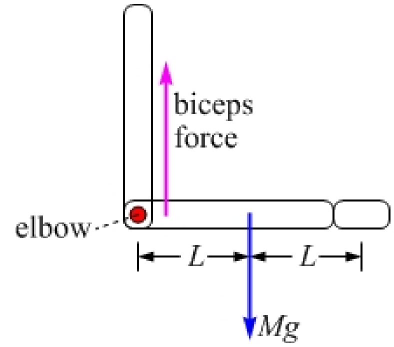
PROBLEM 1 – 20 points



It is often useful to treat the lower arm as a uniform rod of length 2*L* that can rotate about the elbow. The figure shows a simple model of an arm, with the upper arm vertical and the lower arm horizontal.

[10 points] (a) When you are not holding anything in your hand, and your arm is at rest in the position shown, three forces act on your lower arm: the force of gravity (*Mg*), the force exerted by the biceps, and the force exerted at the elbow joint by the humerus (the bone in the upper arm).

(i) Compare the force of gravity with the biceps force. Which has the larger magnitude?

[ ] the force of gravity [**X**] the force from the biceps [ ] neither, they're equal

Briefly justify your answer: **Sum torques around the elbow joint. The force from the humerus applies no torque, so the other two torques must balance (this is equilibrium, so the net torque is zero). Let's say the biceps force is exerted at a distance *d* from the elbow.**

$MgL = F_B d$ **so** $F_B = Mg\dfrac{L}{d}$

Because *L* > *d*, we get $F_B > Mg$

(ii) Compare the force from the biceps with the force from the humerus. Which has the larger magnitude?

[**X**] the force from the biceps [ ] the force from the humerus [ ] neither, they're equal

Briefly justify your answer: **We can take torques around the arm's center-of-gravity, or sum forces. If we add forces we know they add to zero, because this is an equilibrium situation. Because the biceps force is larger than the force of gravity, Mg, the force of the humerus must be directed down. The biceps force has to balance the force of the humerus and the force of gravity together, so the biceps force is the largest of the three.**

[5 points] (b) If you place an object in your hand (at a distance 2*L* from the elbow joint), and your arm remains in the position shown, the force from the biceps increases. Assuming the biceps is attached to the arm at a distance of $d = L/5$ from the elbow joint, by how much does the force from the biceps increase if the object has a weight of $mg = 20.0$ N? Your answer should have units of newtons.

One approach is to take torques about the elbow joint. If counter-clockwise is positive, we get:

Without the ball: $F_B \dfrac{L}{5} - MgL = 0$ **so** $F_B = 5Mg$

With the ball: $F_B^{/} \dfrac{L}{5} - MgL - mg(2L) = 0$ **so** $F_B^{/} = 5Mg + 10mg = 5Mg + 200N$

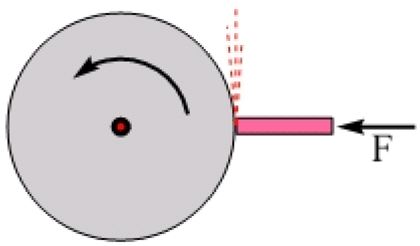
So, the biceps force must increase by 200 N (10 times the weight of the object!).

[5 points] (c) When the 20.0 N object is placed in your hand, what is the change in the force applied by the humerus at the elbow joint? State both the magnitude (in newtons) and the direction of the change, if there is a change.

The simplest way to approach this is to realize that the forces were balanced before the ball was added, and they must still balance now. If we added a 20 N force down and a 200 N force up, that's a net of 180 N up. Therefore the force from the humerus must change by 180 N down (it increases by 180 N, in other words, because the force was already directed down) to keep the forces balanced.

PROBLEM 2 – 15 points

Two identical grinding wheels of mass *m* and radius *r* are initially spinning about their centers. Wheel A has an initial angular speed of ω_i, while wheel B has an initial angular speed of $2\omega_i$. Both wheels are being used to sharpen tools. As shown in the figure, for both wheels the tool is being pressed against the wheel with a force *F* directed toward the center of the wheel, and the coefficient of kinetic friction between the wheel and the tool is μ_K. You are holding the tool firmly so that it does not move tangentially to the wheel.



[3 points] (a) If it takes wheel A a time *T* to come to a stop, how long does it take for wheel B to come to a stop?

Let's start with a constant angular acceleration equation, $\omega = \omega_i + \alpha t$. Wheel A comes to rest, so the final angular velocity is zero. Solving the equation for *T* gives: $T = -\frac{\omega_i}{\alpha}$.

The wheels have the same angular acceleration (it comes from the frictional torque, which is the same for both wheels). We can see that if we double the initial angular speed, the time will also double. So, wheel B comes to rest in a time of 2*T*.

[3 points] (b) Find an expression for *T* in terms of the variables specified above.

What we need to do here is to take our expression for *T* above, and express the angular acceleration in terms of the variables specified above. Starting from Newton's second law for rotation, and defining counter-clockwise as positive, we get:

$$\alpha = \frac{\Sigma\tau}{I} = \frac{-r(\mu_k F)}{\frac{1}{2}mr^2} = \frac{-2\mu_k F}{mr}$$

Substituting this into our expression for *T* gives: $T = -\frac{\omega_i}{\alpha} = +\frac{\omega_i\, mr}{2\mu_k F}$

[3 points] (c) If wheel A rotates through an angle θ before coming to rest, through what angle does wheel B rotate before coming to rest?

Let's start with a constant angular acceleration equation, $\omega^2 = \omega_i^2 + 2\alpha\,\theta$**. Wheel A comes to rest, so the final angular velocity is zero. Solving the equation for** θ **gives:** $\theta = -\frac{\omega_i^2}{2\alpha}$**.**

The wheels have the same angular acceleration, so we can see that if we double the initial angular speed, the angle will quadruple, because the angular speed is squared. So, wheel B comes to rest after rotating through an angle of 2θ.

[3 points] (d) Find an expression for θ in terms of the variables specified above.

Here, we can take the equation we derived in (c) and substitute the expression we derived for the angular acceleration, in part (b).

$$\theta = -\frac{\omega_i^2}{2\alpha} = +\frac{\omega_i^2\, mr}{4\mu_k F}$$

[3 points] (e) If you doubled the value of μ_K, how would that affect the time required to stop wheel A?

We can go back to the equation we derived in part (b), and see that the stopping time is inversely proportional to the coefficient of friction. So, if we double the coefficient of kinetic friction, the time to stop the wheel is reduced by a factor of 2.