Answer to Essential Question 21.9: Every half-period, the string profile is inverted, so at $t = T/2$ the string profile is inverted compared to what it is at $t = 0$, and after a full period ($t = T$), the profile is the same as that as $t = 0$. Halfway between these positions, the string profile is flat, as shown in Figure 21.24.

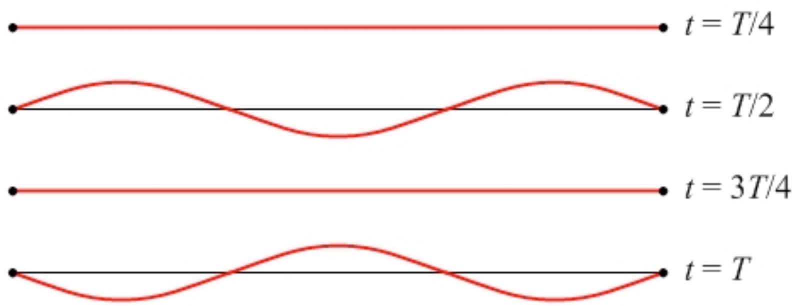

Figure 21.24: The string profile at (a) $t = T/4$, (b) $t = T/2$, (c) $t = 3T/4$, and (d) $t = T$, for the third harmonic.

21-10 Standing Waves in Pipes

Many musical instruments are made from pipes. Such instruments are known as wind instruments. A flute, for instance, is a single pipe in which the effective length can be changed by opening one of several holes in the pipe. In a trombone, the effective length is changed by sliding a tube in or out. In a pipe organ, in contrast, many different pipes, of fixed length, are used, with each pipe having a different fundamental frequency. The connection between all of these instruments is that the effective length of the tube determines the sound the pipe makes.

In contrast with a string instrument, in which a vibrating string sets up a sound wave in the air, the wave in a wind instrument is already a sound wave in a column of air, some of which escapes to make an audible sound. As with strings, however, standing waves produced by reflected waves determine the fundamental frequency of the sound wave produced by a particular pipe. Note that pipes can have both ends open, or have one end open and one end closed. For a sound wave, the open end of a pipe is like a free end, while the closed end of a pipe is like a fixed end. Thus, a pipe with only one end open sounds quite different from a pipe with both ends open, even if the tubes have the same length, because of the different standing waves that are produced by the different reflections in these pipes.

Because an open end acts like a free end for reflection, the standing waves for a pipe that is open at both ends have anti-nodes at each end of the pipe. We can satisfy this condition with standing waves in which an integral number of half-wavelengths fit in the pipe, as shown in parts (a) – (c) of Figure 21.25. This leads to the same equation for standing waves that we had in section 21-9, for the string fixed at both ends.

For a pipe open at both ends: The standing waves produced always have an anti-node at each end of the pipe. The frequencies that produce standing waves in such a pipe are:

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L},$$ (Eq. 21.14: **Standing-wave frequencies for a pipe open at both ends**)

where n is an integer, and L is the effective length of the pipe.

Because an open end acts like a free end, while a closed end acts like a fixed end, the standing waves for a pipe that is open at only one end have anti-nodes at the open end and nodes at the closed end. We can satisfy this condition with standing waves in which an odd integer number of quarter-wavelengths fit in the pipe, as shown in parts (d) – (f) of Figure 21.25. This leads to new equation for the standing-wave frequencies.

For a pipe open at one end only: The standing waves produced have an anti-node at the open end and a node at the closed end. The frequencies that produce standing waves in such a pipe are:

$$f_n = \frac{nv}{4L},$$ (Eq. 21.15: **Standing-wave frequencies for a pipe open at one end**)

where n is an odd integer, and L is the effective length of the pipe.

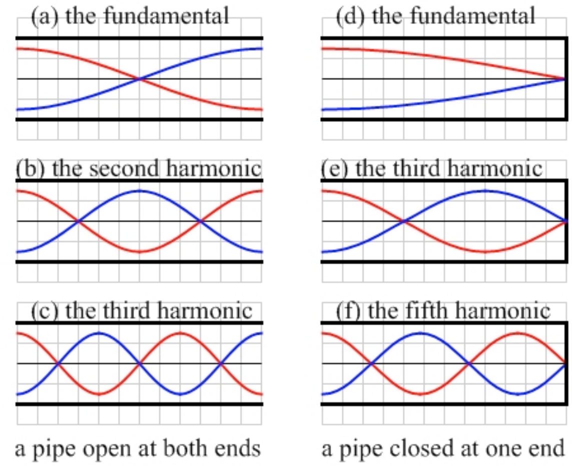

Figure 21.25: (a) – (c) A representation of the standing waves in a pipe that is open at both ends, showing the fundamental (a), and the second (b) and third (c) harmonics. (d) – (f) A similar representation for a pipe that is closed at one end only, showing the fundamental (d), and the two lowest harmonics (e) and (f). For a pipe closed at one end only, the harmonics can only be odd integer multiples of the fundamental. Note that the waves in the pipe are sound waves, which are longitudinal waves. This representation shows the maximum displacement from equilibrium for the air molecules as a function of position along each of the pipes. The standing waves oscillate between the profile shown in red and the profile shown in blue.

EXAMPLE 21.10 – Waves in a pipe

A particular organ pipe has a length of 72 cm, and it is open at both ends. Assume that the speed of sound in air is 340 m/s.

(a) What is the wavelength of the fundamental in this pipe?

(b) What is the corresponding frequency of the fundamental?

(c) If one end of the pipe is now covered, what are the wavelength and frequency of the fundamental?

SOLUTION

(a) For the fundamental, exactly half a wavelength fits in the length of the pipe. Thus, the wavelength is twice the length of the pipe: $\lambda = 2 \times 72$ cm = 144 cm = 1.44 m.

(b) Knowing the speed of sound and the wavelength, we can determine the frequency:

$$f = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.44 \text{ m}} = 236 \text{ Hz}.$$

(c) Covering one end of the pipe means the pipe is open at one end only, so now, for the fundamental, only one-quarter of a wavelength fits in the pipe rather than half a wavelength. This doubles the wavelength of the fundamental to 2.88 m. Doubling the wavelength reduces the frequency by a factor of two, so the new fundamental frequency is 118 Hz.

Related End-of-Chapter Exercises: 31, 33, 37, 38.

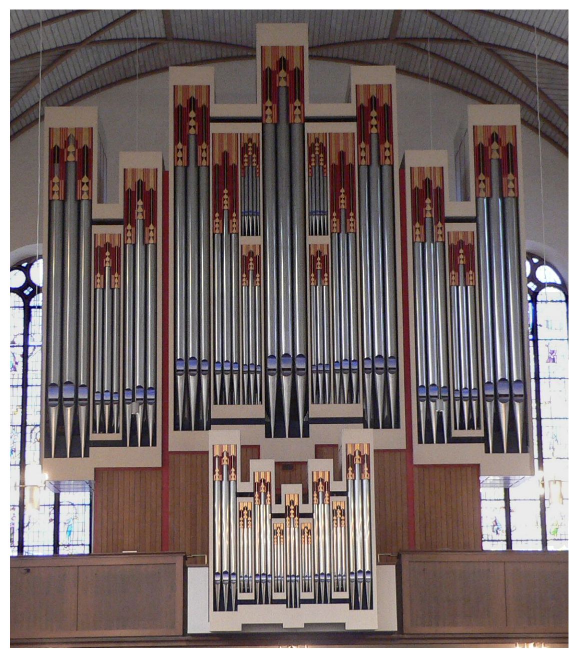

Figure 21.26: The photograph shows a pipe organ in Katharinenkirche, Frankfurt am Main, Germany. Each pipe has a unique frequency.
Photo credit: Wikimedia Commons.

Essential Question 21.10: Musical instruments made from pipes have a variety of pipes or one variable–length pipe. What happens to the fundamental frequency as the pipe length increases?