PROBLEM 1 – 5 points

A box is placed on a horizontal board and then the angle between the board and the horizontal is gradually increased until that angle is 30˚. During this process the box remains at rest on the board.

During this process, while the angle of the board is increasing from 0˚ to 30˚ :

(i) the magnitude of the component of the force of gravity acting on the box that is directed parallel to the slope:

[**X**] increases [ ] decreases [ ] stays the same

(ii) the magnitude of the component of the force of gravity acting on the box that is directed perpendicular to the slope:

[ ] increases [**X**] decreases [ ] stays the same

(iii) the magnitude of the normal force exerted on the box by the board:

[ ] increases [**X**] decreases [ ] stays the same

(iv) the magnitude of the force of friction exerted on the box by the board:

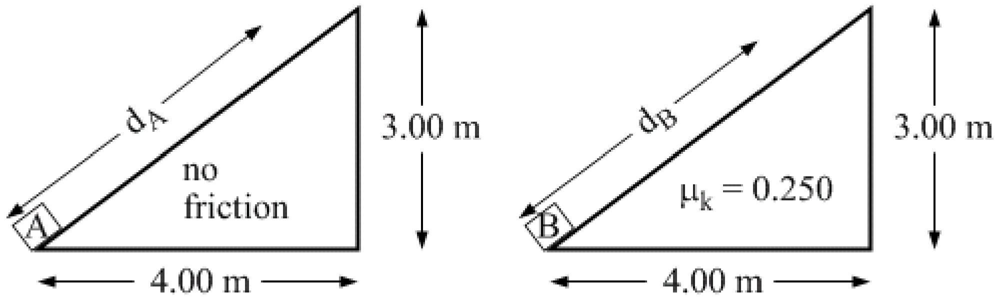
[**X**] increases [ ] decreases [ ] stays the same

(v) the magnitude of the maximum possible force of friction the box could exert on the board:

[ ] increases [**X**] decreases [ ] stays the same

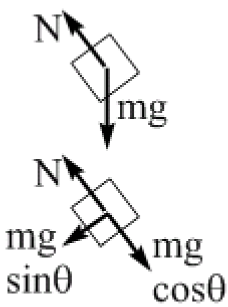
At any given angle, the free-body diagram of the box has a normal force perpendicular to the slope and a component of the force of gravity (mg cosθ) in the opposite direction. These forces must be equal. As the angle increases both these forces decrease in magnitude.
In the direction parallel to the slope the force of static friction is directed up the slope and it must balance the component of the force of gravity acting down the slope (mg sinθ). As the angle increases both these forces increase in magnitude.
Finally, the maximum possible force of static friction is proportional to the normal force, so it does what the normal force does, decreasing in magnitude as the angle increases.

PROBLEM 2 – 20 points



Two identical blocks, A and B, are placed at the bottom of almost-identical ramps and given **initial velocities of 6.00 m/s up their ramps**. Both ramps are in the shape of 3-4-5 triangles, as shown, but block A's ramp is frictionless while the coefficient of kinetic friction between block B and its ramp is 0.250. Both blocks slide up and down their ramps. Use **g = 10.0 m/s^2**.

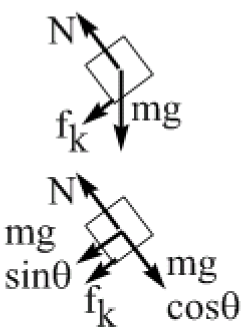
[6 points] (a) Block A travels a distance of d_A up its ramp before turning around. Sketch a free-body diagram of block A as it is sliding up the slope, and use this to determine the distance d_A.



Note that the nice thing about a 3-4-5 triangle is that the sine and cosine are easy numbers to work with. sinθ = 3/5 and cosθ = 4/5. The angle is not 30˚, 45˚, or 60˚.
Defining the positive x-direction as up the slope, apply Newton's Second Law in the x-direction: -mg sinθ = ma,
so a = - g sinθ = -10*(3/5) = -6.0 m/s^2.
To determine the distance traveled use:

$v^2 = v_i^2 + 2a\Delta x$ **so** $\Delta x = \frac{v^2 - v_i^2}{2a} = \frac{0-36}{2*(-6)} = \frac{-36}{-12} = 3.0 \text{ m}$

[10 points] (b) Sketch a free-body diagram of block B as it is sliding up its ramp. For how much time is block B sliding up the ramp?



Applying Newton's Second Law in the y-direction, perpendicular to the slope, gives:
N = mg cosθ

Applying Newton's Second Law in the x-direction, parallel to the slope, gives:
-mg sinθ – f_k = ma

So:

$$a = \frac{-mg\sin\theta - \mu_k N}{m} = \frac{-mg\sin\theta - \mu_k mg\cos\theta}{m} = -g\sin\theta - 0.25g\cos\theta = -6 - 2 = -8 \text{ m/s}^2$$

Substitute this into the equation $v = v_i + at$ **to get** $t = \frac{v - v_i}{a} = \frac{0-6}{-8} = 0.75 \text{ s}$

[4 points] (c) Select all the true statements about this situation from the list below. Grading scheme: +1 for each correct answer, –1 for each incorrect answer (but you can't get less than 0).

[**X**] Block A travels a larger distance up its ramp than does block B.
[**X**] Block A takes the same time to slide up the ramp as it does to slide down.
[   ] Block B takes the same time to slide up the ramp as it does to slide down.
[**X**] Block B's average speed on the way up is larger than its average speed on the way down.
[   ] The time it takes block A to reach its highest point is the same as the time it takes block B to reach its highest point.
[   ] On the way up the slope the net force on block A is zero.
[   ] On the way down the slope the net force on block A is directed up the slope.
[**X**] When the blocks are sliding down their ramps the magnitude of the net force on block A is larger than the magnitude of the net force on block B.

Statement 1 – true because friction on B causes B to stop before A.

Statement 2 – true because A's acceleration is constant

Statement 3 – false because B's acceleration is smaller in magnitude on the way down than on the way up.

Statement 4 – true because the average speed on the way up is 3 m/s, the average of 6 m/s and 0, while on the way down it's less than 3 m/s because it's the average of 0 and something less than 6 m/s.

Statement 5 – false because A takes longer – friction causes B to stop sooner

Statements 6 and 7 – both false, the net force on A is always mg sinθ down the slope

Statement 8 – true because the component of the force of gravity down the slope on B is partly cancelled by the force of friction acting up the slope

PROBLEM 3 – 15 points

Two identical boxes of mass *m* are sliding along a horizontal floor, but both eventually come to rest because of friction. Box A has an initial speed of v_i, while box B has an initial speed of $2v_i$. The coefficient of kinetic friction between each box and the floor is μ_K , and the acceleration due to gravity is *g*.

(a) If it takes box A a time *T* to come to a stop, how much time does it take for box B to come to a stop?

Let's start with the constant-acceleration equation $v = v_i + at$. The final velocity is zero, so solving for *T* gives: $T = -\frac{v_i}{a}$.

Both boxes have the same acceleration (coming from the force of friction, which is the same for both). We can see that doubling the initial speed will double the time, so block B comes to rest in a time of 2*T*.

(b) Find an expression for *T* in terms of the variables specified above.

We can use our expression for T from above, but we must express the acceleration in terms of the given variables. Applying Newton's second law to block A, with the positive direction being the direction of the initial velocity, we get:

$$a = \frac{\Sigma F}{m} = \frac{-F_k}{m} = \frac{-\mu_k F_N}{m} = \frac{-\mu_k mg}{m} = -\mu_k g \ .$$

Substituting this into our expression for time, above, gives: $T = -\frac{v_i}{a} = \frac{v_i}{\mu_k g}$

(c) If box A travels a distance *D* before coming to rest, how far does box B travel before coming to rest?

Let's start with the constant-acceleration equation $v^2 = v_i^2 + 2a(\Delta x)$. The final velocity is zero, so solving for *D* gives: $D = -\frac{v_i^2}{2a}$.

Again, both boxes have the same acceleration. We can see that doubling the initial speed will quadruple the distance, because the speed is squared, so block B travels a distance of 4*D*.

(d) Find an expression for *D* in terms of the variables specified above.

Again, we can substitute our expression for *a* into the equation in (c).

$$D = -\frac{v_i^2}{2a} = \frac{v_i^2}{2\mu_k g}$$

(e) How does *D*, the stopping distance for box A, change if *m* is doubled?

There is no change. As we saw, the factor of mass cancelled out from the equation, so all of our results above our independent of mass. In other words, if we change the mass, it has no effect on the distance (or the time, for that matter).