\left[(11.5\pi\text{ rad})\right] = 0$, because the cosine of $(n + 0.5)\pi$ is always 0. Again, keep your calculator in radians mode to get this answer from your calculator.

21-3 Frequency, Speed, and Wavelength

The speed of a wave depends on the medium the wave is traveling through. If the medium does not change as a wave travels, the wave speed is constant.

As we discussed in section 21-1, in one period, the wave travels one wavelength. Speed is distance over time, so $v = \lambda / T$. The frequency, f, is $1/T$, so the equation relating wave speed, frequency, and wavelength is $v = f\lambda$.

This equation (Equation 21.1), in this form, makes it look like speed is determined by frequency and wavelength, but this is not the case – the speed is determined by the medium. A good example is the speed of a wave on a stretched string. For a string with a tension F_T, a mass m, and a length L, the speed is given by:

$$v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{F_T}{\mu}},$$ (Eq. 21.5: **The speed of a wave on a string**)

where $\mu = m / L$ is the string's mass per unit length.

In general, then, the speed is determined by the medium, the frequency is determined by whatever is producing the wave (such as you, shaking the end of a string back and forth), and the wavelength is determined by Equation 21.1, through the combination of the speed and frequency.

An exception to this rule of thumb is a wave produced by a typical musical instrument. As we will discuss in more detail at the end of the chapter, when you play an instrument you excite a number of frequencies. The size of the instrument (such as the length of a guitar string) then determines the wavelengths of the particular frequencies that are favored. Thus, on a musical instrument, the length of the instrument determines the wavelength, the wave speed is again determined by the properties of the medium, and the combination of the wavelength and speed determines the frequency of the wave.

EXAMPLE 21.3 – Using the equation of motion for a transverse wave

The general equation for a wave traveling on a string is $y = A\cos(\omega t \pm kx)$. In a particular case, the equation is $y = (8.0\text{ cm})\cos\left[(60\text{ rad/s})t + (0.50\text{ rad/m})x\right]$. Determine:

(a) the wave's amplitude, wavelength, and frequency.
(b) the speed of the wave.
(c) the tension in the string, if the string has a mass per unit length of 0.048 kg/m.
(d) the direction of propagation of the wave.
(e) the maximum transverse speed of a point on the string.
(f) What is the displacement of a point at $x = 2.0$ m when $t = 1.0$ s?

SOLUTION

(a) The amplitude of the wave is the A in the equation, which is whatever is multiplying the cosine. In this particular case, the amplitude is 8.0 cm.

The value of k is whatever is multiplying the x, which is 0.50 rad/m. k is proportional to the inverse of the wavelength, with the wavelength given by $\lambda = 2\pi/(0.50 \text{ m}^{-1}) = 4\pi$ m. Note that we can put in or take out the unit of radians whenever we find it to be convenient.

The value of ω is whatever is multiplying the t, which is 60 rad/s. The frequency, f, is related to the angular frequency, ω, by a factor of 2π:

$$f = \frac{\omega}{2\pi} = \frac{60 \text{ rad/s}}{2\pi} = \frac{30}{\pi}\text{Hz} = 9.5 \text{ Hz} .$$

(b) The speed of the wave can be found from the frequency and wavelength:

$$v = f\lambda = (30/\pi) \text{ Hz} \times (4\pi \text{ m}) = 120 \text{ m/s} .$$

(c) The tension in the string can be found by applying equation 21.5. Solving for tension gives:

$$F_T = v^2\mu = (120 \text{ m/s})^2 \times 0.048 \text{ kg/m} = 690 \text{ N} .$$

(d) In the equation describing the wave, the sign of the x-term is positive. This means that the more positive the x value, the sooner the wave reaches that point, so the wave is traveling in the negative x-direction.

(e) What does "maximum transverse speed" mean? It means the maximum y-direction speed of a point on the string. Any point can be used, because every point experiences the same motion, just at different times. To answer the question, remember that every point on the string is experiencing simple harmonic motion. Thus, this is really a harmonic motion question, not a wave question. Returning to what we learned in chapter 12, the maximum speed of a particle experiencing simple harmonic motion is:

$$v_{\max} = A\omega = (8.0 \text{ cm}) \times 60 \text{ rad/s} = (0.08 \text{ m}) \times 60 \text{ rad/s} = 4.8 \text{ m/s} .$$

(f) We can enter the values of x and t right into the equation, giving:

$$\begin{aligned} y &= (8.0 \text{ cm})\cos\left[(60 \text{ rad/s})t + (0.50 \text{ rad/m})x\right] \\ &= (8.0 \text{ cm})\cos\left[(60 \text{ rad/s})(1.0 \text{ s}) + (0.50 \text{ rad/m})(2.0 \text{ m})\right] \\ &= (8.0 \text{ cm})\cos\left[(60 \text{ rad}) + (1.0 \text{ rad})\right] = (8.0 \text{ cm})\cos(61 \text{ rad}) = -2.1 \text{ cm}. \end{aligned}$$

Don't forget to put your calculator into radians mode when you do this calculation.

Related End-of-Chapter Exercises: 14, 15, 40.

Essential Question 21.3: Return to Example 21.3. If the wave's equation of motion was unchanged except for a doubling of the angular frequency, which of the answers in Example 21.3 would change, and how would they change?