EXTENDING EXPLORATION 2.5B

Let's keep investigating the situation described in Exploration 2.5B, in which a bus that, at a time of $t = 0$, has a velocity of +5.0 m/s and is passing through the origin. The bus has a constant acceleration of +2.0 m/s^2.

Step 1 - ***Sketch a graph of the position of the bus as a function of time.*** Let's start by plugging what we know into Equation 2.11, $x = x_i + v_i t + \frac{1}{2} a t^2$. The initial position is $x_i = 0$ (because at $t = 0$ the bus is at the origin); the initial velocity is $v_i = +5.0 \text{ m/s}$; and the acceleration is $a = +2.0 \text{ m/s}^2$. This gives us an equation that tells us where the bus is at any time:

$$x_{bus} = 0 + (5.0 \text{ m/s})t + \frac{1}{2}(2.0 \text{ m/s}^2)t^2 = (5.0 \text{ m/s})t + (1.0 \text{ m/s}^2)t^2 .$$

The graph we want is a picture of this equation. Note that the equation is quadratic, because of the t^2 term, so the graph will be parabolic. The graph is shown in Figure 2.16b.

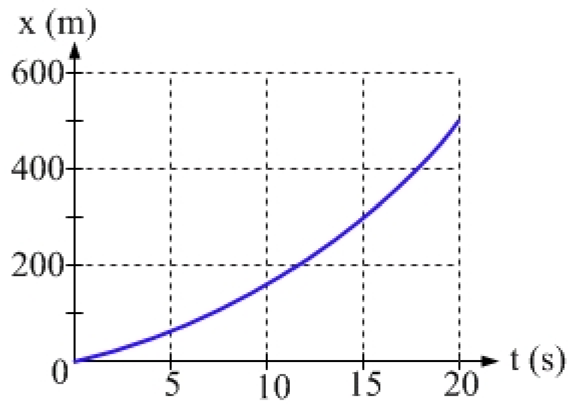

Figure 2.16b: A graph of the position of the bus as a function of time.

Step 2 - ***How is the position graph related to the velocity?*** In exploring constant-velocity motion we found that the velocity is the slope of the position-vs.-time graph. This is the definition of velocity, in fact, so the velocity is still equal to the slope of the position graph for this constant-acceleration situation. Because the velocity is changing, however, we need to distinguish between the instantaneous velocity (the slope of the position versus time curve at a particular instant in time) and the average velocity over a particular time interval (the slope of the straight line connecting two points on the position graph). Figure 2.16c helps make the distinction. The velocity of the bus as a function of time is: $v_{bus} = +5.0 \text{ m/s } + (2.0 \text{ m/s}^2)t$. This follows from Equation 2.9.

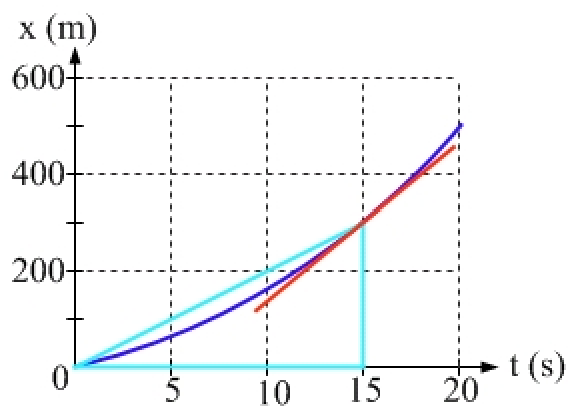

Figure 2.16c: The average velocity of the bus over the first 15 seconds is the displacement (+300 m) divided by the time (15 s), which is +20 m/s. This is the slope of the hypotenuse of the right-angled triangle pictured above. The instantaneous velocity of the bus at t = 15 s, however, is +35 m/s. This is the slope of the tangent (shown in red on the diagram) to the position versus time graph at t = 15 s.

Step 3 - ***How is the velocity graph related to the position?*** The displacement of the bus is +300 m in the first 15 seconds. Let's compare this to the area under the velocity versus time graph over that time interval, which is highlighted in Figure 2.16d.

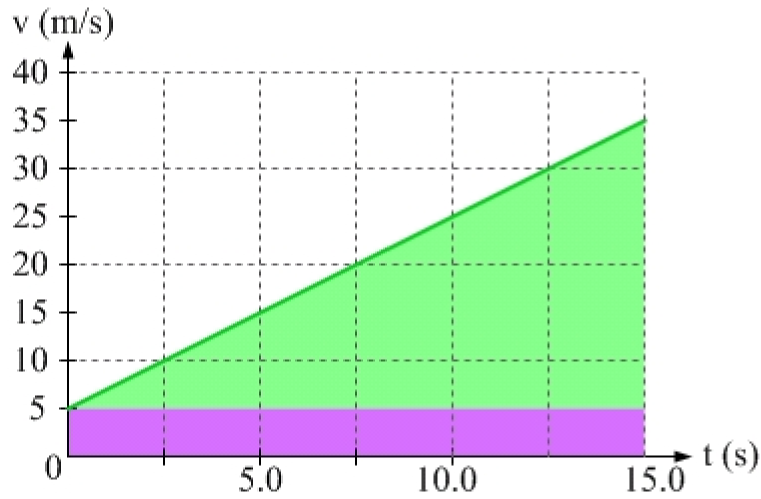

Figure 2.16d: The area under the velocity versus time graph for the first 15 seconds is shown as two regions, a rectangular region (in purple) and a triangular region (in green).

The area of the rectangular region in Figure 2.16d, in purple, is +75 m (+5.0 m/s multiplied by 15 seconds), and corresponds to the $v_i t$ term in the displacement equation, $x - x_i = v_i t + \frac{1}{2}at^2 = (5.0 \text{ m/s})t + (1.0 \text{ m/s}^2)t^2$.

The area of the triangular region, in green, is $\frac{1}{2}\text{base} \times \text{height} = \frac{1}{2}(15 \text{ s})(+30 \text{ m/s}) = +225 \text{ m}$, matching the $\frac{1}{2}at^2$ term in the displacement equation since the base of the triangle is t and the height is $\Delta v = at$.

The lesson here is that the total area under the velocity versus time graph (+75 m + 225 m) is the displacement (+300 m). Also, the average velocity is the displacement over the time, $\bar{v} = \frac{+300 \text{ m}}{15 \text{ s}} = +20 \text{ m/s}$. That's the average of the initial velocity (+5 m/s) and the final velocity (+35 m/s), which is always true for motion with constant acceleration.

Key ideas: The velocity is the slope of the position versus time graph and the displacement is the area under the curve of the velocity versus time graph. For motion with constant acceleration the position versus time graph is parabolic, while the velocity versus time graph is a straight line with a constant slope equal to the value of the constant acceleration.