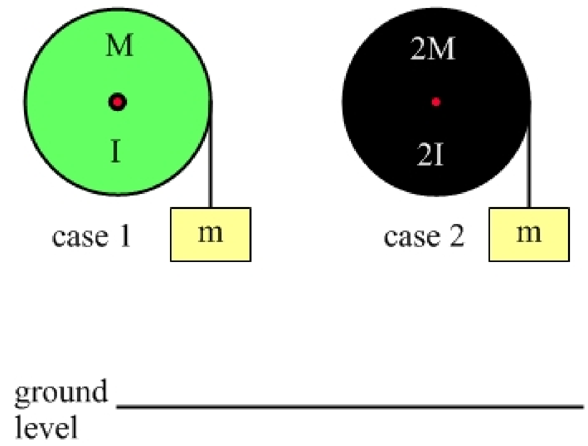
PROBLEM 1 – 15 points



Case 1 and case 2 show two situations of a block hanging from a string wrapped around the outside of a pulley. The blocks are identical, and the pulleys are uniform solid disks of the same radius, but the pulley in case 2 has twice the mass (and therefore twice the moment of inertia) of the pulley in case 1. The systems are released from rest at the same time from the same height above the ground, and in both cases the block accelerates down. Neglect friction.

[3 points] (a) In which case is the acceleration of the block larger?
[**X**] case 1 [ ] case 2 [ ] equal in both cases

Briefly justify your answer: **The force of gravity acting on the block has to accelerate both the block and the pulley. The pulley has twice the rotational inertia in case 2, so that pulley will accelerate more slowly.**

[3 points] (b) In which case is the tension in the string, while the block is falling, larger?
[ ] case 1 [**X**] case 2 [ ] equal in both cases

Briefly justify your answer: **The free-body diagram of the block has a downward force of gravity, and an upward force of tension, with a net downward acceleration. Applying Newton's second law gives** $mg - F_T = ma$ **, so** $F_T = mg - ma$ **. The smaller the acceleration, the larger the tension, and the block has a smaller acceleration in case 2.**

[3 points] (c) In which case is the net torque on the pulley, while the block is falling, larger?
[ ] case 1 [**X**] case 2 [ ] equal in both cases

Briefly justify your answer: **The torque is exerted by the tension in the string. Case 2 has the larger tension, so it also has the larger torque.**

[3 points] (d) In which case does the block have a higher speed after falling through a distance *h*?
[**X**] case 1 [ ] case 2 [ ] equal in both cases

Briefly justify your answer: **Case 1, because the acceleration is larger.**

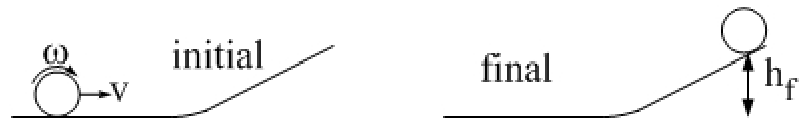
[3 points] (e) In which case does the pulley have a larger rotational kinetic energy, measuring the kinetic energy at the instant the block reaches the ground in each case?
[ ] case 1 [**X**] case 2 [ ] equal in both cases

Briefly justify your answer: **Case 2. The change in gravitational potential energy, -mgh, is the same in the two cases, and total mechanical energy is conserved. Thus, the total kinetic energy is the same in each case as the block's reach the ground (this happens at different times, of course). The total kinetic energy is the sum of the translational kinetic energy and the rotational kinetic energy. Because the translational kinetic energy is larger in case 1, the rotational kinetic energy must be larger in case 2.**

PROBLEM 2 – 20 points

A uniform solid sphere with a mass of M = 5.0 kg and radius R = 20 cm is rolling without slipping on a horizontal surface at a constant speed of 5.0 m/s. It then encounters a ramp, and proceeds to roll without slipping up the ramp. The goal of this problem is to determine the maximum height reached by the sphere on the ramp before it turns around, and to use conservation of energy to do so. Use g = 10 m/s^2.

[3 points] (a) Sketch this situation, showing the sphere in two positions, one at the bottom of the ramp and the other when the sphere reaches its highest point.



[3 points] (b) Start with the usual conservation of energy equation: $K_i + U_i + W_{nc} = K_f + U_f$. Identify all the terms that are zero in this equation, and explain why they are zero.

$U_i = 0$ **- define the zero level to be the lowest level, at the bottom of the ramp.**

$W_{nc} = 0$ **- no non-conservative forces act on the sphere.**

$K_f = 0$ **- the sphere comes to rest for an instant at the highest point.**

[4 points] (c) Write out expressions for the remaining terms. Remember to account for both translational kinetic energy and rotational kinetic energy, if appropriate. Keep everything in terms of variables.

$K_i = U_f$ which gives $\frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = Mgh_f$

[6 points] (d) How far does the sphere roll up the ramp (measuring the vertical distance)? First find an expression for this distance in terms of variables, simplified as much as possible, and then plug in the appropriate values.

Using $I = \frac{2}{5}MR^2$ and $\omega_i = \frac{v_i}{R}$, the rotational kinetic energy becomes:

$$\frac{1}{2}I\omega_i^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{v_i^2}{R^2} = \frac{1}{5}Mv_i^2$$

Substituting this into the energy equation gives: $\frac{1}{2}Mv_i^2 + \frac{1}{5}Mv_i^2 = Mgh_f \quad \Rightarrow \quad \frac{7}{10}v_i^2 = gh_f$

This gives $h_f = \frac{7v_i^2}{10g} = 1.75$ m

[4 points] (e) If a block slides without friction up the ramp, starting at the bottom with the same initial speed as the sphere, which object travels farther up the ramp?

[**X**] the sphere [ ] the block [ ] they travel equal distances

Briefly justify your answer: **Mechanical energy is conserved in both cases, but with the block it is just the translational kinetic energy that gets converted into gravitational potential energy. With the sphere, there is an extra contribution from the rotational kinetic energy.**

PROBLEM 3 – 10 points

A particular horizontal turntable can be modeled as a uniform disk with a mass of 200 g and a radius of 20 cm that rotates without friction about a vertical axis passing through its center. The initial angular speed of the turntable is 2.4 rad/s. A ball of clay, with a mass of 80 g, is dropped from a height of 35 cm above the turntable. It hits the turntable at a distance of 10 cm from the center, and sticks where it hits so that the clay and the turntable rotate together at a new angular speed. Assuming the turntable is firmly supported by its axle so it remains horizontal at all times, find the final angular speed of the turntable-clay system.

Show your work, and explain what you're doing!

First of all, the height from which the clay ball is dropped is irrelevant. The ball's vertical momentum has no effect on the angular momentum around an axis that is also vertical. That vertical momentum is removed from the turntable-ball system by a force exerted by whatever is supporting the turntable.

Our approach here is based on conservation of angular momentum, about the vertical axis that passes through the center of the turntable. The initial angular momentum is all in the turntable:

Initial angular momentum =

$$I_{table}\omega_i = \frac{1}{2}MR^2\omega_i = \frac{1}{2}(0.200 \text{ kg})(0.20 \text{ m})^2(2.4 \text{ rad/s}) = 0.0096 \text{ kg m}^2\text{/s}.$$

The total rotational inertia of the system afterwards is the rotation of the turntable plus the rotational inertia of the clay, which we can use mr^2 to find.

The final angular momentum must be equal to the initial angular momentum, because there is no net torque applied to the system about the vertical axis that passes through the center of the turntable.

$$0.0096 \text{ kg m}^2\text{/s} = I_{total}\omega_f = (I_{table} + I_{ball})\omega_f = \left(\frac{1}{2}MR^2 + mr^2\right)\omega_f$$

Solving for the final angular speed gives:

$$\omega_f = \frac{0.0096 \text{ kg m}^2\text{/s}}{\left(\frac{1}{2}MR^2 + mr^2\right)} = \frac{0.0096 \text{ kg m}^2\text{/s}}{\left(\frac{1}{2}(0.200 \text{ kg})(0.20 \text{ m})^2 + (0.080 \text{ kg})(0.10 \text{ m})^2\right)} = \frac{0.0096 \text{ kg m}^2\text{/s}}{0.0048 \text{ kg m}^2} = 2.0 \text{ rad/s}$$