EXTENDING EXAMPLE 2.8

In Example 2.8, we looked at the situation of a car and bus traveling in neighboring lanes on a straight highway. The positive direction is the direction of travel. The car has a constant velocity of +25.0 m/s, and at t = 0 it is located at +21 meters from the origin. At t = 0, the bus is at the origin, and has a velocity of +5.0 m/s and an acceleration of +2.0 m/s^2. In Example 2.8, we determined how long it took until the bus passed the car. Here, we will extend our analysis to:

(b) Use graphs to find the time when the bus passes the car.
(c) Find how far the vehicles are from the origin when the bus passes the car.
(d) Use graphs to find the time when the vehicles have the same velocity.
(e) Confirm the answer to (d) using constant-acceleration equations.

SOLUTION

We can make use Table 2.3 and Figure 2.16 again, which we used to solve the first three parts of the problem. The subscripts C and B represent the car and bus, respectively.

	Car	**Bus**
Initial position	$x_{iC} = +21\,\text{m}$	$x_{iB} = 0$
Initial velocity	$v_{iC} = +25\,\text{m/s}$	$v_{iB} = +5.0\,\text{m/s}$
Acceleration	$a_C = 0$	$a_B = +2.0\,\text{m/s}^2$

Table 2.3: Summarizing the information that was given about the car and the bus.

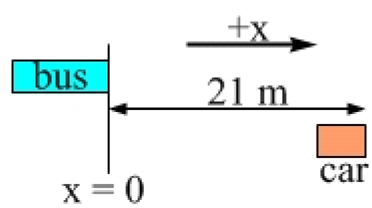

Figure 2.16: A diagram showing the initial positions of the car and the bus, the position of the origin, and the positive direction.

(b) *Use graphs to find the time when the bus passes the car*. Note that the time when the bus passes the car is not the same as the time they have the same velocity. It takes the bus 10 seconds just to reach a speed of 25 m/s, the speed the car was traveling the whole time, so during the first 10 seconds the bus falls farther behind the car. After t = 10 seconds the bus, which continues to accelerate, is traveling faster than the car so it starts to catch up with, and will eventually pass, the car.

The position-vs.-time graphs for the car and bus are plotted on the same set of axes in Figure 2.16b. The graphs intersect at just over 20 seconds. This will be the time when the vehicles the same position, and is the time when the bus passes the car. This result is in agreement with the t = 21 s answer that we solved for in Example 2.8.

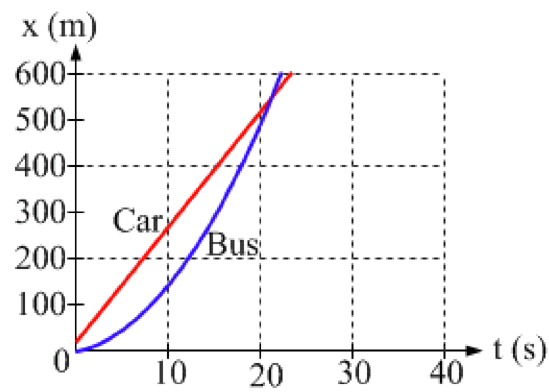

Figure 2.16b: Position-vs.-time graphs for the car and the bus. The graphs cross just beyond 20 seconds, indicating the time and distance from the origin when the bus passes the car.

(c) *Find how far the vehicles are from the origin when the bus passes the car.* Knowing the time when the bus passes the car, we can substitute that time into the equations of motion for the vehicles that we used in Example 2.8. Note that we really only need to determine the position from either the equation of motion for the car or the bus, but it's good to use both because then we can check the answer – both equations should give the same answer.

For the car, at t = 21 s: $x_C = x_{iC} + v_{iC}t + \frac{1}{2}a_C t^2 = +21\text{ m} + (25\text{ m/s})t + 0 = +546\text{ m}$.

For the bus, at t = 21 s: $x_B = x_{iB} + v_{iB}t + \frac{1}{2}a_B t^2 = 0 + (5.0\text{ m/s})t + (1.0\text{ m/s}^2)t^2 = +546\text{ m}$.

These answers agree with one another, as well as with the approximate value that we can read off the graph in Figure 2.16b.

(d) *Use graphs to find the time when the vehicles have the same velocity.* First, does the question make sense? Will there be a time when the two vehicles have the same velocity? Yes, under the conditions stated, there will be one time when they have equal velocities, because the car's velocity stays constant and the velocity of the bus starts below that of the car and gets steadily larger. If we sketch velocity-vs.-time graphs on the same set of axes, as in Figure 2.16d, we can see that the graphs intersect at a time of 10 seconds. This will be the time when the vehicles have the same velocity.

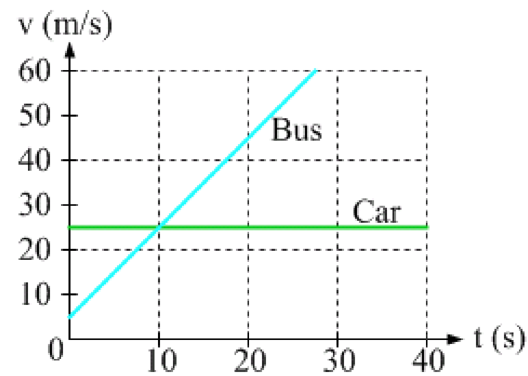

Figure 2.16d: Graphs of the velocity-vs.-time for the bus and the car.

(e) *Confirm the answer to (d) using constant-acceleration equations.* Let's see if the equations give the same result. If we use Equation 2.7 twice, once for each vehicle, we will come up with expressions giving the velocity of each as a function of time. We can then set the equations equal to one another, and solve for the time when the velocities are equal.

For the car: $\vec{v}_C = \vec{v}_{iC} + \vec{a}_C t = +25\text{ m/s} + 0 = +25\text{ m/s}.$

For the bus: $\vec{v}_B = \vec{v}_{iB} + \vec{a}_B t = +5.0\text{ m/s} + (2.0\text{ m/s}^2)t.$

At what time are the two velocities equal? Set the equations equal and solve for this time, which we can call t_2:

$$+25\text{ m/s} = +5.0\text{ m/s} + (2.0\text{ m/s}^2)t_2$$
$$\left(2.0\text{ m/s}^2\right)t_2 = +25\text{ m/s} - 5.0\text{ m/s} = 20\text{ m/s}$$
$$t_2 = \frac{20\text{ m/s}}{2.0\text{ m/s}^2} = 10\text{ s}$$, confirming what we found using the graphs.