20-1 Magnetic Flux

Let's begin by introducing the concept of flux. Flux means something quite different in physics than it does in everyday conversation. In physics, the flux through an area is simply a measure of the number of field lines passing through an area. In chapter 16, for instance, we could have defined an electric flux, a measure of the number of electric field lines passing through an area, in a way analogous to the following definition of magnetic flux. As we will see in section 20-2, magnetic flux turns out to play a crucial role in the generation of electricity.

Magnetic flux is a measure of the number of magnetic field lines passing through an area. The symbol we use for flux is the Greek letter capital phi, Φ. The equation for magnetic flux is:

$$\Phi = BA\cos\theta ,$$ (Equation 20.1: **Magnetic flux**)

where θ is the angle between the magnetic field $\vec{B}$ and the area vector $\vec{A}$. The area vector has a magnitude equal to the area of a surface, and a direction perpendicular to the plane of the surface. The SI unit for magnetic flux is the weber (Wb). 1 Wb = 1 T m^2.

EXAMPLE 20.1 – Determining the magnetic flux

A rectangular piece of stiff paper measures 20 cm × 25 cm. You hold the piece of paper in a uniform magnetic field that has a magnitude of 4.0×10^{-3} T. For each situation below, sketch a diagram showing the magnetic field and the paper, and determine the magnitude of the magnetic flux through the paper, when the magnitude of the flux is (a) maximized, (b) minimized, and (c) halfway between its maximum and minimum value.

SOLUTION

(a) How should we hold the paper so that the largest number of field lines pass through it? As shown in Figure 20.1, we hold it so that the plane of the paper is perpendicular to the direction of the magnetic field. We can also understand this orientation by considering equation 20.1. To maximize the flux with an area vector of constant magnitude and a field of constant magnitude, we need to maximize the factor of $\cos\theta$. The factor of $\cos\theta$ reaches its maximum magnitude of 1 when θ, the angle between the area vector and the magnetic field, is either 0° or 180°. In other words, the area vector must be parallel to the magnetic field, which is the case when the plane of the paper is perpendicular to the magnetic field.

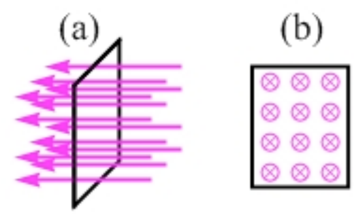

Figure 20.1: To maximize the magnetic flux through a flat area, orient the area so the plane of the area is perpendicular to the direction of the magnetic field. (a) shows a perspective view, while (b) shows the view looking along the field lines. In this case, the area vector is in the same direction as the field lines.

Because $\cos\theta$ has a magnitude of 1, the magnitude of the maximum flux equals the area multiplied by the magnetic field:

$$\Phi_{max} = AB = 0.20 \text{ m} \times 0.25 \text{ m} \times (4.0\times10^{-3} \text{ T}) = 2.0\times10^{-4} \text{ T m}^2 .$$

(b) The factor of $\cos\theta$ in equation 20.1 can be zero. Thus, the minimum magnitude of the magnetic flux is zero ($\Phi_{min} = 0$). How do we hold the paper so that there is no magnetic flux? As shown in Figure 20.2, if the plane of the paper is parallel to the magnetic field, no field lines pass through the paper and the magnetic flux is zero.

Figure 20.2: There is no flux when the plane of the area is parallel to the field. (a) shows a perspective view, while (b) shows the view looking along the field lines. In this case, the area vector is perpendicular to the field lines.

(c) Starting from the situation in Figure 20.1, tilting the loop by 60° (see Figure 20.3) gives a factor of $\cos\theta$ of ½, halfway between its maximum and minimum value. In this case the magnetic flux is $\Phi = 1.0\times10^{-4}$ T m^2, half its value from part (a).

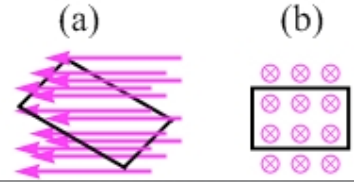

Figure 20.3: Tilting the loop from the orientation in Figure 20.1 reduces the flux. (a) shows a perspective view, while (b) shows the view along the field lines.

EXPLORATION 20.1 – Ranking situations based on flux

The four areas in Figure 20.4 are in a magnetic field. The field has a constant magnitude, and is directed into the page in the left half of the region and out of the page in the right half. Rank the areas based on the magnitude of the net flux passing through them, from largest to smallest. Note that, when we calculate net flux, field lines passing in one direction through an area cancel an equal number of field lines passing in the opposite direction through an area.

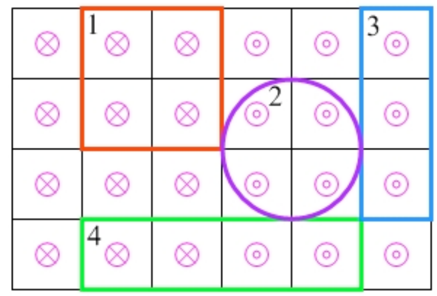

Figure 20.4: Four different regions in a magnetic field. The field has the same magnitude everywhere, but it is directed into the page in the left half of the field and out of the page in the right half.

Region 1 is tied with region 4 for the largest area, but all the field lines in region 1 pass through in the same direction, giving region 1 the largest-magnitude flux. In contrast, the net flux through region 4 is zero because the flux through the right half of region 4 cancels the flux through the right side. The field lines in regions 2 and 3 all pass through in the same direction. Region 3 has an area of 3 boxes, while region 2 has an area of πr^2, where the radius is 1 unit, so region 2 has an area of π boxes. Because π is larger than 3, the magnetic flux through region 2 is larger than that through region 3. Thus, ranking by flux magnitude gives 1 > 2 > 3 > 4.

Key idea for net flux: In calculating net flux, field lines passing one way through an area cancel an equal number of field lines passing in the opposite direction through that area.
Related End-of-Chapter Exercises: 1, 2, 4, 41.

An aside: Electric Flux and Gauss' Law

We did not mention electric flux when we talked about electric field, but we can define electric flux in an analogous way to magnetic field. Electric flux is a measure of the number of electric field lines passing through a surface. The equation for electric flux is:

$$\Phi_E = EA\cos\theta ,$$

where θ is the angle between the electric field $\vec{E}$ and the area vector $\vec{A}$.

There is a law called Gauss' Law, which says that the net electric flux passing through a closed surface is proportional to the net charge enclosed by that surface. Using the correct proportionality constant, Gauss' Law can be used to calculate electric fields in highly-symmetric situations. It is interesting to note that the analogous law in a magnetic situation, Gauss' Law for magnetism, is not nearly so useful. Because magnetic field lines are always continuous loops, the net magnetic flux passing through a closed surface is always zero – if a magnetic field line emerges from a surface, it must re-enter the surface at some other location, giving a net flux for that field line of zero, to ensure that the field line is a continuous loop.

Essential Question 20.1: Return to the situation described in Exploration 20.1, and shown in Figure 20.4. If we define out of the page as the positive direction for magnetic flux, rank the four areas by their net flux, from most positive to most negative.