16-8 Gauss' Law

In Chapter 16, we stated the equation for the electric field from a point charge. However, we can actually derive the point charge equation, as well as equations for the electric field in other situations, by applying Gauss' law.

Gauss' law is often stated in the form of an integral. However, if you actually have to integrate when applying Gauss' law, you're probably not doing it correctly! What we generally do is to apply Gauss' law in highly symmetric situations. Taking advantage of symmetry allows us to use Gauss' law in a form that does not require an integral. Such use of symmetry is often used in physics to transform complicated problems into problems that are easier to solve.

Applying Gauss' law in highly symmetric situations. In symmetric situations, Gauss' law can be stated as follows: the product of the surface area of an enclosed volume and the electric field at the surface of that volume is equal to the charge enclosed by that volume divided by a constant, ε_0, the permittivity of free space.

$$A E = \frac{q_{enc}}{\varepsilon_0},$$ 　　　　(Equation 16.5: **Gauss' law in situations of high symmetry**)

Written in this form, the equation applies as long as:
* the electric field is the same magnitude at all points on the surface, and
* the electric field is perpendicular to the surface at all locations.

The product of the electric field and the area is called the **electric flux** – it is a measure of the number of electric field lines passing through the area.

If the enclosed charge is positive, the electric field is directed out through the surface. If the enclosed charge is negative, the electric field is directed in through the surface. The permittivity of free space, ε_0, is related to k, the constant we used in Coulomb's law, in the following way:

$$k = \frac{1}{4\pi \varepsilon_0} = 9.0 \times 10^9 \text{ N m}^2/\text{C}^2.$$ 　　(Eq. 16.6: **Connecting k to the permittivity of free space**)

EXPLORATION 16.8 – Finding the electric field from a point charge
The description of Gauss' law above probably sounds a little complicated, so let's apply it to the situation of the electric field from a point charge.

Step 1 – *Figure 16.8A shows a small ball with a charge of +Q embedded at the center of (a) a sphere, and (b) a cube. Do either of these situations meet the conditions necessary to apply Equation 16.5, to calculate the electric field from a point charge?*

Figure 16.8A: Two-dimensional representations of a charged ball, which we treat as a point charge, embedded at the center of (a) a sphere, and (b) a cube.
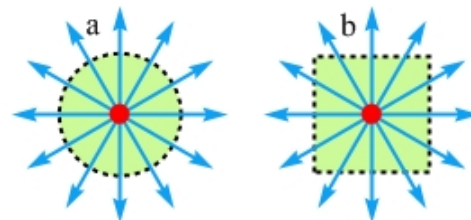
The sphere works well – the electric field is perpendicular to the sphere's surface at all points and, because all points on the surface are the same distance from the point charge, the electric field at every point on the surface has the same magnitude.

On the other hand, the cube fails on both conditions. The electric field is perpendicular to the cube's surface at some points, but not at all points. In addition, different points on the cube's surface are different distances away from the charged ball, so the magnitude of the electric field is different at different points.

Step 2 – *Figure 16.8B shows a second sphere, in which the charged ball is not at the center of the sphere. Does this sphere meet the conditions of Gauss' law?*
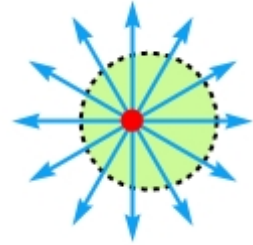
Figure 16.8B: In this sphere, the charged ball is not at the center of the sphere it is enclosed in.

No, the second sphere does not meet the conditions. To take advantage of our formulation of Gauss' law, the situation has to be as symmetric as possible. If the charged ball is at the center of the sphere, the situation is highly symmetric. Placing the ball off-center ruins the symmetry – the second sphere suffers from the same issues as the cube in Step 1.

Step 3 – *Use Gauss' law to find an expression for the electric field, due to the point charge, at the surface of the spherical volume in part (a) of Figure 16.8A. Assume the spherical volume has a radius r.* The area, A, in Equation 16.5, is the surface area of the enclosing surface. In this case, we use the expression for the surface area of a sphere of radius r, $A = 4\pi r^2$. Thus, Equation 16.5 becomes:

$$4\pi r^2 E = \frac{q_{enc}}{\varepsilon_0} = \frac{+Q}{\varepsilon_0}.$$

Solving for the electric field, and bringing in Equation 16.6, gives:

$$E = \frac{+Q}{4\pi r^2 \varepsilon_0} = +\frac{kQ}{r^2}.$$

This is completely consistent with Equation 16.4, the expression for the electric field from a point charge.

Key ideas about applying Gauss' law: In highly symmetric situations, we can apply a simplified version of Gauss' law to calculate the electric field. We did that in this Exploration to derive the equation we used in Chapter 16 for the electric field from a point charge.
Related End of Chapter Exercises: see the *Essential Physics* web site.

*Essential Question 16.8***:** In Chapter 16, we learned that the electric field inside a conductor is zero when the conductor is at static equilibrium. Figure 16.8C shows a solid metal sphere of radius R. The metal sphere has a net charge of $+8q$ on it, and the charge on the sphere is in static equilibrium. We also draw a spherical volume, of radius $r = R/2$, inside the metal sphere. The centers of the metal sphere and the spherical volume are the same. Using Gauss' law, determine the charge enclosed by the spherical volume. Comment on how your answer changes if the radius of the spherical volume increases. (See the *Essential Physics* web site for the answer.)
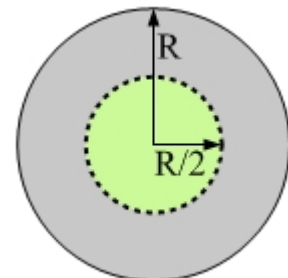
Figure 16.8C: A metal sphere with a net charge of $+8q$, and a spherical volume that has half the radius of the metal sphere.