Answer to Essential Question 21.1: Both representations are needed. The wavelength is found from the graph of displacement versus position, while the period is found from the graph of displacement versus time. Both the wavelength and the period are needed to find the wave speed.

21-2 The Connection with Simple Harmonic Motion

Consider a single frequency transverse wave, like the one shown in Figure 21.4. There is clearly a connection between this wave and simple harmonic motion, because each part of the string experiences simple harmonic motion. Thus, for each part of the string we can use an equation like we used to describe simple harmonic motion, $y = A\cos(\omega t)$ or $y = A\sin(\omega t)$. These equations are good starting points, but they are not sufficient to describe what every point on the string is doing. For instance, at $t = 0$, the equation $y = A\cos(\omega t)$ gives $y = +A$, and only three pieces of the string, marked with dots, have $y = +A$.

Every point on the string does reach $y = +A$, but not at $t = 0$. For a given point, therefore, we can introduce something called a phase angle, θ, so the equation reflects the position (and the direction of the velocity) of the point at $t = 0$. Thus, each point has an equation of the form $y = A\cos(\omega t + \theta)$, with every point having a unique θ. Having a different equation to describe every point works, but it is cumbersome. Let's see if we can be more efficient in describing the wave mathematically.

First, consider a point on the string just to the right of the left-most point. A point just to the right of the left-most point does exactly what the left-most point does, just at a slightly later time. Thus, θ for that point is a small negative number, reflecting the small delay in the motion compared to the left-most point. Figure 21.5 shows graphs of the displacement versus time for two different sets of points, one set that is at $y = +A$ in the top picture, and the other set which is at $y = 0$, but moving in the positive y-direction, in the top picture. Note that the motion for the second set of points is delayed compared to the first, with a delay proportional to the distance between the points.

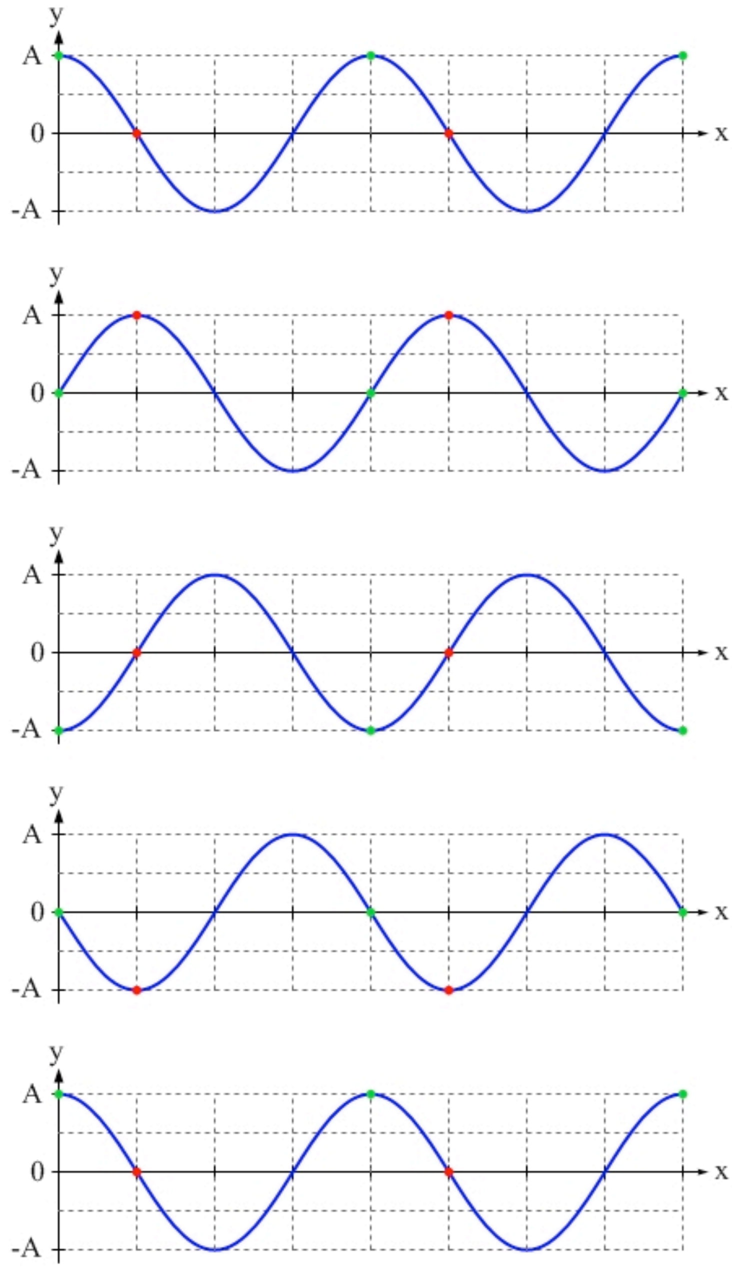

Figure 21.4: This figure, like Figure 21.2, shows fives pictures of a string, separated by equal time intervals. Time increases down the page, so the wave is traveling to the right.

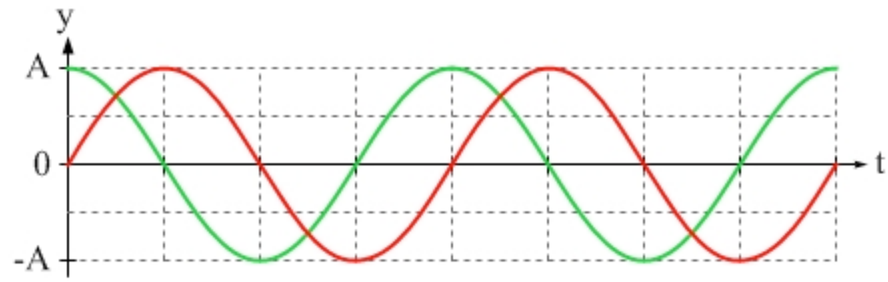

Figure 21.5: Graphs of the displacement versus time for two sets of points, one set initially at $y = +A$, and the other set initially at $y = 0$ and moving in the $+y$ direction.

As x increases, the delay increases, and we find that the phase angle is, in fact, proportional to x, the distance of a point from the left- most point. Thus, we can say that, in the case of a wave traveling in the positive x-direction, $\theta = -kx$, where k is some constant.

If we can identify what the constant k is, we will be finished with our mathematical description. Let's focus now on the point exactly one wavelength to the right of the left-most point. These two points are in phase with one another, which means that whatever one of them does, the other does at the same time. Thus, the equation $y = A\cos(\omega t)$, for the left-most point (at $x = 0$), must agree with the equation for the second point, $y = A\cos(\omega t - k\lambda)$, at $x = \lambda$. Changing the value inside a cosine by a multiple of 2π produces the same result, and because this point is the first point to the right of $x = 0$ that is in phase with the point at $x = 0$, we must have $k\lambda = 2\pi$.

The constant k is known as the **wave number**, and is given by:

$$k = \frac{2\pi}{\lambda}.$$ (Equation 21.2: **the wave number**)

The wave number is, in some sense, the spatial equivalent of the angular frequency. The angular frequency is given by:

$$\omega = \frac{2\pi}{T}.$$ (Equation 21.3: **the angular frequency**)

Note that we now have a single equation that describes the wave. The equation tells us the displacement from equilibrium of each point in the medium at any value of t we might be interested in.

$$y = A\cos(\omega t \pm kx),$$ (Equation 21.4: **Equation of motion for a transverse wave**)

where the plus sign is used when the wave is traveling in the negative x-direction, and the minus sign is used when the wave is traveling in the positive x-direction.

Related End-of-Chapter Exercises: 13, 17, 18, 41.

Essential Question 21.2: In a particular case, the equation of motion of a transverse wave is:

$$y = (8.0 \text{ mm})\cos\left[(3\pi \text{ rad/s})t + (2\pi \text{ rad/m})x\right].$$

Determine the displacement of a point at $x = 2.0$ m at (a) $t = 0$, and (b) $t = 2.5$ s.