PROBLEM 1 – 10 points

Stephanie loves to kayak. When she kayaks on a large lake, which has no current, she paddles a distance of 8.0 km in 2.0 hours at a constant speed.

One day, Stephanie decides to kayak on a river that has a constant current of 2.0 km/h. Stephanie paddles against the current for a while, and then turns around and travels with the current for a while, returning to the same point from which she started. This trip takes a total time of 2.0 hours (the same time as her trip on the lake), and Stephanie always paddles at the same constant speed, relative to the water, that she maintains when kayaking on the lake.

All the questions below pertain to Stephanie's trip on the river.

[5 points] (a) The total distance Stephanie travels on this trip is …

[**X**] less than 8.0 km [ ] 8.0 km [ ] more than 8.0 km

Briefly explain your answer. Give us a conceptual explanation, not a calculation.

The current subtracts from Stephanie's net speed for the first part of the trip, and adds to it, by the same amount, on the way back. The distance covered in both parts of the trip is the same, so Stephanie paddles against the current for more time, going slowly, than she paddles with the current, going faster. Because Stephanie spends more time being slowed by the current, her average speed is less than her average speed when there is no current. The times are the same for her trip on the lake and on the river, so with a lower average speed on the river she travels a shorter distance.

[3 points] (b) How much time did Stephanie paddle before turning around?

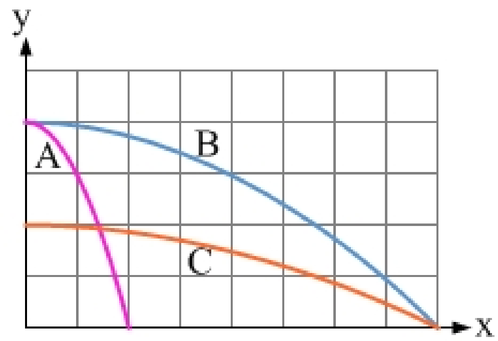
Against the current, Stephanie's net speed is 4 km/h – 2 km/h = 2 km/h.
With the current, Stephanie's net speed is 4 km/h + 2 km/h = 6 km/h.
The times for the two parts of the trip add up to two hours: $t_1 + t_2 = 2 \text{ hours}$**.**
The distances are the same, where distance is speed * time, so
$(2 \text{ km/h})t_1 = (6 \text{ km/h})t_2 \;\text{ so }\; t_1 = 3t_2$
This gives $3t_2 + t_2 = 4t_2 = 2 \text{ hours}, \quad \text{so} \quad t_2 = 0.5 \text{ hours}$ **and** $t_1 = 1.5 \text{ hours}$**.**
Stephanie travels 1.5 hours before turning around.

[2 points] (c) How far, relative to her starting point (which is a fixed point on the riverbank) did Stephanie travel before turning around?

$$\text{distance} = (2 \text{ km/h})t_1 = (2 \text{ km/h}) \times (1.5 \text{ h}) = 3 \text{ km}$$

Check your answer! Verify that the distance on the way back also equals 3 km by multiplying 6 km/h by 0.5 hours (that does equal 3 km), and that the total distance, 6 km, agrees with the answer in part (a).

PROBLEM 2 – 10 points



The trajectories of three projectiles, A, B, and C, are shown in the figure. All three projectiles are launched horizontally, with no initial vertical component of velocity. Projectile C is launched from a lower point than are projectiles A and B, but C travels the same distance horizontally as projectile B. All three projectiles are influenced only by gravity after launch.

[4 points] (a) Rank the projectiles based on their times of flight, from largest to smallest.

[**X**] A=B>C [ ] A>B>C [ ] B>A>C [ ] B>C>A [ ] B=C>A

Briefly justify your answer: **The time of flight depends only on what happens vertically. The vertical accelerations (g) and initial velocity vertically (zero) are the same for all three projectiles, so it is only the height that matters, where starting higher gives a longer time. A and B are equal because they fall through the same height.**

[2 points] (b) Rank the projectiles based on the magnitude of the vertical component of their velocity just before reaching the ground (the *x* axis), from largest to smallest.

[**X**] A=B>C [ ] A>B>C [ ] B>A>C [ ] B>C>A [ ] B=C>A

Again, it comes down to how the projectiles compare vertically, so the answer here is the same as in (a).

[2 points] (c) Rank the projectiles based on the magnitude of the horizontal component of their velocity, from largest to smallest.

[ ] A=B=C [ ] B=C>A [ ] B>C>A [**X**] C>B>A [ ] C>B=A

A's horizontal velocity is less than that of B, because the times are the same for A and B but A travels less distance horizontally. C's horizontal velocity is larger than B's, because B and C travel the same distance horizontally but C covers that distance in less time.

[2 points] (d) Rank the projectiles based on the magnitude of their accelerations while they are in flight, from largest to smallest.

[**X**] A=B=C [ ] A=B>C [ ] A>B>C [ ] B>A>C [ ] B>C>A

The acceleration is due to gravity for all three.

PROBLEM 3 – 10 points

A projectile is launched from ground level on flat ground. The initial velocity has a magnitude of v_i at an angle of 60° above the horizontal. The horizontal component of the initial velocity is v_{ix} and the vertical component of the initial velocity is v_{iy}.

Answer TRUE or FALSE for the following statements, which deal with the special case in which the projectile starts and ends at the same height.

(a) Keeping v_i fixed but increasing the launch angle by 5 degrees will increase the time of flight.

[**X**] TRUE [ ] FALSE

Increasing the launch angle makes the projectile go higher, increasing the time of flight.

(b) Keeping v_i fixed but increasing the launch angle by 5 degrees will increase the range.

[ ] TRUE [**X**] FALSE

This statement is true if you start from 30°, but false if you start from 60°. In general, changing the angle so that it gets farther from 45° decreases the range.

(c) Keeping v_{iy} fixed and increasing v_{ix} will increase the time of flight.

[ ] TRUE [**X**] FALSE

v_{ix} **has no bearing on the time of flight. It is the *y*-component of the initial velocity that matters.**

(d) Keeping v_{ix} fixed and increasing v_{iy} will increase the time of flight.

[**X**] TRUE [ ] FALSE

Increasing v_{iy} **takes the projectile higher, increasing the time of flight.**

(e) Keeping v_{ix} fixed and increasing v_{iy} will increase the range.

[**X**] TRUE [ ] FALSE

If we calculate the range as v_{ix} **multiplied by the time of flight, increasing** v_{iy} **increases the time of flight, thereby increasing the range.**

PROBLEM 4 – 10 points

You are flying a small plane from Boston to Buffalo, which is located 660 km due west of Boston. Immediately after taking off you point your plane due west, set the autopilot to cruise at a speed of 220 km/h relative to the air, and then you take a nap. You wake up later and, after checking your watch, you expect to be directly over your destination. Instead, you find yourself 240 km south and 480 km west of Boston, close to Harrisburg, Pennsylvania instead.

[5 points] (a) Assuming the autopilot did exactly what you told it to do, what was the average velocity of the wind acting on your airplane during the flight? You can express this in terms of its components.

What you need:

$x = x_o + v_{eff,x}t$ $\qquad$ $y = y_o + v_{eff,y}t$

$v_{eff,x} = v_{plane,x} + v_{wind,x}$ $\qquad$ $v_{eff,y} = v_{plane,y} + v_{wind,y}$

For this question you should treat the x components and y components separately. We will define west as the positive x direction and south as the positive y direction.
You also need to get as much information as you can from the question.
For example, here you are told that you wake up when you were supposed to arrive at Buffalo, from this statement you can compute how long you have traveled:
$x = v_{plane,x}\, t$
$t = x / v_{plane,x} = 660/220 = 3$ **hours**
Remember that the effective velocity of the plane will be the velocity of the plane itself, plus the velocity of the wind (added as vectors).

Now you are able to find $v_{x,wind}$ and $v_{y,wind}$:
$x = (v_{plane,x} + v_{wind\ x})\, t = v_{plane,x}t + v_{wind\ x}t$
$v_{wind,x} = (x - v_{plane,x}t)/t = (480 - 660)/3 = -60$ **km/h**

And the same reasoning for the y component:
$y = (v_{plane,y} + v_{wind\ y})\, t = v_{plane,y}t + v_{wind\ y}t$
$v_{wind,y} = (y - v_{plane,y}t)/t = (240 - 0)/3 = 120$ **km/h**

[5 points] (b) Sketch a vector diagram to support your calculations above.