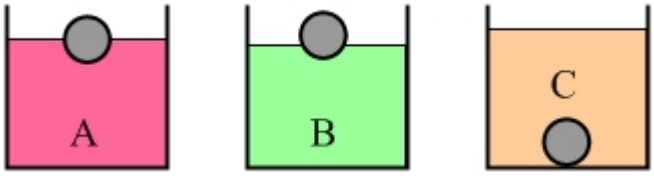
PROBLEM 1 – 15 points

You have three identical balls, and three beakers containing different liquids. Each ball has a density of 600 kg/m^3. You place one ball in each beaker.
In beaker A the ball floats with 50% of its volume submerged.
In beaker B the ball floats with 25% of its volume submerged.
In beaker C the ball sinks to the bottom.

[3 points] (a) Sketch these three situations.



[3 points] (b) Rank the liquids based on their densities, from largest to smallest. Use > and/or = signs in your ranking, such as B>A=C.

B>A>C. Because the ball sinks in C, that liquid must be less dense than the ball. We know liquids A and B are more dense than the ball, because the ball floats in those. Therefore C is the fluid with the lowest density. Comparing A and B, the larger the density of the fluid the less volume is displaced, so B must be the most dense.

[3 points] (c) Compare situations A and B. In which situation does the ball experience a larger buoyant force? The ball experiences a larger buoyant force in

[ ] beaker A [ ] beaker B [**X**] neither – the buoyant forces are equal

Briefly justify your answer: **Draw a free-body diagram of the ball. In each case the buoyant force has to balance the force of gravity. Since the force of gravity is the same the buoyant forces must be equal.**

[3 points] (d) Compare situations B and C. In which situation does the ball experience a larger buoyant force? The ball experiences a larger buoyant force in

[**X**] beaker B [ ] beaker C [ ] neither – the buoyant forces are equal

Briefly justify your answer: **Again the answer comes from comparing free-body diagrams. In B the buoyant force balances the force of gravity. In C, when the ball is resting on the bottom of the beaker, the sum of the buoyant force and the normal force balances the force of gravity. Since the force of gravity acting on the ball is the same in both cases the buoyant force is larger in B.**

[3 points] (e) What is the density of the liquid in beaker A?

1200 kg/m^3. From the free-body diagram we have $mg = F_B$. We can express the object's mass in terms of its density, and we can also use Archimedes' principle to express the buoyant force in terms of the density of the fluid and the volume displaced. This gives:

$\rho_{ball} V_{ball} g = \rho_A V_{disp} g = \rho_A \left(0.5 V_{ball}\right) g$.

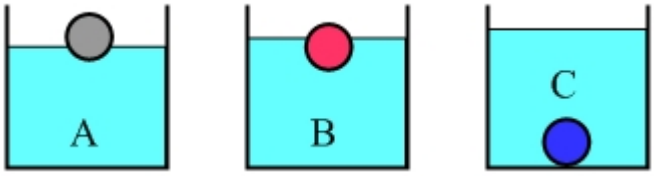
Solving for the density of the fluid gives:

$$\rho_A = \frac{\rho_{ball} V_{ball}}{0.5 V_{ball}} = \frac{\rho_{ball}}{0.5} = 2\rho_{ball} = 2\left(600\,\text{kg/m}^3\right) = 1200\,\text{kg/m}^3 .$$

PROBLEM 2 – 10 points

You have three balls, which have equal volumes but different densities, and three beakers of water. Water has a density of 1000 kg/m^3. You place one block in each beaker.
Ball A floats with 30% of its volume submerged.
Ball B floats with 70% of its volume submerged.
Ball C sinks to the bottom.

[2 points] (a) Sketch these three situations.



[2 points] (b) Rank the balls based on their densities, from largest to smallest. Use > and/or = signs in your ranking, such as B>A=C.

C>B>A. Ball C sinks, so its density is greater than that of water. Ball B floats 70% submerged, so its density is 70% of the density of water, while ball A's density is only 30% that of water.

[3 points] (c) Compare balls A and B. Which ball experiences a larger buoyant force?

[ ] ball A [**X**] ball B [ ] neither – the buoyant forces are equal

Briefly justify your answer: **One justification is that the buoyant force is proportional to the volume of fluid displaced, and ball B displaces more fluid than ball A does. Another justification is that the buoyant force balances the force of gravity acting on the ball. The ball's have the same volume, so ball B, being more dense, has a larger mass, and thus experiences a larger force of gravity. Thus, the buoyant force acting on B must be larger than that acting on A.**

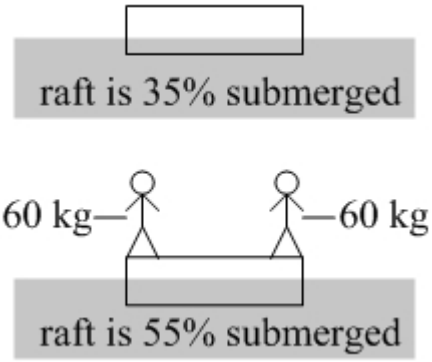
[3 points] (d) When **ball C** is resting at the bottom of its beaker it experiences a buoyant force of 50 N, directed up. Use this information to solve for the mass of **ball A**. Use g = 10 m/s^2.

The buoyant force acting on C, which is 100% submerged, is 50 N. Note that this does not mean that ball C has a mass of 5 kg! All we know is that ball C's mass is greater than or equal to 5 kg.
The buoyant force acting on A, which is 30% submerged, must be 30% of the buoyant force acting on C, because the buoyant force is proportional to the volume of fluid displaced. 30% of 50 N is 15 N.
For ball A, the upward buoyant force of 15 N balances the downward force of gravity, so *mg* for ball A is 15 N.
For ball A, the mass is $m_A = \frac{m_A g}{g} = \frac{15\text{ N}}{10\text{ m/s}^2} = 1.5\text{ kg}$**.**

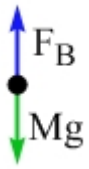
PROBLEM 3 – 10 points

When there are no people standing on a large wooden raft that is floating on a lake, the raft floats with 35% of its volume submerged.

When two people, each with a mass of 60 kg, stand on the raft, the raft floats with 55% of its volume submerged.

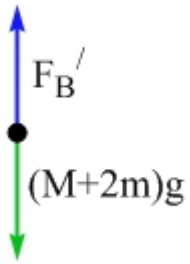


[2 points] (a) Sketch the free-body diagram for the raft when it has nobody on it.



The upward buoyant force balances the downward force of gravity.

[2 points] (b) Sketch the free-body diagram for the system of the raft and the two people.



The upward buoyant force, which is larger than above, balances the combined weight of the system. You don't include any normal forces between the raft and the people because they are internal forces in the system – the upward normal force applied by the raft on a person is cancelled by the downward normal force applied to the raft by the person. *M* = mass of the raft; *m* = mass of a person.

[2 points] (c) If one more person, with a mass of 60 kg, gets on the raft, what percentage of the raft will be submerged?

The key here is to recognize that the buoyant force is proportional to the volume displaced. If adding two people causes the volume displaced to go from 35% to 55% (increasing it by 20 percentage points), one person would increase it by 10 percentage points. So, with one more person, the volume displaced goes to 65%.

[2 points] (d) What is the maximum number of people, each with a mass of 60 kg, that the raft can support without sinking? (Your answer should be an integer.)

We have 65% to work with. If one person corresponds to 10%, six people corresponds to 60%. With the 35% from the raft by itself, six people on the raft will cause the raft to be submerged by 35% + 60% = 95%. Any more people and it sinks.

[2 points] (e) Calculate the mass of the raft.

Here we can do a ratio, recognizing that 10% is associated with one 60 kg person.

$$\frac{10\%}{60\text{ kg}} = \frac{35\%}{m_{raft}} \qquad \textbf{leads to} \qquad m_{raft} = 3.5 \times 60\text{ kg} = 210\text{ kg}$$