Answer to Essential Question 1.7: Yes, you could have an equation with no real solutions. In that case when you applied the quadratic formula you would get a negative under the square root, while the graph would still be parabolic but would not cross (or touch) the *x*-axis.

Chapter Summary

Essential Idea

Physics is the study of how things work, and in analyzing physical situations we will try to apply a logical, systematic approach. Some of the basic tools we will use include:

Units

Our primary set of units is the système international (SI), based on meters, kilograms, and seconds, and four other base units. SI is widely accepted in science worldwide, and convenient because conversions are based on powers of ten. Converting between units is straightforward if you know the appropriate conversion factor(s).

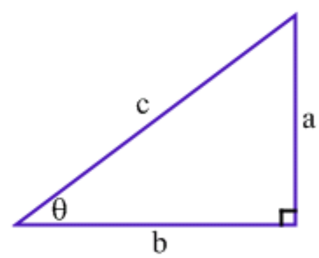
Significant Figures

Three useful guidelines to follow when rounding off include:

1. Round off only at the end of a calculation when you state the final answer.
2. When you multiply or divide, round your final answer to the smallest number of significant figures in the values going into the calculation.
3. When adding or subtracting, round your final answer to the smallest number of decimal places in the values going into the calculation.

Trigonometry

In a right-angled triangle we use the following relationships:



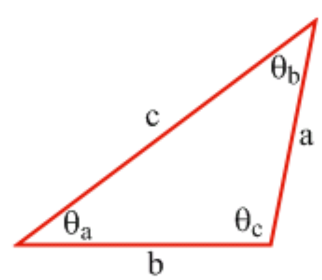
$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{a}{c}; \quad \cos\theta = \frac{adjacent}{hypotenuse} = \frac{b}{c}; \quad \tan\theta = \frac{opposite}{adjacent} = \frac{a}{b}.$$

We relate the three sides using: $c^2 = a^2 + b^2$. (Eq. 1.1: **The Pythagorean Theorem**)

Many triangles do not have a 90˚ angle. For a general triangle, such as that in Figure 1.3, if we know the length of two sides and one angle, or the length of one side and two angles, we can use the Sine Law and the Cosine Law to find the other sides and angles.

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}.$$ (Equation 1.2: **Sine Law**)

$$c^2 = a^2 + b^2 - 2ab\cos\theta_c.$$ (Equation 1.3: **Cosine Law**)



Vectors

A vector is a quantity with both a magnitude and a direction. Vectors can be added geometrically (drawn tip-to-tail), or by using components.

A unit vector is a vector with a length of one unit. A unit vector is denoted by having a carat on top, which looks like a hat, like $\hat{x}$ (pronounced "x hat").

A vector can be stated in unit-vector notation or in magnitude-direction notation.

A Method for Adding Vectors Using Components

1. Draw a diagram of the situation, placing the vectors tip-to-tail to show how they add geometrically.
2. Show the coordinate system on the diagram, in particular showing the positive direction(s).
3. Make a table showing the *x* and *y* components of each vector you are adding together.
4. In the last line of this table, find the components of the resultant vector by adding up the components of the individual vectors.

Algebra and Dimensional Analysis

Dimensional analysis can help check the validity of an equation. Units must be the same for values that are added or subtracted, as well as the same on both sides of an equation.

A quadratic equation in the form $ax^2 + bx + c = 0$ can be solved by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

(Equation 1.4: **The quadratic formula**)

End-of-Chapter Exercises

Exercises 1 – 10 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. You can convert back and forth between miles and kilometers using the approximation that 1 mile is approximately 1.6 km. (a) Which is a greater distance, 1 mile or 1 km? (b) How many miles are in 32 km? (c) How many kilometers are in 50 miles?

2. (a) How many significant figures are in the number 0.040 kg? (b) How many grams are in 0.040 kg?

3. You have two numbers, 248.0 cm and 8 cm. Rounding off correctly, according to the rules of significant figures, what is the (a) sum, and (b) product of these two numbers?

4. Figure 1.14 shows an 8-15-17 right-angled triangle. For the angle labeled θ in the triangle, express (as a ratio of integers) the angle's (a) sine, (b) cosine, and (c) tangent.

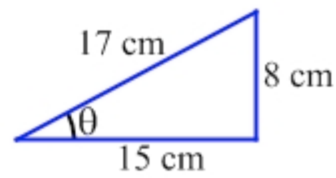


Figure 1.14: An 8-15-17 triangle, for Exercise 4.

5. You are adding two vectors by breaking them up into components. Your friend is adding the same two vectors, but is using a coordinate system that is rotated by 40° with respect to yours. Assuming you both follow the component method correctly, which of the following do the two of you agree on and which do you disagree about? (a) The magnitude of the resultant vector. (b) The direction of the resultant vector. (c) The *x*-component of the resultant vector. (d) The *y*-component of the resultant vector. (e) The angle between the *x*-axis and the resultant vector.

6. Three vectors are shown in Figure 1.15, along with an *x*-*y* coordinate system. Find the *x* and *y* components of (a) $\vec{A}$, (b) $\vec{B}$, and (c) $\vec{C}$.

7. The vectors $\vec{A}$ and $\vec{B}$ are specified in Figure 1.15. What is the magnitude and direction of the vector $\vec{A} + \vec{B}$?

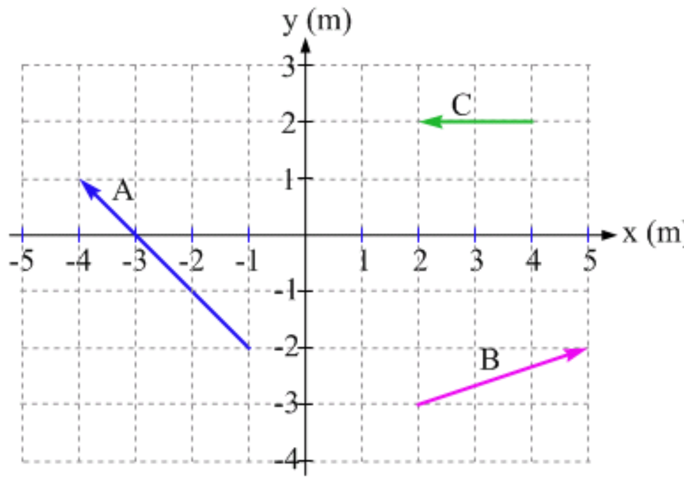


Figure 1.15: The vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, for Exercises 6 and 7.

8. You have two vectors, one with a length of length 4 m and the other with a length of 7 m. Each can be oriented in any direction you wish. If you add these two vectors together what is the (a) largest-magnitude resultant vector you can obtain? (b) smallest-magnitude resultant vector you can obtain?

9. You have three vectors, with lengths of 4 m, 7 m, and 9 m, respectively. If you add these three vectors together, what is the (a) largest-magnitude resultant vector you can obtain? (b) smallest-magnitude resultant vector you can obtain?

10. You have three vectors, with lengths of 4 m, 7 m, and 15 m, respectively. If you add these three vectors together what is the (a) largest-magnitude resultant vector you can obtain? (b) smallest-magnitude resultant vector you can obtain?

Exercises 11 – 17 deal with unit conversions.

11. Using the conversion factors you find in some reference (such as on the Internet), convert the following to SI units. In other words, express the following in terms of meters, kilograms, and/or seconds. (a) 1.00 years (b) 1.00 light-years (c) 8.0 furlongs (d) 12 slugs (e) 26 miles, 385 yards (the length of a marathon).

English Unit	Metric Unit
1 inch	2.54 cm
1 foot	______ cm
1 foot	______ m
______ feet	1 m
1 mile	______ km
______miles	1 km

Table 1.3: A table of conversion factors for length units.

12. Using the fact that 1 inch is precisely 2.54 cm, fill in Table 1.3 to create your own table of conversion factors for various length units.

13. If someone were to give you a 50-carat diamond, what would be its mass in grams?

14. Fill in Table 1.4 to create your own table of conversion factors for various mass units.

English Unit	Metric Unit
1 ounce	28.35 g
1 lb.	______ g
1 lb.	______ kg
______ lbs.	1 kg
1 stone	______ kg
______stones	1 kg

Table 1.4: A table of conversion factors for mass units.

15. (a) Which is larger, 1 acre or 1 hectare? (b) If you own a plot of land that has an area of exactly 1 hectare and it is square, what is the length of one of its sides, in meters? (c) If your 1-hectare lot is rectangular with a width of 20 m, how long is it?

16. What is your height in (a) inches? (b) cm? What is your mass in (c) pounds? (d) kg?

17. Firefighters using a fire hose can spray about 1.0 x 10^2 gallons of water per minute on a fire. What is this in liters per second? Assume the firefighters are in the USA. (Why is this assumption necessary?)

Exercises 18 – 26 deal with various aspects of vectors and vector components.

18. In Exploration 1.4, we expressed the vector $\vec{A}$ in terms of its components. Assuming the magnitude of each component is known to three significant figures, express $\vec{A}$ in terms of its magnitude and direction.

19. Using the result of Exercise 18 to help you, and aided by Figure 1.7, use the Sine Law and/or the Cosine Law to determine the magnitude and direction of the vector $\vec{C}$ shown in Figure 1.7. The vectors $\vec{A}$ and $\vec{B}$ are defined in Exploration 1.4. Hint: you may find it helpful to use geometry to first determine the angle between the vectors $\vec{A}$ and $\vec{B}$. Show your work.

20. Two vectors $\vec{Q}$ and $\vec{R}$ can be expressed in unit-vector notation as follows: $\vec{Q} = (3.0\text{ m})\hat{x} + (4.0\text{ m})\hat{y}$ and $\vec{R} = (5.0\text{ m})\hat{x} - (7.0\text{ m})\hat{y}$. Express the following in unit-vector notation: (a) $6\vec{Q}$ (b) $6\vec{Q} - 4\vec{R}$ (c) $4\vec{R} - 6\vec{Q}$.

21. Repeat Exercise 20, but express your answers in magnitude-direction format instead.

22. See Exercise 20 for the definitions of the vectors $\vec{Q}$ and $\vec{R}$. (a) Is it possible to solve for the number *a* in the equation $\vec{Q}+a\vec{R}=0$? If it is possible, then solve for *a*; if not, explain why not. (b) How many different values of *b* are there such that the sum $\vec{Q}+b\vec{R}$ has only an *x*-component? Find all such values of *b*.

23. Three vectors are shown in Figure 1.16, along with an *x*-*y* coordinate system. Use magnitude-direction format to specify vector (a) $\vec{A}$ (b) $\vec{B}$ (c) $\vec{C}$.

24. The vectors $\vec{A}$ and $\vec{C}$ are shown in Figure 1.16. Consider the following vectors: 1. $\vec{A}$; 2. $\vec{C}$; 3. $\vec{A}+\vec{C}$; 4. $\vec{A}-\vec{C}$. Rank those four vectors by their magnitude, from largest to smallest. Use notation such as 3 > 2 = 4 > 1.

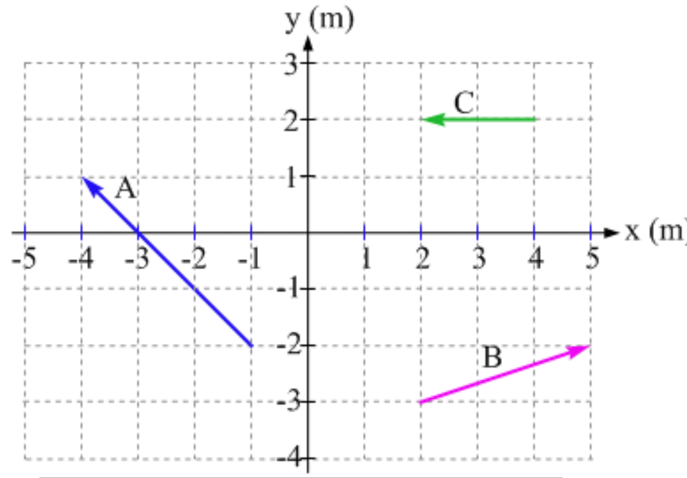


Figure 1.16: The vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, for Exercises 23 – 31.

25. Three vectors are shown in Figure 1.16. (a) Use the geometric method of vector addition (add vectors tip-to-tail) to draw the vector representing $\vec{B}+\vec{C}$. (b) Specify that resultant vector in unit-vector notation.

26. Three vectors are shown in Figure 1.16. (a) Use the geometric method of vector addition (add vectors tip-to-tail) to draw the vector representing $\vec{A}+\vec{B}+\vec{C}$. (b) Specify that resultant vector in unit-vector notation. (c) Specify that resultant vector in magnitude-direction notation.

Exercises 27 – 31 are designed to give you practice in applying the component method of vector addition. For each exercise, start with the following: (a) Draw a diagram showing how the vectors add geometrically (the tip-to-tail method). (b) Show the coordinate system (we'll use the standard coordinate system shown in Figure 1.16 above). (c) Make a table showing the *x* and *y* components of each vector you're adding together. (d) In the last row of the table, find the components of the resultant vector.

In addition to the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ shown in Figure 1.16, let's make use of these two vectors: the vector $\vec{D}=(-6.00\text{ m})\hat{x}+(-2.00\text{ m})\hat{y}$, and the vector $\vec{E}$ with a magnitude of 5.00 m at an angle of 30° above the positive *x*-axis.

27. Find the resultant vector representing $\vec{A}+\vec{B}+\vec{C}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in magnitude-direction notation.

28. Find the resultant vector representing $\vec{A}+\vec{D}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in unit-vector notation.

29. Find the resultant vector representing $\vec{C} - \vec{E}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in magnitude-direction format.

30. Find the resultant vector representing $\vec{B} + \vec{D}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in unit-vector notation. (f) State the resultant vector in magnitude-direction format.

31. Repeat Exercise 30, but now use an *x-y* coordinate system that is rotated so the positive *x*-direction is the direction of the vector $\vec{B}$.

Exercises 32 – 36 involve applications of the physics concepts addressed in this chapter.

32. In 1999, NASA had a high-profile failure when it lost contact with the Mars Climate Orbiter as it was trying to put the spacecraft into orbit around Mars. Do some research and write a paragraph or two about what was responsible for this failure, and how much the project cost.

33. In 1983, an Air Canada Boeing 767 airplane was nicknamed "The Gimli Glider." Discuss the events that led to the plane getting this nickname, and how they relate to the topics in this chapter.

34. One way to travel from Salt Lake City, Utah, to Billings, Montana, is to first drive 660 km north on interstate 15 to Butte, Montana, and then drive 370 km east to Billings on interstate 90. If you did this trip by plane, instead, traveling in a straight line between Salt Lake City and Billings, how far would you travel?

35. In the sport of orienteering, participants must plan carefully to get from one checkpoint to another in the shortest possible time. In one case, starting at a particular checkpoint, Sam decides to take a path that goes west for 600 meters, and then go northeast for 400 meters on another path to reach the next checkpoint. Between the same two checkpoints, Mary decides to take the shortest distance between the two checkpoints, traveling off the paths through the woods instead. What is the distance Mary travels along her route, and in what direction does she travel between the checkpoints?

36. You throw a ball almost straight up, with an initial speed of 10 m/s, from the top of a 20-meter-high cliff. The approximate time it takes the ball to reach the base of the cliff can be found by solving the quadratic equation $-20 \text{ m} = (10 \text{ m/s})t - (5.0 \text{ m/s}^2)t^2$. Solve the equation to find the approximate time the ball takes to reach the base of the cliff.

General problems and conceptual questions

37. Prior to the 2004 Boston Marathon, the Boston Globe newspaper carried a story about a running shoe called the Nike Mayfly. According to the newspaper, the shoes were designed to last for 62.5 miles. Does anything strike you as odd about this distance? If so, what?

38. Do the following calculations make any sense? Why or why not? If any make sense, what could they represent?

(a) $a = 17\frac{\text{m}}{\text{s}} \times 3.2\text{ s}$ (b) $b = 17\frac{\text{m}}{\text{s}} + 3.2\text{ s}$ (c) $c = \frac{751\text{ m}}{\cos\left(23^{\circ}\right)}$ (d) $d = (751\text{ m}) - \cos\left(23^{\circ}\right)$.

39. The distance from Dar es Salaam, Tanzania to Nairobi, Kenya, is 677 km, while it is 1091 km from Dar es Salaam to Kampala, Uganda. (a) Using these numbers alone, can you determine the distance between Nairobi and Kampala? Briefly justify your answer. (b) Using only these numbers, what is the minimum possible distance between Nairobi and Kampala? (c) What is the maximum possible distance?

40. Use the information given in Exercise 39, combined with the fact that it is 503 km from Nairobi to Kampala, to construct a triangle with the three cities at the vertices. What is the angle between the two lines that meet at (a) Dar es Salaam? (b) Nairobi? (c) Kampala?

41. Figure 1.17 shows overhead views of similar sections of two different cities where the blocks are marked out in a square grid pattern. In City A, the streets run north-south and east-west, while in City B, the streets are at some angle with respect to those in City A. In City A, Anya intends to walk from the lower left corner (marked by a blue dot) to the upper right corner (marked by a yellow dot). In City B, Boris will walk a similar route between the colored dots, ending up due north of his starting point. For both cities, use a coordinate system where positive x is east and positive y is north. (a) Assuming Anya goes along the streets, marked in black, choose a route for Anya to follow that involves her changing direction only once. Express her route in unit-vector notation. (b) How many blocks does she travel? (c) Assuming Boris goes along the streets, choose a route for Boris to follow that involves him changing direction only once. Break Boris' trip into two parts, the first ending at the corner where he changes direction and the second starting there, and express each part as a vector in unit-vector notation. (d) What do you get when you add the two vectors from part (c)?

42. Return to the situation described in Exercise 39, where for City B we used a coordinate system where positive x is east and positive y is north. Comment on the relative advantages and disadvantages of that coordinate system over one in which the coordinate axes are aligned parallel to the streets.

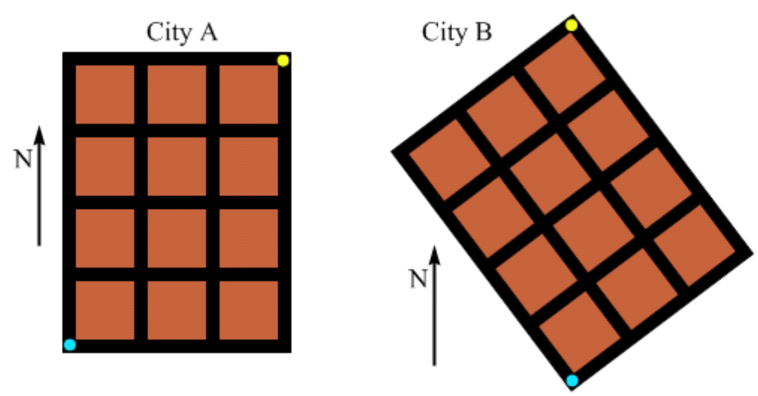


Figure 1.17: Overhead views of a 3 block by 4 block region in two different cities, for Exercises 41 and 42.

43. The top of a mountain is 2100 m north, 3300 m west, and 2300 m vertically above the initial location of a mountain climber. (a) What is the straight-line distance between the top of the mountain and the climber? (b) Later in the climb, the climber finds that she is 1200 m south, 900 m east, and 1100 m vertically below the top of the mountain. What is the minimum distance she has traveled from her starting point? (c) Checking her handheld GPS (global positioning system) receiver, she finds she has actually traveled a distance 2.5 times larger than the answer to part (b). How is this possible?

44. In Figure 1.18, four successive moves are shown near the end of a chess match. First, Black moves his Pawn (P); then, White moves her Queen (Q); then Black moves his Knight (K); and White moves her Rook (R). Using a traditional *x*-*y* coordinate system with positive *x* to the right and positive *y* up, we can express the movement of the Queen as +4 units in the *x*-direction and -4 units in the *y*-direction. Using similar notation express the movement of the (a) Pawn; (b) Knight; and (c) Rook.

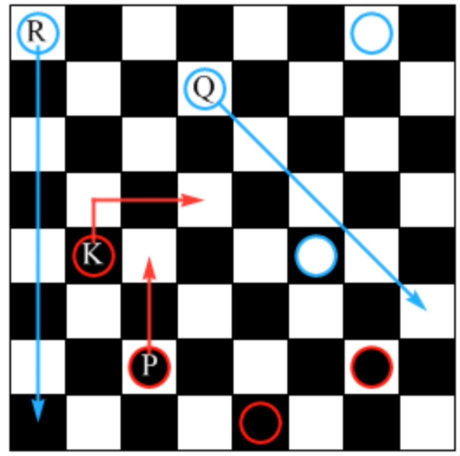


Figure 1.18: Four successive moves in a chess match.

45. Solve for *x* in the following expressions:

(a) $5x - 7 = 2x + 5$. (b) $3 = \frac{4}{x} + \frac{4}{2x}$.

46. Often, when we solve a one-dimensional motion problem, which we will spend some time doing in the next chapter, we need to solve a quadratic equation to find the time when some event happens. For instance, solving for the time it takes a police officer to catch a speeding motorist could involve solving an equation of the form:

$$\left(2\text{ m/s}^2\right)t^2 - \left(10\text{ m/s}\right)t - 100\text{ m} = 0,$$

where the *m* stands for meters and *s* stands for seconds. (a) What are the two possible solutions for *t*, the time when the police officer catches up to the motorist? (b) Which solution is the one we want to keep as the solution to the problem?

47. In the optics section of this book, we will use an equation to relate the position of an image formed by a lens (known as the image distance, d_i) to the position of the object with respect to the lens (the object distance, d_o) and the focal length, *f*, of the lens. (a) Use dimensional analysis to determine which of the following three equations could correctly relate those three lengths.

Equation 1: $d_i = \frac{d_o - f}{d_o f}$; Equation 2: $d_i = \frac{d_o f}{d_o - f}$; Equation 3: $d_i = \frac{d_o f - f}{d_o}$.

If you think any of the above are dimensionally incorrect, explain why. (b) If you found one or more of the three equations to be dimensionally correct, does this guarantee that the equation is the correct way to relate these three lengths? Explain.

48. Three students are having a conversation. Comment on each of their statements.

Ruben: The question says, what's the magnitude of the resultant vector obtained by adding a vector of length 3 units to a vector of length 4 units. That's just 7 units, right?

Marta: I think it is 5 units, because you can make a 3-4-5 triangle.

Kaitlyn: It depends on what direction the vectors are in. If the 3 and the 4 are in the same direction, you get 7 when you add them. If you have the 3 and the 4 perpendicular to one another, you get the 3-4-5 triangle. I think you can get a resultant of anything between 5 units and 7 units, depending on the angle between the 3 and the 4.