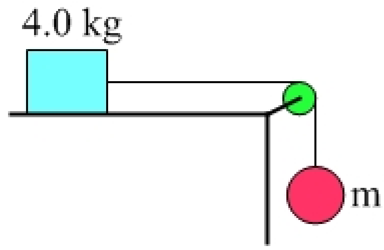
PROBLEM 1 – 10 points

A block with a mass of 4.0 kg is connected to a ball, which has an unknown mass *m*, by means of a string that passes over a frictionless pulley. Assume the mass of the pulley is negligible. The coefficient of kinetic friction between the block and the horizontal surface is $\mu_K = 0.050$. The system is released from rest, and the block and ball accelerate. When the ball has dropped through a height of 2.0 m its speed is 4.0 m/s.
Use g = 10 m/s^2.



[4 points] (a) Calculate the work done by friction on the block over the 2.0 m distance moved by the block.

Here we can use the expression for the work done by a force: $W = Fd\cos\theta$**.**

The displacement of the block is 2.0 m to the right. The force is the force of kinetic friction, which is:
$F_K = \mu_K F_N = \mu_K mg = 0.050 \times (4.0\text{ kg}) \times (10\text{ m/s}^2) = 2.0\text{ N}$**. This is directed to the left, so the angle between the force and the displacement is 180°.**
The work done by friction is therefore: $W = (2.0\text{ N})(2.0\text{ m})\cos(180°) = -4.0\text{ J}$**.**

[6 points] (b) Calculate the mass of the ball.

There are a number of ways this can be done, including finding the acceleration and using forces. Seeing as we just found the work done by friction, however, let's do this using conservation of energy. Start by writing out the five-term equation for energy conservation.

$$U_i + K_i + W_{nc} = U_f + K_f$$

We'll apply this to the ball + block system.

- **The block does not move vertically, so we can neglect the block's gravitational potential energy.**
- **Let's define the zero for the ball's gravitational potential energy so its final potential energy is zero and its initial potential energy is** $mg(+2.0\text{ m})$
- **There is no initial kinetic energy.**
- **The work term is the answer from part (a)**
- **Remember that both objects move, so there are two kinetic energy terms, one for each object, in the end.**

This all gives: $mg(2.0\text{ m}) - 4.0\text{ J} = \frac{1}{2}m(4.0\text{ m})^2 + \frac{1}{2}(4.0\text{ kg})(4.0\text{ m})^2$

$$(20\text{ J/kg})m - 4.0\text{ J} = (8\text{ J/kg})m + 32\text{ J}$$

$$(12\text{ J/kg})m = 36\text{ J}$$

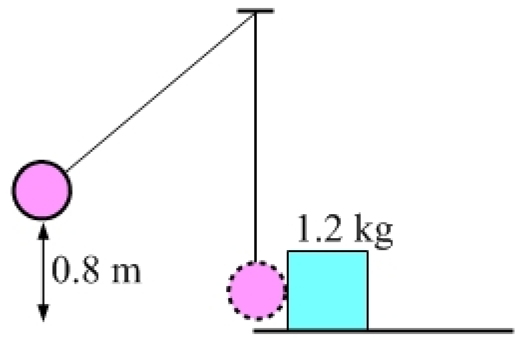
$$m = \frac{36\text{ J}}{12\text{ J/kg}} = 3.0\text{ kg}$$

PROBLEM 2 – 15 points

A ball with an unknown mass *m* is tied to a string and hung from the ceiling so that it rests against a block that has a mass of 1.2 kg. The ball is pulled back, raising it by 0.80 m, and released from rest. The ball then swings down and collides with the block. After the collision the block has a velocity of 1.0 m/s to the right on the frictionless surface, while the ball rebounds to a height of 0.20 m.
Use g = 10 m/s^2.

[3 points] (a) What is the velocity of the ball just before the collision?

Here we can apply energy conservation. Defining the zero for gravitational potential energy as the lowest point leads to $mgh = \frac{1}{2}mv^2$**. Solving for the speed gives the familiar result that:** $v = \sqrt{2gh}$**.**
In this case we get $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.8} = \sqrt{16} = 4.0\,\mathrm{m/s}$
We are asked for a velocity so we also need to specify a direction. The velocity is 4.0 m/s to the right.

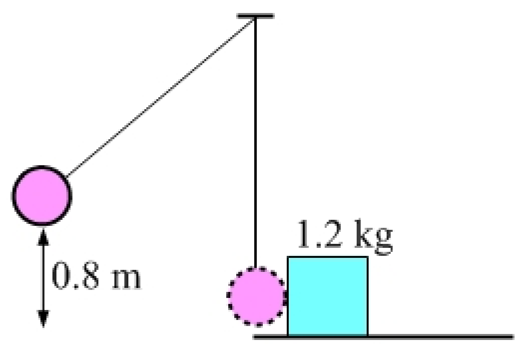


[3 points] (b) What is the velocity of the ball just after the collision?

Applying the same method as in (a) gives:

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.20} = \sqrt{4} = 2.0\,\mathrm{m/s}$$

We are asked for a velocity so we also need to specify a direction. The velocity is 2.0 m/s to the left.



[5 points] (c) What is the mass of the ball?

Now we need to apply momentum conservation. This means writing out an equation of the form:

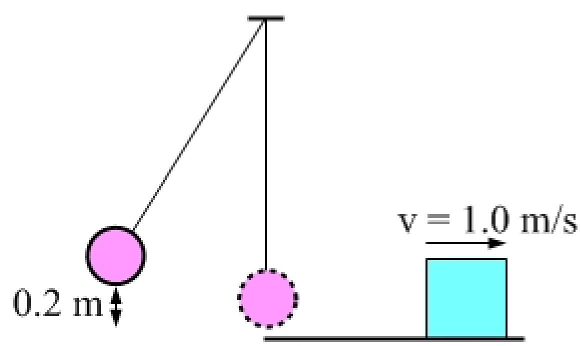
$$m_{ball}\vec{v}_{i,ball} + m_{block}\vec{v}_{i,block} = m_{ball}\vec{v}_{f,ball} + m_{block}\vec{v}_{f,block}.$$

Choose right to be the positive direction. Using + and – signs in the equation to indicate the direction gives:

$$+(4.0\,\mathrm{m/s})m_{ball} = -(2.0\,\mathrm{m/s})m_{ball} + (1.2\,\mathrm{kg})(1.0\,\mathrm{m/s}).$$

$$+(6.0\,\mathrm{m/s})m_{ball} = +1.2\,\mathrm{kg\ m/s}.$$

$$m_{ball} = \frac{1.2\,\mathrm{kg\ m/s}}{6.0\,\mathrm{m/s}} = 0.20\,\mathrm{kg}.$$



[4 points] (d) What kind of collision is this?

[ ] elastic [**X**] inelastic [ ] completely inelastic

Justify your answer: **There are (at least) two ways to justify this. One is to use the definition of elasticity, the ratio of the relative speed after the collision to the relative speed before the collision. Before the collision the block is at rest and the ball's velocity is 4.0 m/s to the right, so the relative speed is 4.0 m/s. Afterwards the ball is moving at 2.0 m/s left while the block is moving at 1.0 m/s right, for a relative speed of 3.0 m/s. The ratio is ¾ = 0.75, which is less than 1. An elasticity less than 1 is classified as an inelastic collision, but it can't be completely inelastic since the two objects do not stick together.**

A more popular way to justify this answer is to work out the kinetic energy immediately before and immediately after the collision.

Before the collision we get: $K_i = \frac{1}{2} m_{ball} v_{i,ball}^2 = \frac{1}{2}(0.2 \text{ kg})(4.0 \text{ m/s})^2 = 1.6 \text{ J}$

After the collision:

$$K_i = \frac{1}{2} m_{ball} v_{f,ball}^2 + \frac{1}{2} m_{block} v_{f,block}^2 = \frac{1}{2}(0.2 \text{ kg})(2.0 \text{ m/s})^2 + \frac{1}{2}(1.2 \text{ kg})(1.0 \text{ m/s})^2 = 0.4 \text{ J} + 0.6 \text{ J} = 1.0 \text{ J}$$

Because the total kinetic energy after the collision is less than the kinetic energy before the collision, the collision is inelastic (but, again, it can't be completely inelastic).

PROBLEM 3 – 10 points

In a collision between two carts in the physics lab, the situation is as follows:

Before the collision, cart A, with mass *m*, has a velocity of *v* to the right.
Before the collision, cart B, with a mass 2*m*, has a velocity of *v* to the left.

After the collision cart A has a velocity of *v* to the left.

[4 points] (a) What is the velocity of cart B after the collision?

Cart B is at rest after the collision.

Consider the momentum before and after the collision. Remember that momentum is a vector, so let's take to the left to be positive.
The total momentum before the collision is $-mv + 2mv = +mv$ **.**

After the collision, cart A has a momentum of $+mv$ **.**

To conserve momentum, cart B must be at rest after the collision.

[4 points] (b) Is kinetic energy conserved in this collision? Briefly justify your answer.

No, kinetic energy is not conserved in this case. Cart A has the same kinetic energy before and after the collision, but B's kinetic energy went from something to nothing. Overall, therefore, there is less kinetic energy after the collision than before.

[2 points] (c) Which cart exerts more force on the other during the collision?

[ ] Cart A exerts a larger-magnitude force on B than B exerts on A.

[ ] Cart B exerts a larger-magnitude force on A than A exerts on B.

[**X**] The two carts exert forces of equal magnitudes on one another.

By Newton's Third Law, the force that A exerts on B is equal in magnitude, and opposite in direction, to the force that B exerts on A.