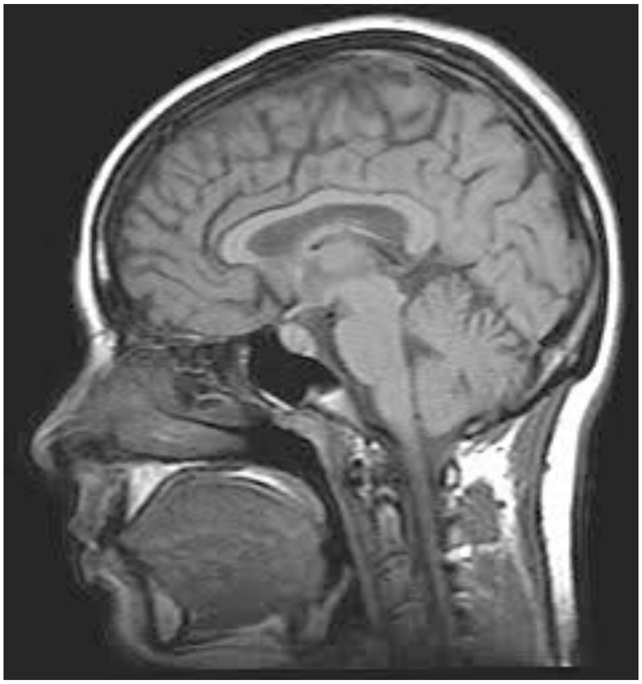

This image shows shows structures in a particular plane inside a person's head, obtained with a magnetic resonance imaging system. Such images are commonly used in hospitals for diagnostic purposes.

Image credit: public-domain image.

Chapter 29 – The Nucleus

CHAPTER CONTENTS

To understand chemical reactions, we can generally neglect the nucleus and focus on the electrons in the atom. To understand things like where the Sun's energy comes from, the production of nuclear power, and what radioactivity is all about, we need to go farther down into the very heart of the atom, into the nucleus.

In this final chapter of the book, we will investigate puzzles such as what holds a nucleus together; what does the most famous equation in physics, $E = mc^2$, have to do with the nucleus; and, how can we use signals from nuclei in our bodies to obtain an MRI, as in the picture above.

Finally, note that, although this chapter marks the end of the book, there is a vast universe out there, and we have only just scratched the surface of all there is to know about how the universe works. This is an exciting time to be alive, because there are plenty of interesting physics questions still to be solved. What is dark energy? Is string theory a good model of how the universe is constructed? Will anyone invent a material that is superconducting at room temperature? What did the early universe look like? How can physics be applied most effectively to treat cancer? It is also an interesting time to be alive because there are a number of important experiments coming on-line that will allow us to probe many of these questions more deeply than ever. With the base of physics knowledge and understanding you have built up now, you should be well-prepared to keep reading about new developments in physics as they come about during your lifetime.

29-1 What Holds the Nucleus Together?

Before answering the question of what keeps the nucleus together, let's go over the basics of what a nucleus is. A nucleus is at the heart of the atom. It is positively charged, because it contains the atom's protons. It also contains the neutrons, which have no net charge. We actually have a word, **nucleons**, for the particles in a nucleus. The atomic mass number, A, for a nucleus is the total number of nucleons (protons and neutrons) in a nucleus. The atomic number, Z, for a nucleus is the number of protons it contains.

The protons and neutrons can be modeled as tiny balls that are packed together into a spherical shape. The radius of this sphere, which represents the nucleus, is approximately:

$$r \approx (1.2\times10^{-15}\text{ m})A^{1/3} .$$ (Eq. 29.1: **The radius of a nucleus**)

This is almost unbelievably small, orders of magnitude smaller than the radius of the atom.

An atom of a particular element contains a certain number of protons. For instance, in the nucleus of a carbon atom there are always 6 protons. If the number of protons changes, we get a new element. There can be different numbers of neutrons in the nucleus, however. Carbon atoms, for instance, are stable if they have six neutrons (by far the most abundant form of natural carbon), seven neutrons, or eight neutrons. These are known as **isotopes** of carbon – they all have six protons, but a different number of neutrons.

The general notation for specifying a particular isotope of an element is shown in Figure 29.1. The notation is somewhat redundant – by definition, aluminum is the 13$^{\text{th}}$ element in the periodic table, and thus has an atomic number of $Z = 13$. A little redundancy is not a bad idea, however – it will help us keep things straight later when we write equations for radioactive decay processes and for nuclear reactions.

Figure 29.1: The general notation for specifying a particular isotope is shown at left, where X represents the chemical symbol for the element, A is the atomic mass number (the number of protons + neutrons) and Z is the atomic number (the number of protons). A specific case, the isotope aluminum-13. is shown at right.

$${}^{A}_{Z}\mathrm{X} \qquad\qquad {}^{27}_{13}\mathrm{Al}$$

The masses and charges of the three basic building blocks of atoms are shown below in Table 29.1. In addition to specifying the mass in kilograms, the mass is also shown in atomic mass units (u). By definition, 1 atomic mass unit is 1/12$^{\text{th}}$ the mass of a neutral carbon-12 atom.

$1\text{ u} = 1.660540\times10^{-27}\text{ kg}$.

Particle	Charge	Mass (kg)	Mass (u)
neutron	0	1.674929×10^{-27} kg	1.008664
proton	$+e$	1.672623×10^{-27} kg	1.007276
electron	$-e$	9.109390×10^{-31} kg	0.00054858

Table 29.1: The charge and mass of the neutron, proton, and electron. Recall that $e = 1.602\times10^{-19}$ C.

Related End-of-Chapter Exercises: 1, 41, 42.

EXPLORATION 29.1 – Holding the nucleus together

Let's explore what it is that holds a nucleus together, starting by considering whether what holds the nucleus together is a force we already know about.

Step 1 – ***First, apply Coulomb's Law to calculate the electrostatic force between two protons that are separated by 1×10^{-15} m. Can this force hold the nucleus together?*** Substituting the appropriate values into Coulomb's Law gives:

$$F = \frac{k(+e)(+e)}{r^2} = \frac{(9.0\times10^{9})(1.602\times10^{-19}\text{ C})^2}{(1.0\times10^{-15}\text{ m})^2} = 231\text{ N}\,.$$

This is a repulsive force, so it certainly cannot be the force holding the nucleus together – the electrostatic repulsion between the protons is trying to spread the protons apart.

Step 2 – ***Let's use Newton's Law of Universal Gravitation to find the gravitational force between the two protons. Is the gravitational force sufficient to keep the nucleus together?*** Substituting the appropriate values into the law of gravitation gives:

$$F = \frac{Gmm}{r^2} = \frac{(6.67\times10^{-11}\text{ N m}^2/\text{kg}^2)(1.6726\times10^{-27}\text{ kg})^2}{(1.0\times10^{-15}\text{ m})^2} = 1.87\times10^{-34}\text{ N}\,.$$

This gravitational force is an attractive force, but at about 36 orders of magnitude less than the electrostatic force, it is negligible compared to the electrostatic force.

Step 3 – ***Are any of the other forces we have encountered already in this book responsible for holding the nucleus together?*** Other forces we have dealt with include forces of tension and friction, and normal forces. These forces are macroscopic manifestations of forces between charges, however, and they do not apply at the microscopic level of the nucleus. Another force we looked at is the magnetic force applied on charged objects but, as we discussed in Chapter 26, magnetism can actually be interpreted as another manifestation of the electrostatic force.

Step 4 – ***What do you conclude about the force responsible for holding the nucleus together?*** Our conclusion must be that there is a force we have not yet discussed that is responsible for holding the nucleus together. The basics of this force are given in the box below.

The force that holds a nucleus together is the **nuclear force**, a short-range force between nucleons. At very small separations, the nuclear force is repulsive, keeping the protons and neutrons from getting too close to one another. In general, however, protons and neutrons exert attractive forces on other protons and neutrons via the nuclear force, up to a separation distance of about 1.3×10^{-15} m (at larger separations the nuclear force is negligible). To keep a nucleus together, the net attraction from all the nuclear forces between nucleons must balance the net repulsion from all the protons mutually repelling one another.

Key idea: Nuclei are held together, against the electrostatic repulsion trying to tear them apart, by the nuclear force, a (generally) attractive, but short-range, force between nucleons (neutrons and protons). **Related End-of-Chapter Exercise: 16.**

Essential Question 29.1: Your friend says that two isotopes of carbon are specified by ${}^{12}_{6}\text{C}$ and ${}^{14}_{8}\text{C}$. Do you agree with your friend? Explain why or why not.

Answer to Essential Question 29.1: The isotope ${}^{12}_{6}\mathrm{C}$ is certainly correct – it is the isotope of carbon that makes up 98.9% of naturally occurring carbon on Earth. On the other hand, ${}^{14}_{8}\mathrm{C}$ can not be correct. The lower number represents the number of protons in the nucleus, and carbon atoms all have 6 protons. The correct notation for carbon-14 is ${}^{14}_{6}\mathrm{C}$.

29-2 E = mc²

As we discussed in Section 29-1, the atomic mass unit is defined as 1/12th the mass of a neutral carbon-12 atom. Table 29.1, copied from Section 29-1, shows the mass of a neutron, proton, and electron in both kilograms and atomic mass units. The data seems to be contradictory. We will explore, and resolve, that apparent contradiction in Exploration 29.2.

Table 29.1: The charge and mass of the neutron, proton, and electron.

Particle	Charge	Mass (kg)	Mass (u)
neutron	0	1.674929×10^{-27} kg	1.008664
proton	+e	1.672623×10^{-27} kg	1.007276
electron	–e	9.109390×10^{-31} kg	0.00054858

EXPLORATION 29.2 – Nuclear binding energy

Step 1 – ***How many neutrons, protons, and electrons are in a neutral carbon-12 atom?*** A neutral carbon-12 atom has six neutrons, six protons, and six electrons.

Step 2 – ***Find the total mass of the individual constituents of a neutral carbon-12 atom. Compare this total mass to the mass of a neutral carbon-12 atom.***
The total mass of six neutrons, six protons, and six electrons is:

$$\begin{array}{rl} & 6\times1.008664 \text{ u} \\ + & 6\times1.007276 \text{ u} \\ + & 6\times0.00054858 \text{ u} \\ \hline = & 12.098931 \text{ u} \end{array} .$$

The total mass of a carbon-12 atom is, by definition, exactly 12 u. This seems to be a contradiction – how can the masses of the individual constituents of the atom add up to more than the mass of the atom itself?

To resolve the contradiction, we apply the most famous equation in physics, $E = mc^2$. The idea is that the missing mass, amounting to 0.098931 u, is converted to energy, in the form of the binding energy that holds the atom together.

The difference in mass between the individual constituents of an atom and the atom itself is known as the **mass defect**. The mass that is "missing" from the atom is the atom's binding energy (almost of this binding energy is associated with the nucleus, rather than the electrons). The conversion from mass to energy is done using Einstein's famous equation:

$E = mc^2$,   (Equation 29.2: **The equivalence of mass and energy**)

where $c = 2.998 \times 10^8$ m/s is the speed of light in vacuum.

Step 3 – ***Use Equation 29.2 to determine the energy, in MeV (mega electron volts) associated with a mass of 1 u.*** Let's first convert 1 u, expressed in units of kilograms, to joules.

$$E = mc^2 = (1.660539\times10^{-27}\ \text{kg})(2.998\times10^{8}\ \text{m/s})^2 = 1.492493\times10^{-10}\ \text{J}.$$

We can now convert to MeV using the conversion factor 1 MeV = 1.602176×10^{-13} J.

$$1.492493\times10^{-10}\ \text{J}\times\frac{1\ \text{MeV}}{1.602176\times10^{-13}\ \text{J}} = 931.5\ \text{MeV}.$$

Step 4 – ***Use the conversion factor derived in step 3 to find the binding energy, in MeV, of a carbon-12 atom, as well as the binding energy per nucleon.*** The conversion factor we just derived is that 1 u is equivalent to 931.5 MeV of energy. Using this conversion factor to convert our value for the mass defect of carbon-12 from part (b), 0.098931 u, we get

$$0.098931\ \text{u}\times\frac{931.5\ \text{MeV}}{1\ \text{u}} = 92.15\ \text{MeV}.$$

To more easily compare different atoms, the binding energy for an atom is often expressed in terms of the binding energy per nucleon. Carbon-12 has 12 nucleons, so dividing the 92.15 MeV binding energy by 12 nucleons gives 7.68 MeV per nucleon.

Key idea: The binding energy for an atom is its mass defect (the difference between the total mass of the atom's neutrons, protons, and electrons and the mass of the atom), converted to energy. Electron binding energies are measured in electron volts, whereas atomic binding energies are measured in millions of electron volts – almost all the binding energy for an atom is associated with the energy holding the nucleus together.
Related End-of-Chapter Exercises: 2, 3, 13 – 15, 17, 18, 43.

Comparing chemical energy to nuclear energy

Much of the energy we use on a daily basis comes from chemical energy, such as through the burning of fossil fuels. Burning a gallon of gasoline in a car engine, for instance, typically produces about 1.3×10^{8} J of energy. However, the energy obtained comes from changes in bonds between atoms, and this is associated with the electron energy levels. Breaking and forming bonds (generally associated with carbon atoms, when we're talking about fossil fuels) typically frees up a few electron volts at a time.

The relatively modest energy output, per unit mass, available from chemical energy contrasts with the much larger energy output, per unit mass, available from nuclear energy sources. Converting mass directly to energy gives enormous amounts of energy, but it is not feasible to convert all mass to energy. It is possible to convert a fraction of the mass associated with a nucleus into energy - as we will discuss in more detail later in this chapter, this can be done through the processes of nuclear fusion or nuclear fission (nuclear power plants make use of nuclear fission). Changes in the nucleus through these processes generally produce millions of electron volts per nucleus, several orders of magnitude more than is obtained from sources of chemical energy. Tapping into the vast sources of energy associated with the nucleus is why nuclear power is so attractive.

Essential Question 29.2: The power output of the Sun is about 4 x 10^{23} J/s. This energy comes from converting mass into energy. How much mass is the Sun losing every second?

Answer to Essential Question 29.2: Solving for the mass converted to energy within the Sun every second gives $m = E/c^2 = (4\times10^{23}\ \text{J})/(9\times10^{16}\ \text{m}^2/\text{s}^2) \approx 4\times10^{6}\ \text{kg}$. This is a huge mass, but it represents a tiny fraction of the Sun's mass of 2.0×10^{30} kg.

29-3 Radioactive Decay Processes

In general, there are three types of radioactive decay processes, named after the first three letters of the Greek alphabet, alpha, beta, and gamma. In the alpha and beta decay processes, a nucleus emits a particle, or a collection of particles, turning into a nucleus of a different element. The gamma decay process is more analogous to what happens in an atom when an electron drops from a higher energy level to a lower energy level, emitting a photon. Gamma decay occurs when a nucleus makes a transition from a higher energy level to a lower energy level, emitting a photon in the process. Because nuclear energy levels are generally orders of magnitude farther apart than are electron energy levels, however, the photon released in a gamma decay process is very high energy, and falls in the gamma ray region of the electromagnetic spectrum.

Radioactive decays can happen spontaneously when the products resulting from the decay process are more stable than the original atom or nucleus. In any kind of radioactive decay process, a number of conservation laws are satisfied, as explained in the box below.

All nuclear reactions and decays satisfy a few different conservation laws. First of all, the process can generally be viewed as a super-elastic collision, and thus linear momentum is conserved. Kinetic energy is generally not conserved, but any excess or missing kinetic energy can be explained in terms of a conversion of mass into kinetic energy. Charge must also be conserved in a reaction or a decay. In addition to the preceding guidelines, the number of nucleons (the number of protons plus neutrons) must also be conserved, a law known as conservation of nucleon number.

Alpha decay

An alpha particle is a helium nucleus, two protons and two neutrons, which is particularly stable. Heavy nuclei can often become more stable by emitting an alpha particle – this process is known as **alpha decay**. Equation 29.3 describes alpha decay, in which a nucleus with a generic chemical symbol X_1, with atomic mass number *A* and atomic number *Z*, transforms into a second nucleus, X_2, with an atomic number of *A*–4 and atomic number *Z*–2. The number of neutrons, protons, and electrons (assuming all three atoms are neutral) is the same on both sides of the equation.

$${}^{A}_{Z}X_1 \Rightarrow {}^{A-4}_{Z-2}X_2 + {}^{4}_{2}\text{He}.$$ (Equation 29.3: **General equation for alpha decay**)

A particular example of alpha decay is the transformation of uranium-238 into thorium-234.

$${}^{238}_{92}\text{U} \Rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}.$$ (Equation 29.4: **An alpha decay example**)

Table 29.3 in Section 29-8 gives the masses of a number of isotopes. The atomic masses of uranium-238, thorium-234, and helium-4 are 238.050786 u, 234.043596 u, and 4.002603 u, respectively. The total mass on the right side of Equation 29.4 is 238.046199 u, which is lower in mass, by 0.004587 u, than the mass of the uranium-238. How do we explain this mass difference?

The missing mass is converted to kinetic energy, which is shared by the two atoms after the decay. Using our conversion factor of 931 MeV/u, 0.004587 u corresponds to 4.273 MeV of kinetic energy, most of which is carried away by the helium nucleus after the decay.

Beta-minus decay

There are two kinds of beta decay, beta-plus and beta-minus. A beta-minus particle is familiar to us – it is an electron – so let's examine beta-minus decay first. The general equation for beta-minus decay, which takes the nucleus one step up the periodic table, is

$${}^{A}_{Z}X_1 \Rightarrow {}_{Z+1}^{\;\;A}X_2^{+} + {}^{\;\,0}_{-1}\mathrm{e}^{-} + \overline{\nu}_e . \qquad \text{(Eq. 29.5: \textbf{General equation for beta-minus decay})}$$

The beta-minus decay process can be viewed as one of the neutrons in the nucleus decaying into a proton and an electron (the electron is symbolized by ${}^{\;\,0}_{-1}\mathrm{e}^{-}$). The last term on the right-hand side of Equation 29.5 represents an electron anti-neutrino. In the early 20th-century, analysis of beta-minus decay processes seemed to indicate a violation of energy conservation and of momentum conservation. In 1930, Wolfgang Pauli proposed that the missing energy and momentum was being carried away by a particle that was very hard to detect, which Enrico Fermi called the **neutrino** (little neutral one). Pauli was proven correct, and we now know that the Sun emits plenty of neutrinos, which interact so rarely that the majority of neutrinos incident on the Earth pass right through without interacting at all! Note that, when comparing the masses on the two sides of the decay, you can neglect the mass of the anti-neutrino, and looking up the mass of the neutral version of the nucleus on the right accounts for the electron, because the atom on the right is positively charged.

An example of beta-minus decay is the decay of thorium-234 into protactinium-234.

$${}^{234}_{\;90}\mathrm{Th} \Rightarrow {}^{234}_{\;91}\mathrm{Pa}^{+} + {}^{\;\,0}_{-1}\mathrm{e}^{-} + \overline{\nu}_e . \qquad \text{(Eq. 29.6: \textbf{A specific example of beta-minus decay})}$$

Beta-plus decay

A beta-plus particle is a positron, which is the antimatter version of the electron. It has the same mass and the same magnitude charge as the electron, but the sign of its charge is positive. A beta-plus decay takes the nucleus one step down the periodic table.

$${}^{A}_{Z}X_1 \Rightarrow {}_{Z-1}^{\;\;A}X_2^{-} + {}^{\;\,0}_{+1}\mathrm{e}^{+} + \nu_e . \qquad \text{(Eq. 29.7: \textbf{General equation for beta-plus decay})}$$

The neutrino in this case is an electron neutrino (there are two other kinds of neutrino, each with an antimatter version). In this case, when comparing the masses on the two sides of the decay, you can neglect the mass of the neutrino. In addition to the mass of the neutral version of the nucleus on the right, you need to add two electron masses, one for the extra electron (the atom is negatively charged) and one for the positron, which has the same mass as the electron.

An example of beta-plus decay is the decay of astatine-210 into polonium-210.

$${}^{210}_{\;85}\mathrm{As} \Rightarrow {}^{210}_{\;84}\mathrm{Po}^{-} + {}^{\;\,0}_{+1}\mathrm{e}^{+} + \nu_e . \qquad \text{(Eq. 29.8: \textbf{A specific example of beta-plus decay})}$$

Gamma decay

In gamma decay, the atom does not turn into anything different, as the nucleus simply decays from a higher-energy state to a lower-energy state. Using an asterisk to denote the higher state, the general equation for a gamma decay is

$${}^{A}_{Z}X_1^{*} \Rightarrow {}^{A}_{Z}X_1 + \gamma . \qquad \text{(Equation 29.9: \textbf{General equation for gamma decay})}$$

Related End-of-Chapter Exercises: 4 – 6, 23, 45.

Essential Question 29.3: If a carbon-13 atom (${}^{13}_{\;6}\mathrm{C}$) experienced alpha decay, what would it decay into? Use the atomic mass data in Table 29.3 in Section 29-8 to help you explain why carbon-13 will not spontaneously undergo alpha decay.

Answer to Essential Question 29.3: If carbon-13 experienced alpha decay, the process would be written ${}^{13}_{6}\mathrm{C} \Rightarrow {}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He}$. There are 13 nucleons and 6 protons on both sides of the reaction. Looking up the relevant masses in Table 29.3, we find that the total mass after the reaction is larger than the mass of the carbon-13 atom. Spontaneous reactions occur when the total mass of the decay products is less than the mass of the initial atom – in that case, the missing mass shows up as the kinetic energy of the decay products. Thus, carbon-13 will not spontaneously exhibit alpha decay, because it does not make sense from the perspective of energy conservation.

EXPLORATION 29.3A – Beta-minus bookkeeping

Let's analyze the beta-minus process given above in some detail, so we can make sense of the statement about not having to add the mass of an electron when we do the bookkeeping necessary to determine the mass that is converted to kinetic energy. Here's the beta-minus process we are considering, the decay of thorium-234 into protactinium-234.

$${}^{234}_{90}\mathrm{Th} \Rightarrow {}^{234}_{91}\mathrm{Pa}^{+} + {}^{0}_{-1}\mathrm{e}^{-} + \bar{\nu}_e . \quad \text{(Eq. 29.6: \textbf{A specific example of beta-minus decay})}$$

Step 1 – *How many neutrons, protons, and electrons are in the neutral thorium-234 atom?*
A neutral thorium-234 atom has 144 neutrons, 90 protons, and 90 electrons.

Step 2 – *Given that, in beta-minus decay, a neutron turns into a proton, an electron, and an anti-neutrino, how many neutrons, protons, and electrons should we expect to have to account for after the decay?*
Afterwards, we will have lost a neutron and gained one proton and one electron, so we have to account for 143 neutrons, 91 protons, and 91 electrons.

Step 3 – *What happens to the electron that is created in the decay process? How many electrons will the protactinium atom have after the decay?*
The electron produced in the decay has enough energy to be emitted from the atom (this is why radioactive materials are dangerous, because they are emitting energetic particles), so the protactinium has the same number of electrons, 90, that the thorium started with. This is why the protactinium is labeled with a + charge, because it has 91 protons and only 90 electrons.

Step 4 – *When we look up the mass of protactinium-234 in the table, how many electrons does it include?*
The table gives the mass of the neutral version of the atom, so it accounts for 91 electrons. That turns out to be exactly the number we need to account for, 90 electrons in the positively-charged protactinium, and 1 more emitted from the atom. So, looking up the mass of the neutral form of protactinium means we have already accounted for the mass of the electron that is emitted from the nucleus. Note that we don't need to worry about the mass of the anti-neutrino. Neutrinos and anti-neutrinos have such a small mass that we have yet to be able to determine the mass accurately.

Step 5 – *How much energy is emitted in this particular decay?*
Looking up the masses in the table, we get a mass for thorium-234 of 234.043596 u, while protactinium-234 has a mass of 234.043302 u. Subtracting the protactinium mass from the thorium mass gives a missing mass of 0.000294 u. Using the conversion factor 931.5 MeV/u, to convert to energy, gives us an energy of 274 keV. Almost all this energy is carried off in the form of kinetic energy by the electron and the anti-neutrino (the protactinium has a very small fraction of the kinetic energy).

EXPLORATION 29.3B – Beta-plus bookkeeping

Now, we will do a similar analysis of a beta-plus process, to see how we do the bookkeeping necessary to determine the mass that is converted to kinetic energy in that case. The specific beta-plus decay process we are considering is the decay of astatine-210 into polonium-210.

$${}^{210}_{85}\mathrm{As} \Rightarrow {}^{210}_{84}\mathrm{Po}^{-} + {}^{0}_{+1}\mathrm{e}^{+} + \nu_e . \quad \text{(Eq. 29.8: } \textbf{A specific example of beta-plus decay}\text{)}$$

Step 1 – ***How many neutrons, protons, and electrons are in the neutral astatine-210 atom?***
A neutral astatine-210 atom has 125 neutrons, 85 protons, and 85 electrons.

Step 2 – ***Given that, in beta-plus decay, a proton turns into a neutron, a positron, and a neutrino, how many neutrons, protons, electrons, and positrons should we expect to have to account for after the decay?***
Afterwards, we will have lost a proton and gained one neutron and one positron, so we have to account for 126 neutrons, 84 protons, 85 electrons, and one positron.

Step 3 – ***How many electrons will the polonium atom have after the decay?***
The decay has no impact on the number of electrons, so the polonium has the same number of electrons, 85, that the astatine started with. This is why the polonium is labeled with a negative charge, because it has 84 protons and 85 electrons.

Step 4 – ***When we look up the mass of polonium-210 in the table, how many electrons does it include? What else do we need to account for, in addition to the mass of polonium-210?***
The table gives the mass of the neutral version of the atom, so it accounts for 84 electrons. There is one additional electron to account for, as well as the positron. The positron, being the anti-matter equivalent of the electron, has the same mass as the electron, so we also need to add 2 electron masses (one for the extra electron, and one for the positron) to correctly account for all the mass there is after the decay. Once again, the neutrino has a negligible mass.

Step 5 – ***How much energy is emitted in this particular decay?***
Looking up the masses in the table, we get a mass for astatine-210 of 209.987148 u, while polonium-210 has a mass of 209.9828737 u. Adding in two electron masses (each with a mass of 0.00054858 u) brings the total mass of the products to 209.9839709 u. Subtracting the total mass afterwards from the astatine mass gives a missing mass of 0.003177 u. Using the conversion factor 931.5 MeV/u, to convert to energy, gives us an energy of 2.96 MeV. Again, almost all this energy is carried off in the form of kinetic energy by the positron and the neutrino.

Key idea: In a beta-minus decay, looking up the mass of the neutral version of the product atom accounts for the electron, because the atom on the right is positively charged. In a beta-plus decay, calculating the mass of the products correctly requires adding two electron masses, in addition to the neutral version of the product atom, to account for one electron and the positron.

Related End-of-Chapter Exercises: 20, 21.

29-4 The Chart of the Nuclides

A nuclide is an atom that is characterized by what is in its nucleus. In other words, it is characterized by the number of protons it has, and by the number of neutrons it has. Figure 29.2 shows the chart of the nuclides, which plots, for stable and radioactive nuclides, the value of Z (the atomic number), on the vertical axis, and the value of N (the number of neutrons) on the horizontal axis.

If you look at a horizontal line through the chart, you will find nuclides that all have the same number of protons, but a different number of neutrons. These are all nuclides of the same element, and are known as isotopes (equal proton number, different neutron number).

On a vertical line through the chart, the nuclides all have the same number of neutrons, but a different number of protons. These nuclides are known as isotones (equal neutron number, different proton number).

It is interesting to think about how the various decay processes play a role in the chart of the nuclides. Only the nuclides shown in black are stable. The stable nuclides give the chart a line of stability that goes from the bottom left toward the upper right, curving down and away from the Z = N line as you move toward higher N values. There are many more nuclides that are shown on the chart but which are not stable - these nuclides decay by means of a radioactive decay process.

In general, the dominant decay process for a particular nuclide is the process that produces a nucleus closer to the line of stability than where it started. In an alpha decay process, the resulting nuclide (usually referred to as the daughter nuclide), has a Z value that is 2 less than the parent nuclide, and the N value is also 2 less than that of the parent nuclide. This is because the alpha particle takes away two protons and two neutrons. On the chart of the nuclides, therefore, an alpha decay process moves the nucleus two spaces down and two spaces left. Thus, nuclides that decay via alpha decay are found mainly at the upper right of the chart. Because the line of stability curves down as the number of nucleons increases, as well as because, above a certain number of nucleons, there are no stable nuclides, a decay process that moves a nucleus down and to the left on the chart tends to bring that nucleus closer to the line of stability.

Beta-plus decay, on the other hand, increases the neutron number by one and decreases the proton number by one. In other words, a beta-plus decay moves a nucleus down and to the right on the chart. Nuclides that decay via beta-plus decay, therefore, are generally found above and to the left of the line of stability.

The beta-minus decay is, in many ways, the opposite of beta-plus. Beta-minus decay decreases the neutron number by one and increases the proton number by one, moving a nucleus up and to the left on the chart. Nuclides that decay via beta-plus decay, therefore, are generally found below and to the right of the line of stability.

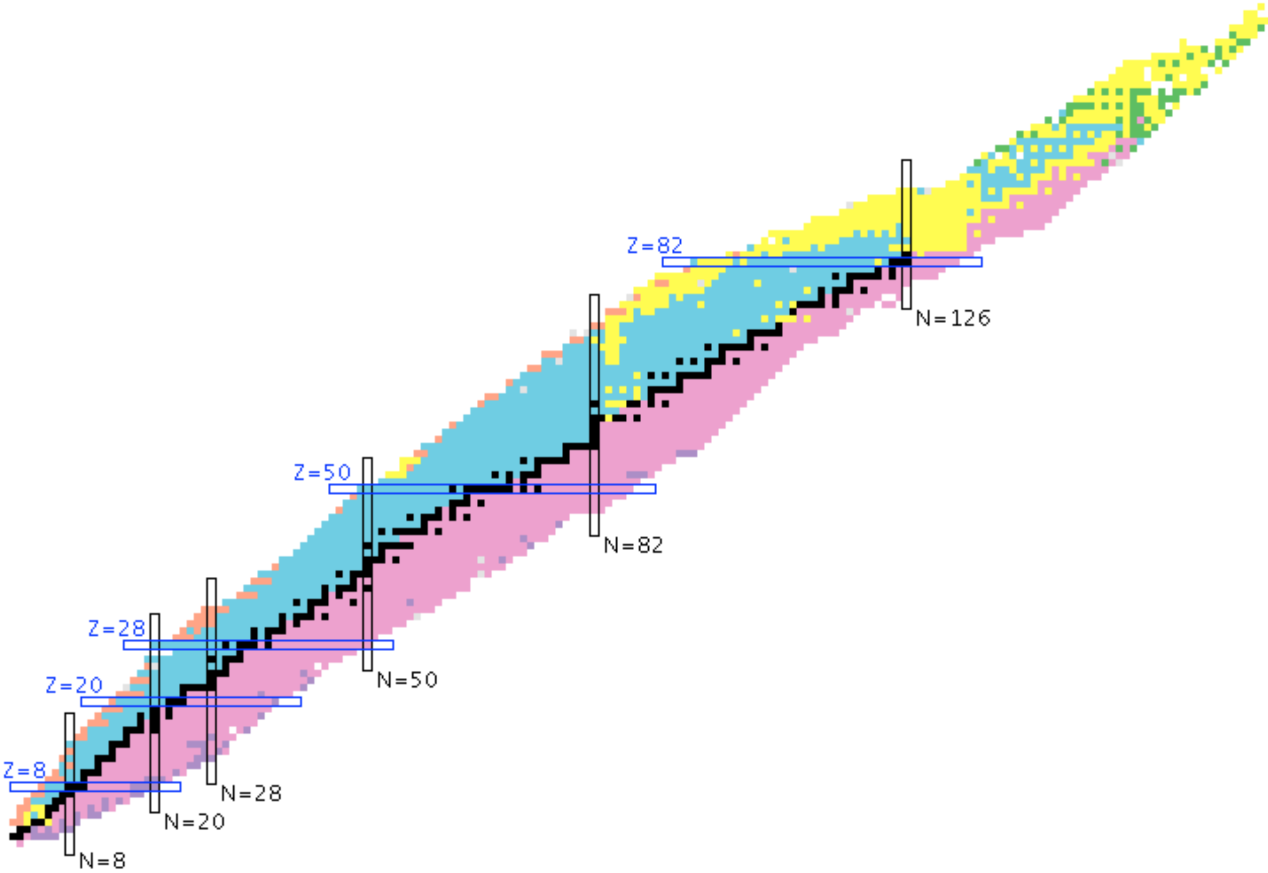

Figure 29.2: The chart of the nuclides, plotted with Z (the atomic number) on the vertical axis, and N (the number of neutrons) on the horizontal axis. Nuclides in black are stable - all other nuclides exhibit radioactive decay. The stable nuclides start out following the line Z = N (at bottom left), but then, as the number of nucleons increases, stable nuclides gradually increase the number of neutrons they have relative to their number of protons. Viewed in color, the chart is color-coded according to the nuclides primary decay mode. Above the line of stability, the chart is mainly light blue - those nuclides decay via beta-plus decay. Below the line of stability, the chart is mainly purple - those nuclides decay via beta-minus decay. At the top right of the chart, many of the nuclides are shown in yellow - those nuclides decay via the alpha decay process. In general, a radioactive decay will produce another nuclide that is closer to the line of stability than the original nuclide was. Also in general, the closer the nuclide is to the line of stability, the more stable it is, and the longer its half-life is. See Section 29-5 for an explanation of half-life. Chart of Nuclides from National Nuclear Data Center, using information extracted from the Chart of Nuclides database, http://www.nndc.bnl.gov/chart/

Related End-of-Chapter Exercises: 60 – 62.

Essential Question 29.4: Sodium-23, which has 11 protons and 12 neutrons, is stable. Sodium-22, however, is unstable. What type of radioactive decay process would you expect sodium-22 to undergo? What is the daughter nuclide that is produced in this decay?

Answer to Essential Question 29.4: Sodium-22 is to the left of sodium-23 on the chart of the nuclides, so we expect it to decay via beta-plus decay, taking the nucleus down and to the right on the chart. In fact, sodium-22 is a well-known positron emitter; it does decay via the beta-plus process. Through beta-plus decay, sodium-22, with 11 protons and 11 neutrons, produces the nuclide with 10 protons and 12 neutrons. That is neon-22, which happens to be stable.

29-5 Radioactivity

A **radioactive nucleus** is a nucleus that will spontaneously undergo radioactive decay. For an individual radioactive nucleus, it is not possible to predict precisely when the nucleus will decay. However, the statistics of radioactive decay are well understood, so for a sample of radioactive material containing a large number of nuclei, we can accurately predict the fraction of radioactive nuclei that will decay in a particular time interval.

For a given isotope, we can generally look up the **half-life**. The half-life is the time it takes for half of a large number of nuclei of this particular isotope to decay. Half-lives can vary widely from isotope to isotope. For instance, the half-life of uranium-238 is 4.5 billion years; the half-life of carbon-14 is 5730 years; and the half-life of oxygen-15 is about 2 minutes.

For a particular sample of material, containing one isotope of radioactive nuclei, the number of radioactive decays that occur in a particular time interval is related to the half-life of that isotope, but it is also proportional to the number of radioactive nuclei in the sample. The time rate of decay is given by

$$\frac{\Delta N}{\Delta t} = -\lambda N \,, \qquad \text{(Equation 29.10: \textbf{Decay rate for radioactive nuclei})}$$

where N is the current number of radioactive nuclei, and the **decay constant** λ is related to the half-life, $T_{1/2}$, by the equation

$$\lambda = \frac{\ln(2)}{T_{1/2}} = \frac{0.693}{T_{1/2}} \,. \qquad \text{(Eq. 29.11: \textbf{The connection between decay constant and half-life})}$$

A process in which the rate that a quantity decreases is proportional to that quantity is characterized by exponential decay. The exponential equation that describes the number of a particular radioactive isotope that remain after a time interval t is

$$N = N_i \, e^{-\lambda t} \,, \qquad \text{(Equation 29.12: \textbf{The exponential decay of radioactive nuclei})}$$

where N_i is a measure of the initial number of radioactive nuclei (the number at $t = 0$).

EXAMPLE 29.5 – Calculations for exponential decay

A particular isotope has a half-life of 10 minutes. At $t = 0$, a sample of material contains a large number of nuclei of this isotope. How much time passes until (a) 80% of the original nuclei remain? (b) 1/8th of the original nuclei remain? See if you can answer part (b) in your head, without using Equation 29.12.

SOLUTION

(a) Let's first determine approximately what the answer is. We know that in 10 minutes (one half-life), 50% of the nuclei would decay. To get 20% of the nuclei to decay must take less than half of a half-life, so the answer should be something like 3 or 4 minutes. In Equation 29.12, we can use a number of different measures for N and N_i, as long as we are consistent. For instance, we can measure them both in terms of mass (e.g., grams), or use the actual number of nuclei, or express them as a percentage or fraction. It makes sense to use fractions or percentage here, because the question was posed in terms of percentage. Thus, we're looking for the time when N = 80% of N_i, or when $N = 0.8\, N_i$. Note that the units on λ and t must match one another, but they can be completely different from the units on N and N_i. Applying Equation 29.12, we get:

$0.8 N_i = N_i\, e^{-\lambda t}$, which becomes $0.8 = e^{-\lambda t}$.

The inverse function of the exponential function is the natural log, so to bring down the t term we can take the natural log of both sides. Thus, $\ln(0.8) = -\lambda t$.

Bringing in the half-life via Equation 29.11, we get:

$$t = \frac{-\ln(0.8)}{\lambda} = \frac{-\ln(0.8)}{\ln(2)} T_{1/2} = \frac{-\ln(0.8)}{\ln(2)} (10 \text{ min}) = 3.2 \text{ min}.$$

(b) If the fraction of nuclei remaining can be expressed as 1 divided by a power of 2, the time taken is an integer multiple of the half life (the integer being equal to the power to which 2 is raised). Table 29.2 illustrates the idea.

Number of half-lives	Fraction of radioactive nuclei remaining
0	$1/(2^0) = 1$
1	$1/(2^1) = 1/2$
2	$1/(2^2) = 1/4$
3	$1/(2^3) = 1/8$
4	$1/(2^4) = 1/16$
5	$1/(2^5) = 1/32$

Table 29.2: The table shows the relationship between the number of half-lives that have passed and the fraction of radioactive nuclei remaining. In general, if the fraction remaining is expressed as $1/(2^x)$ (where x does not have to represent an integer), the amount of time that has passed is x multiplied by the half-life.

In our example, if 1/8th of the nuclei remain, three half-lives must have passed. Three half-lives is 30 minutes, in this case.

A graph of Equation 29.11 is shown in Figure 29.3. This figure, as it must be, is also consistent with the data in Table 29.2.

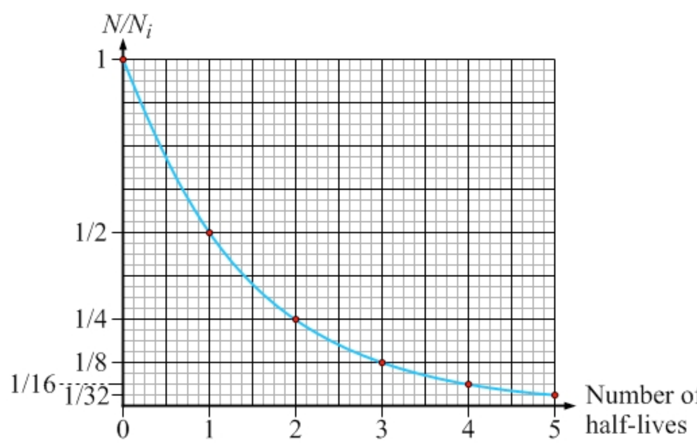

Figure 29.3: A graph of the exponential decay of radioactive nuclei, showing the fraction of nuclei remaining as a function of the number of half-lives that have elapsed.

Related End-of-Chapter Exercises: 7–9, 48–54.

Essential Question 29.5: Equation 29.10 can be re-arranged to $\Delta N = -\lambda N(\Delta t)$. This form of the equation gives a reasonable approximation of the number of decayed nuclei as long as Δt is much smaller than what?

Answer to Essential Question 29.5: It is best to use Equation 29.12 to find the number of nuclei remaining after a particular time interval. Equation 29.10 is designed to give the decay rate at a particular instant in time; however, it will give a good approximation of the number of decays that have taken place in a time interval if the time interval is much smaller than the half life.

29-6 Nuclear Fusion and Nuclear Fission

In Section 29-2, we calculated the mass defect for carbon-12, and used that information to determine the binding energy per nucleon for carbon-12. The basic process is:

- Calculate the total mass of the individual neutrons, protons, and electrons in an atom.
- Calculate the mass defect for the atom by subtracting the atomic mass from the total mass of the individual constituents.
- Convert the mass defect from mass to energy, using the conversion factor 931.5 MeV / u. This represents the atom's binding energy, which is almost all in the nucleus.
- Divide the binding energy by the number of nucleons (neutrons plus protons) to find the average binding energy per nucleon.

Following the procedure above, we obtain the graph in Figure 29.4, showing the average binding energy per nucleon for a variety of common isotopes. The most stable isotope, having the largest binding energy per nucleon, is nickel-62, followed closely by iron-58 and iron-56.

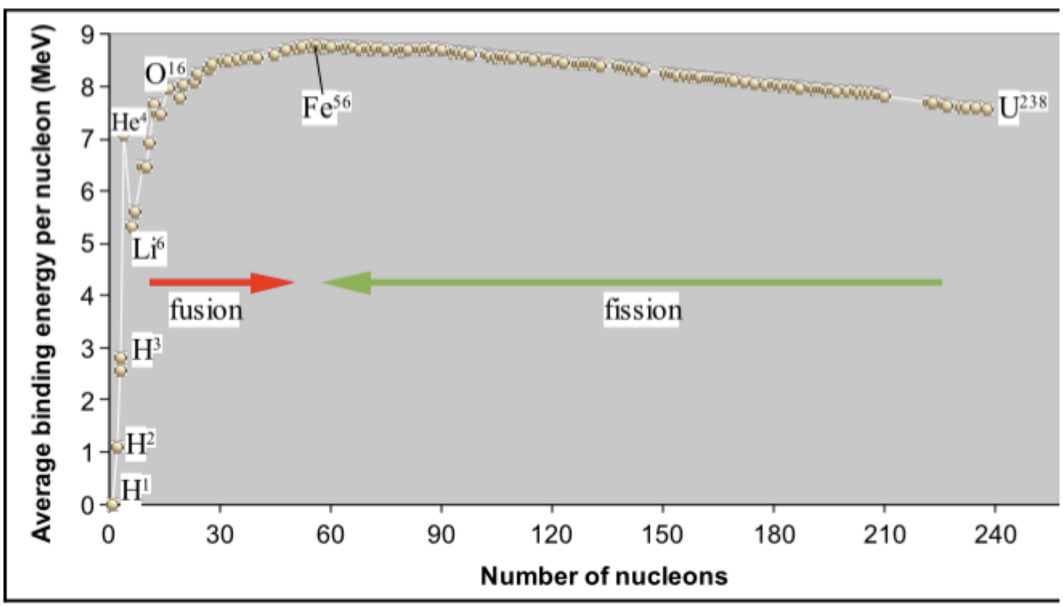

Figure 29.4: A graph of the average binding energy per nucleon for most common isotopes.

Nuclear fusion

The larger the average binding energy per nucleon, the more stable a particular isotope is. Thus, light elements (less than 50 nucleons, say) can, in general, become more stable by joining together to form a nucleus that has more binding energy per nucleon. This process is known as **nuclear fusion**, and it releases a significant amount of energy. Nuclear fusion is the process by which the Sun generates its energy, for instance. Currently, the Sun is made up mostly of hydrogen, which is gradually fusing together to become helium. When the hydrogen is used up, the helium atoms will fuse together with one another, or with other light atoms, to form heavier nuclei. All of these fusion reactions, resulting in atoms with more binding energy per nucleon, produce energy. The process continues for billions of years until the Sun's atoms fuse together to become nickel and iron, which are the most stable elements of all. At this point, the fusion reactions will cease, because the peak of stability will have been reached, and the Sun will, essentially, die of old age.

An example fusion reaction is the fusion of deuterium and tritium, which are both isotopes of hydrogen, into helium. This reaction may well form the basis, in future, of controlled fusion reactions that generate energy in a fusion reactor.

$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \Rightarrow {}^{5}_{2}\mathrm{He} \Rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}. $$

The products of this reaction, the helium-4 atom and the neutron, carry away 17.6 MeV between them, 3.5 MeV for the helium atom and 14.1 MeV for the neutron. This is an enormous amount of energy compared to the several eV of energy produced in a typical chemical reaction.

Nuclear fission
At the heavy end of the scale in Figure 29.4, those nuclei with a large number of nucleons can reach a more stable state by splitting apart into smaller pieces that have a higher average energy per nucleon. This process is known as **nuclear fission**, and it is used in nuclear reactors, in well-controlled reactions, to generate energy. In uncontrolled reactions, a chain of fission reactions can occur so quickly that nuclear meltdown occurs, or a nuclear bomb explodes.

An example of a fission reaction that may occur in a nuclear reactor is

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \Rightarrow {}^{236}_{92}\mathrm{U} \Rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\left({}^{1}_{0}\mathrm{n}\right). $$

Note that the reaction is triggered by bombarding the uranium-235 atom with a neutron, temporarily creating a uranium-236 atom that quickly splits into krypton, barium, and three more neutrons. These neutrons can go on to cause more uranium-235 atoms to split apart. In a controlled reaction, the rate at which reactions occur should be constant, so if it takes one neutron to start a reaction, only one of the product neutrons (on average) are allowed to go on to produce further reactions. The other neutrons are absorbed by a moderator inside a reactor, which could be heavy water or a boron control rod embedded in the reactor core.

A typical fission reaction, like the one shown, above releases on the order of 200 MeV. As with typical fusion reactions, this is millions of times larger than the energy released by burning oil or gas in a chemical reaction, explaining why nuclear power is so appealing in comparison to the burning of fossil fuels. That huge advantage has to be weighed against the negative aspects of nuclear energy, including the fact that nuclear reactors produce radioactive waste products that must be handled carefully and stored securely for rather long times.

Related End-of-Chapter Exercises: 10, 11, 29 – 33.

Essential Question 29.6: The fission reaction shown above is just one possible way that a uranium-235 atom, bombarded with a neutron, can split up. What is the missing piece in another of the many possible reactions, which is shown here?

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \Rightarrow {}^{236}_{92}\mathrm{U} \Rightarrow ? + {}^{94}_{38}\mathrm{Sr} + 2\left({}^{1}_{0}\mathrm{n}\right). $$

Answer to Essential Question 29.6: The reaction must satisfy conservation of nucleon number (there are 236 nucleons) and conservation of charge (in this case, there are 92 protons). For both sides of the reaction to have 236 nucleons and 92 protons, the missing piece must have 140 nucleons and 54 protons. The element with 54 protons is xenon so the missing piece is ${}^{140}_{54}\text{Xe}$.

29-7 Applications of Nuclear Physics

Radiocarbon dating

One well-known application of nuclear physics is the use of carbon-14, which is radioactive, to determine the age of artifacts made from materials that used to be alive, such as bowls made of wood. The idea is that when the tree from which the wood was taken was alive, it was exchanging carbon with the atmosphere, and the ratio of carbon-14 to non-radioactive carbon-12 in the wood matched the corresponding ratio in the atmosphere. This ratio is maintained at an approximately constant value of 1 carbon-14 atom to every 10^{12} carbon-12 atoms by cosmic rays that turn nitrogen-14 in the atmosphere into carbon-14. After a tree dies or is cut down, the carbon-14 in the wood decays, decreasing the carbon-14 to carbon-12 ratio. The more time passes, the smaller the carbon-14 to carbon-12 ratio, so the carbon-14 to carbon-12 ratio can be used to estimate the time that has passed since the tree was cut down. This process is called **radiocarbon dating**.

A famous artifact dated using radiocarbon dating is the Shroud of Turin, which was long believed to be the burial cloth of Jesus Christ. Tests on small samples of the fabric indicate that the fabric dates not from 2000 years ago, however, but from around 700 years ago instead.

Carbon-14 decays via the beta-minus process, with a half-life of 5730 years, so radiocarbon dating works well for artifacts with an age ranging from several hundred years old to about 60000 years old, an age range corresponding to a reasonable fraction of the carbon-14 half-life to several times the carbon-14 half-life. To date much older artifacts, such as dinosaur bones, which are about 100 million years old, carbon-14 would not be appropriate, because so many carbon-14 half-lives would have passed in 100 million years that the amount of carbon-14 in the sample would be negligibly small. A similar process to radiocarbon dating can be carried out, however, using an isotope with a much longer half-life than carbon-14.

EXAMPLE 29.7 – Dating a bowl

While on an archeological dig, you uncover an old wooden bowl. With your radiation detector, you measure 560 counts per minute coming from the bowl, each of these counts corresponding to the beta-minus decay of a carbon-14 atom into a nitrogen-14 atom (your detector picks up the fast-moving electron that is emitted from each of these decays). Using the same kind of wood, but from a tree that was recently cut down, you make a similar bowl of the same shape and mass as the one you unearthed at the excavation site. This bowl registers 800 counts per minute in your detector. Estimate the age of the old bowl you unearthed.

SOLUTION

Our working assumption is that the old bowl would also have emitted 760 counts per minute originally, when it was first made from wood from a tree that had been recently cut down. This assumes the ratio of carbon-14 to carbon-12 in our atmosphere has remained constant over time, which is approximately true. Accurate dating involves correcting for effects such as the fluctuation of carbon-14 to carbon-12 in the atmosphere in the past, but we can get a good estimate of the age of the bowl without these corrections. Applying Equation 29.12 gives:

$$560 = 800 e^{-\lambda t}, \quad \text{which becomes} \quad \frac{560}{800} = 0.7 = e^{-\lambda t} .$$

Taking the natural log of both sides gives $\ln(0.7) = -\lambda t$. Bringing in the half-life via Equation 29.11, we get:

$$t = \frac{-\ln(0.7)}{\lambda} = \frac{-\ln(0.7)}{\ln(2)} T_{1/2} = \frac{-\ln(0.7)}{\ln(2)} (5730 \text{ y}) = 2900 \text{ y} .$$

Thus, the site you are exploring contains artifacts from approximately 3000 years ago.

Radiation therapy

A common method of treating cancer is with radiation therapy, which involves treatment with ionizing radiation associated with nuclear decays. For instance, x-rays can be directed at a tumor within the body, damaging the DNA of the cancer cells with the energy deposited by the photons. It is hard to avoid damaging healthy cells with this process, but if the tumor is targeted from various directions the energy deposited in the tumor can be maximized while the damage to surrounding healthy cells is minimized.

Another example of radiation therapy is in the treatment of prostate cancer, in which small radioactive rods, called seeds, are embedded in the prostate, damaging the DNA of the cancer cells with the energy that comes from the decay. The seeds may contain, for example, iodine-125 or palladium-103, which are emitters of gamma rays or x-rays.

Medical imaging – PET and MRI

Two more applications of nuclear physics are **positron emission tomography** (PET) and **magnetic resonance imaging** (MRI). If you get a PET scan (like that in Figure 29.5), you must first take in an isotope that is a positron emitter. A common example is to use fluorodeoxyglucose (FDG), in which the fluorine atom is fluorine-18, which decays via the beta-plus (positron) process with a half-life of about two hours – this short half-life minimizes your exposure to radiation. FDG is taken up by cells that use glucose, so FDG is useful for studying cells with significant glucose uptake, such as those in the brain or in a cancer tumor.

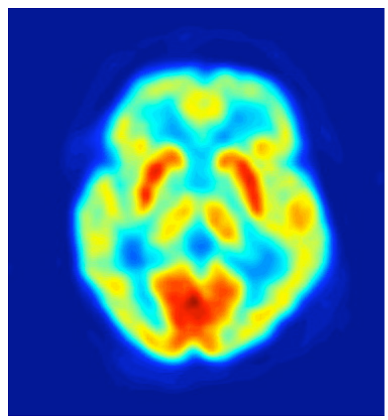

Figure 29.5: An image of a human brain obtained with positron emission tomography. The red and blue areas correspond to high and low positron activity, respectively. Image credit: Jens Langner, via Wikimedia Commons.

When a fluorine-18 atom decays, it emits a positron. The positron and a nearby electron annihilate one another, turning into two high-energy photons (gamma rays), which exit the body in almost exactly opposite directions. If the photons are detected by detectors surrounding the body, the path of the photons can be determined. After many such photon pairs have been detected, areas of high glucose uptake in the body can be reconstructed.

A different process is at work in MRI, in which hydrogen nuclei in our bodies (in water and lipid molecules, in particular) are excited by strong magnetic fields. Signals from these nuclei are then detected, and the signals can be used to create an image of what is going in inside a body being scanned. Such scans are often used to diagnose soft-tissue injuries. The image that opens this chapter shows such an MRI scan.

Related End-of-Chapter Exercises: 12, 26, 27, 35, 58, 59.

Essential Question 29.7: The ratio of carbon-14 to carbon-12 in a wooden implement, unearthed at an archeological dig, is only ¼ as large as it is in a growing tree today. Estimate its age.

Answer to Essential Question 29.7: To reduce the level to ¼ = ½ × ½ of its original value, two half-lives must have passed. Two half-lives for carbon-14 represents 11000 – 12000 years.

29-8 A Table of Isotopes

In doing calculations of mass defect, or nuclear decays and reactions, we need to know the atomic mass(es) of the atom or atoms involved. This information is shown in Table 29.3.

Z, Element	Isotope : Atomic mass
1, Hydrogen	$^{1}_{1}H$: 1.00782503 u
	$^{2}_{1}H$: 2.01410178 u
	$^{3}_{1}H$: 3.01604928 u, (β^-)
2, Helium	$^{4}_{2}He$: 4.00260325 u
3, Lithium	$^{6}_{3}Li$: 6.01512279 u
	$^{7}_{3}Li$: 7.01600455 u
4, Beryllium	$^{9}_{4}Be$: 9.0121822 u
5, Boron	$^{10}_{5}B$: 10.0129370 u
	$^{11}_{5}B$: 11.0093054 u
6, Carbon	$^{12}_{6}C$: 12.00000000 u
	$^{13}_{6}C$: 13.00335484 u
	$^{14}_{6}C$: 14.00324199 u, (β^-)
7, Nitrogen	$^{14}_{7}N$: 14.00307400 u
8, Oxygen	$^{15}_{8}O$: 15.0030656 u, (β^+)
	$^{16}_{8}O$: 15.99491462 u
	$^{18}_{8}O$: 17.9991610 u
9, Fluorine	$^{18}_{9}F$: 18.0009380 u, (β^+)
	$^{19}_{9}F$: 18.99840322 u
10, Neon	$^{20}_{10}Ne$: 19.99244018 u
11, Sodium	$^{22}_{11}Na$: 21.9944364 u, (β^+)
	$^{23}_{11}Na$: 22.98976928 u
12, Magnesium	$^{24}_{12}Mg$: 23.98504170 u
13, Aluminum	$^{27}_{13}Al$: 26.98153863 u
14, Silicon	$^{28}_{14}Si$: 27. 97692653 u

Z, Element	Isotope : Atomic mass
15, Phosphorus	$^{31}_{15}P$: 30.97376163 u
16, Sulfur	$^{32}_{16}S$: 31.97207100 u
17, Chlorine	$^{35}_{17}Cl$: 34.96885268 u
	$^{37}_{17}Cl$: 36.96590259 u
18, Argon	$^{40}_{18}Ar$: 39.96238312 u
19, Potassium	$^{39}_{19}K$: 38.96370668 u
	$^{41}_{19}K$: 40.96182576 u
20, Calcium	$^{40}_{20}Ca$: 39.96259098 u
21, Scandium	$^{45}_{21}Sc$: 44.9559119 u
	$^{46}_{21}Sc$: 45.9551719 u, (β^-)
22, Titanium	$^{46}_{22}Ti$: 45.9526316 u
	$^{48}_{22}Ti$: 47.9479463 u
23, Vanadium	$^{51}_{23}V$: 50.9439595 u
24, Chromium	$^{52}_{24}Cr$: 51.9405075 u
	$^{53}_{24}Cr$: 52.9406494 u
25, Manganese	$^{55}_{25}Mn$: 54.9380451 u
26, Iron	$^{56}_{26}Fe$: 55.9349375 u
	$^{58}_{26}Fe$: 57.9332756 u
27, Cobalt	$^{59}_{27}Co$: 58.9331950 u
28, Nickel	$^{58}_{28}Ni$: 57.9353429 u
	$^{62}_{28}Ni$: 61.9283451 u
29, Copper	$^{63}_{29}Cu$: 62.9295975 u
	$^{65}_{29}Cu$: 64.9277895 u
30, Zinc	$^{64}_{30}Zn$: 63.9291422 u

Table 29.3: A table of selected isotopes and their atomic masses, taken from data made available by the Lawrence Berkeley Laboratory. Note that the mass, in atomic mass units, of an electron or a positron is 0.00054858 u. The neutron mass is 1.008664 u, and the proton mass is 1.007276 u.

Z, Element	Isotope : Atomic mass
31, Gallium	${}^{69}_{31}$Ga : 68.9255736 u ${}^{71}_{31}$Ga : 70.9247013 u
32, Germanium	${}^{74}_{32}$Ge : 73.9211778 u ${}^{76}_{32}$Ge : 75.9214026 u, (β^-)
33, Arsenic	${}^{75}_{33}$As : 74.9215965 u
34, Selenium	${}^{80}_{34}$Se : 79.9165213 u
35, Bromine	${}^{79}_{35}$Br : 78.9183371 u ${}^{81}_{35}$Br : 80.9162906 u
36, Krypton	${}^{84}_{36}$Kr : 83.911507 u ${}^{85}_{36}$Kr : 84.9125273 u, (β^-)
37, Rubidium	${}^{85}_{37}$Rb : 84.91178974 u
38, Strontium	${}^{88}_{38}$Sr : 87.9056121 u
39, Yttrium	${}^{89}_{39}$Y : 88.905848 u
40, Zirconium	${}^{90}_{40}$Zr : 89.9047044 u ${}^{92}_{40}$Zr : 91.9050408 u
41, Niobium	${}^{93}_{41}$Nb : 92.906378 u
42, Molybdenum	${}^{98}_{42}$Mo : 97.9054082 u
43, Technetium	${}^{99}_{43}$Tc : 98.9062547 u, (β^-)
44, Ruthenium	${}^{102}_{44}$Ru : 101.9043493 u
45, Rhodium	${}^{103}_{45}$Rh : 102.905504 u
46, Palladium	${}^{106}_{46}$Pd : 105.903486 u
47, Silver	${}^{107}_{47}$Ag : 106.905097 u ${}^{109}_{47}$Ag : 108.904752 u
48, Cadmium	${}^{112}_{48}$Cd : 111.902758 u
49, Indium	${}^{115}_{49}$In : 114.903878 u, (β^-)

Z, Element	Isotope : Atomic mass
50, Tin	${}^{120}_{50}$Sn : 119.902195 u
51, Antimony	${}^{121}_{51}$Sb : 120.9038157 u
52, Tellurium	${}^{130}_{52}$Te * : 129.9062244 u, (β^-)
53, Iodine	${}^{127}_{53}$I : 126.904473 u ${}^{131}_{53}$I : 130.9061246 u, (β^-)
54, Xenon	${}^{132}_{54}$Xe : 131.9041535 u
55, Cesium	${}^{133}_{55}$Cs : 132.90545193 u
56, Barium	${}^{138}_{56}$Ba : 137.9052472 u ${}^{141}_{56}$Ba : 140.914411 u, (β^-)
82, Lead	${}^{208}_{82}$Pb : 207.9766521 u
83, Bismuth	${}^{209}_{83}$Bi : 208.9803987 u, (α)
84, Polonium	${}^{210}_{84}$Po : 209.9828737 u, (α)
85, Astatine	${}^{210}_{85}$At : 209.987148 u, (β^+)
86, Radon	${}^{222}_{86}$Rn : 222.0175777 u, (α)
87, Francium	${}^{223}_{87}$Fr : 223.019736 u, (β^-)
88, Radium	${}^{226}_{88}$Ra : 226.0254098 u, (α)
89, Actinium	${}^{227}_{89}$Ac : 227.027752 u, (β^-)
90, Thorium	${}^{232}_{90}$Th : 232.0380553 u, (α)
91, Protactinium	${}^{231}_{91}$Pa : 231.0358840 u, (α)
92, Uranium	${}^{235}_{92}$U : 235.0439299 u, (α) ${}^{238}_{92}$U : 238.0507882 u, (α)
93, Neptunium	${}^{237}_{93}$Np : 237.0481734 u, (α)
94, Plutonium	${}^{238}_{94}$Pu : 238.0495599 u, (α) ${}^{239}_{94}$Pu : 239.0521634 u, (α)
95, Americium	${}^{241}_{95}$Am : 241.0568291 u, (α)

Table 29.3, continued: Elements 57 – 81 are emitted to make room for the high-Z elements, which are rather radioactive. Radioactive isotopes are indicated with the decay process in brackets after the mass. *Tellurium-130 actually undergoes double-beta decay, in which two neutrons become protons by emitting electrons.

Related End-of-Chapter Exercises: 19 – 22, 56, 57.

Essential Question 29.8: Table 29.3 shows that krypton-85 experiences beta-minus decay. What does krypton-85 decay into?

Answer to Essential Question 29.8: Beta-minus decay increases the number of protons by 1, without changing the number of nucleons. Thus, beta-minus decay always takes the element one step up the periodic table. In addition to an electron and an anti-neutrino, krypton-85 decays into rubidium-85.

Chapter Summary

Essential Idea: The Nucleus.

Atomic nuclei are almost unimaginably tiny, yet the energy associated with nuclei is orders of magnitude larger than that associated with the electrons in an atom. Nuclear reactions, tapping into the energy of the nucleus, power the Sun (and other stars) as well as nuclear reactors (as well as nuclear bombs).

Holding the nucleus together

To hold a nucleus together, the mutual repulsion between protons in the nucleus is balanced by the nuclear force, which is associated with the interaction between nucleons (neutrons and protons). The nuclear force is a short-range force that is generally attractive.

To find the binding energy for an atom, we first find the mass defect, which is the difference between the total mass of the individual constituents of the atom and the mass of the atom itself. The mass defect is converted to energy via Einstein's famous equation, $E = mc^2$. This results in the mass to energy conversion factor:

1 u is equivalent to 931.5 MeV.

Radioactive decay processes

All radioactive decay processes conserve nucleon number and charge. The general equations describing specific processes are:

$${}_{Z}^{A}X_1 \Rightarrow {}_{Z-2}^{A-4}X_2 + {}_{2}^{4}\text{He}.$$ (Equation 29.3: **General equation for alpha decay**)

$${}_{Z}^{A}X_1 \Rightarrow {}_{Z+1}^{A}X_2 + {}_{-1}^{0}\text{e}^- + \overline{\nu}_e.$$ (Eq. 29.5: **General equation for beta-minus decay**)

$${}_{Z}^{A}X_1 \Rightarrow {}_{Z-1}^{A}X_2 + {}_{+1}^{0}\text{e}^+ + \nu_e.$$ (Eq. 29.7: **General equation for beta-plus decay**)

$${}_{Z}^{A}X_1^* \Rightarrow {}_{Z}^{A}X_1 + \gamma.$$ (Equation 29.9: **General equation for gamma decay**)

In the alpha and beta decay processes, the nucleus becomes a nucleus of a different element. In gamma decay, the nucleus simply drops from a higher-energy state to a lower-energy state, in a manner analogous to that of an electron making a transition from one electron energy level to a lower level. An alpha particle is a helium atom; a beta-minus particle is an electron; and a beta-plus particle is a positron.

Radioactivity

The rate at which N radioactive nuclei decay is:

$$\frac{\Delta N}{\Delta t} = -\lambda N .$$ (Equation 29.10: **Decay rate for radioactive nuclei**)

The equation that describes the exponential decay in the number of nuclei of a particular radioactive isotope as a function of time t is

$$N = N_i\, e^{-\lambda t}, \qquad \text{(Equation 29.12: \textbf{The exponential decay of radioactive nuclei})}$$

where N_i is a measure of the initial number of radioactive nuclei (the number at $t = 0$).

The decay constant, λ, is related to the half-life, $T_{1/2}$, (the time for half of the nuclei to decay) by

$$\lambda = \frac{\ln(2)}{T_{1/2}} = \frac{0.693}{T_{1/2}}. \qquad \text{(Eq. 29.11: \textbf{The connection between decay constant and half-life})}$$

Nuclear fusion and nuclear fission

The most stable nuclei (those with the highest average binding energy per nuclei) are nickel-62, iron-58 and iron-56. Nuclei that are lighter than these most stable nuclei can generally become more stable (increasing the average binding energy per nuclei) by joining together with other light nuclei – this process is known as nuclear fusion.

Very heavy nuclei, in contrast, can generally become more stable by splitting apart, usually into two medium-sized nuclei and a few neutrons. This process is known as nuclear fission. The fission of uranium-235, driven by the bombardment of the uranium-235 atoms with neutrons, is exploited in a nuclear reactor to produce nuclear energy, while the fission of plutonium-239 is what drives the explosion of a nuclear bomb.

End-of-Chapter Exercises

Exercises 1 – 12 are mainly conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. The symbol for the isotope iron-56 is ${}^{56}_{26}\mathrm{Fe}$. A neutral iron-56 atom has how many (a) protons? (b) neutrons? (c) nucleons?

2. Which of these numbers is larger, the mass of an iron-56 atom, or the total mass of the individual constituents (neutrons, protons, and electrons) of an iron-56 atom? Briefly explain your answer.

3. If you could convert 1 kg of matter entirely to energy, how much energy would you get?

4. (a) Fill in the blank to complete this decay process: ${}^{226}_{88}\mathrm{Ra} \Rightarrow \underline{\quad} + {}^{4}_{2}\mathrm{He}$. (b) What kind of decay is this? (c) Based on the fact that this decay process happens spontaneously, which side of the equation do you expect to have more mass? Why?

5. (a) What kind of radioactive decay process gives rise to a positron? (b) What is the electric charge of a positron? (c) Complete this sentence: The mass of a positron is the same as the mass of ________________ .

6. Fill in the blanks to complete the following decay processes:
(a) $\underline{\quad} \Rightarrow {}^{15}_{7}\mathrm{N} + {}^{0}_{+1}\mathrm{e}^{+} + \nu_e$. (b) ${}^{46}_{21}\mathrm{Sc} \Rightarrow \underline{\quad} + {}^{0}_{-1}\mathrm{e}^{-} + \overline{\nu}_e$. (c) ${}^{60}_{28}\mathrm{Ni}^{*} \Rightarrow \underline{\quad} + \gamma$.

7. A particular sample contains a large number of atoms of a certain radioactive isotope, which has a half life of 1 hour. After 5 hours, approximately what percentage of the original radioactive nuclei remain?

8. You have two samples of radioactive material. At $t = 0$, sample A contains a large number of nuclei that have a half-life of 10 minutes, while sample B contains exactly the same number of a different kind of nuclei, which have a half-life of 40 minutes. (a) In which sample is the rate at which the nuclei decay larger, at $t = 0$? Briefly justify your answer. Find the ratio of the number of nuclei that have decayed in sample A to the number that have decayed in sample B after (b) $t = 40$ minutes, (c) $t = 1$ year (feel free to make a reasonable approximation in part (c)).

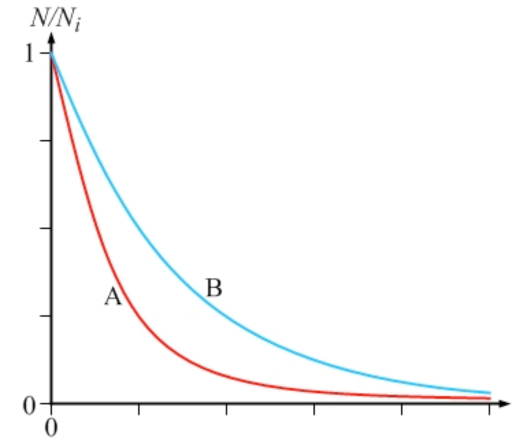

Figure 29.6: A graph of the ratio of the number of undecayed nuclei to the initial number of nuclei for samples of two different radioactive isotopes, labeled A and B, for Exercise 9.

9. Figure 29.6 shows a graph of the ratio of the number of undecayed nuclei to the initial number of nuclei for samples of two different radioactive isotopes, labeled A and B. Which isotope has the larger (a) decay constant? (b) half-life?

10. In stars that are more massive than the Sun, one of the primary methods of fusing hydrogen into helium is the CNO (carbon-nitrogen-oxygen) cycle, which is a sequence of fusion reactions. One of the reactions in the CNO cycle, which liberates about 5 MeV of energy, is shown below. Identify the isotope represented by the question mark.
$${}^{1}_{0}\mathrm{n} + {}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \Rightarrow ? + {}^{4}_{2}\mathrm{He}.$$

11. How many neutrons are produced in the following fission reaction for a uranium-235 atom that combines with a neutron?
$${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \Rightarrow {}^{236}_{92}\mathrm{U} \Rightarrow {}^{139}_{52}\mathrm{Te} + {}^{94}_{40}\mathrm{Zr} + ?\left({}^{1}_{0}\mathrm{n}\right).$$

12. Modern human activities have altered the ratio of carbon-14 to carbon-12 in the atmosphere. A good example of this is the burning of fossil fuels, which contain carbon that is millions of years old. The half-life of carbon-14 is about 5700 years, while carbon-12 is stable. Based on this information, has all of our burning of fossil fuels increased or decreased the carbon-14 to carbon-12 ratio in the atmosphere? Explain your answer.

Exercises 13 – 18 involve applications of $E = mc^2$. For some of these exercises you will probably want to make use of the data in Table 29-3 in Section 29-8.

13. For an oxygen-16 atom, calculate (a) the mass defect, in atomic mass units, (b) the total binding energy, in MeV, and (c) the average binding energy per nucleon.

14. (a) What is the difference, in terms of constituents, between a carbon-12 atom and a carbon-13 atom? (b) Compare the mass of a carbon-13 atom to the mass of a carbon-12 atom plus the mass of the extra particle(s) that makes up a carbon-13 atom compared to a carbon-12 atom. Is there a difference? If so, why?

15. In almost all nuclei, there are more than 2 nucleons, and thus there is more than one pair of nucleons interacting via the nuclear force. The situation is much simpler in a deuterium (${}^{2}_{1}H$) atom, in which the binding energy is associated with the attractive nuclear force between the single neutron and the single proton. For the deuterium atom, calculate (a) the mass defect, in atomic mass units, (b) the binding energy, in MeV, which is almost entirely associated with the nuclear force between the proton and neutron.

16. In general, for a nucleus to be stable, the attractive forces between nuclei must balance the repulsive forces associated with the charged protons. A helium-4 atom is particularly stable. To help understand why this is, consider how many pairs of (a) interacting protons, and (b) interacting nucleons there are in a helium-4 atom. (c) Briefly explain why your answers to (a) and (b) support the idea that the helium-4 atom is particularly stable.

17. Lead-208 is stable, which is relatively rare for a high-mass nucleus. For ${}^{208}_{82}Pb$, calculate (a) the mass defect, in atomic mass units, (b) the total binding energy, in MeV, and (c) the average binding energy per nucleon.

18. Nickel-62 has the largest average binding energy per nucleon of any isotope. Calculate the average binding energy per nucleon for nickel-62.

Exercises 19 – 24 involve radioactive decay processes. Make use of the data in Table 29-3 in Section 29-8.

19. Plutonium-239 (${}^{239}_{94}Pu$) decays via alpha decay. (a) Write out the decay equation for plutonium-239. (b) Calculate the energy released in the decay of one Pu-239 atom.

20. Scandium-46 (${}^{46}_{21}Sc$) decays via the beta-minus process. (a) Write out the complete decay equation for scandium-46. (b) Calculate the energy released in the decay of one scandium-46 atom.

21. A certain isotope decays via the beta-plus process to neon-22 (${}^{22}_{10}Ne$). (a) Write out the complete decay equation for this situation. (b) The mass of a neon-22 atom is 21.99138511 u. Calculate the energy released in this beta-plus decay process.

22. (a) According to Table 29.3, what does krypton-85 decay into? (b) Write out the decay equation for krypton-85. (c) Calculate the energy released in this decay process.

23. Silver-111, ${}^{111}_{47}Ag$, which has an atomic mass of 110.905291 u, is radioactive, spontaneously decaying via either alpha decay, beta-plus decay, or beta-minus decay. The masses of the possible decay products are 110.904178 u for ${}^{111}_{48}Cd$, 110.907671 u for ${}^{111}_{46}Pd$, and 106.906748 u for ${}^{107}_{45}Rh$. (a) Explain how you can use these numbers to determine the spontaneous decay process for silver-111. (b) Write out the complete decay reaction for the spontaneous decay of silver-111.

Exercises 24 – 28 involve radioactivity.

24. Oxygen-15 has a half-life of 2 minutes. Nuclear activity is often measured in units of becquerels (Bq), which are the number of nuclear decays per second. If the activity of a sample of oxygen containing some oxygen-15 is 64×10^6 Bq at $t = 0$, what is the activity level of the sample at (a) $t = 4$ minutes, (b) $t = 16$ minutes, and (c) $t = 20$ minutes?

25. Fluorine-20 decays to neon-20 with a half-life of 11 seconds. At $t = 0$, a sample contains 120 grams of fluorine-20 atoms. How many grams of fluorine-20 atoms remain in the sample at (a) $t = 5.0$ seconds, (b) $t = 30$ seconds, and (c) $t = 1.0$ minutes?

26. A supply of fluorodeoxyglucose (FDG) arrives at a positron emission tomography clinic at 8 am. At 5 pm, at the end of the working day, the activity level of the FDG has dropped significantly because of the 110 minute half-life of the radioactive fluorine. Considering equal masses of FDG at 8 am and 5 pm, by what factor has the activity level been reduced by 5 pm?

27. Four hours after being injected with a radioactive isotope for a positron emission tomography scan, the activity level of the material injected into the patient has been reduced by a factor of 8 compared to its initial value (at the time of the injection). After another four hours elapses, will the activity level be reduced by another factor of 8? Briefly justify your answer.

28. After 3 hours, the activity level of a sample of a particular radioactive isotope, which decays into a stable isotope, has fallen to 20% of its initial value. Calculate the half-life of this isotope.

Exercises 29 – 33 involve nuclear fusion and nuclear fission. Make use of the data in Table 29-3 in Section 29-8.

29. (a) What is the second object produced in the following fusion reaction? ${}^{2}_{1}\text{H} + {}^{6}_{3}\text{Li} \Rightarrow {}^{7}_{3}\text{Li} + \underline{\quad\quad}$. (b) How much energy is produced in this reaction?

30. In Exercise 10, we examined one of the fusion reactions in the CNO (carbon-nitrogen-oxygen) cycle, which is a primary method of fusing hydrogen into helium in massive stars. The primary reactions in the CNO cycle are shown in Figure 29.7. Two more of the fusion reactions in the CNO cycle are ${}^{1}_{1}\text{H} + {}^{13}_{6}\text{C} \Rightarrow {}^{14}_{7}\text{N}$ and ${}^{1}_{1}\text{H} + {}^{14}_{7}\text{N} \Rightarrow {}^{15}_{8}\text{O}$. Calculate the energy released in each of these fusion reactions.

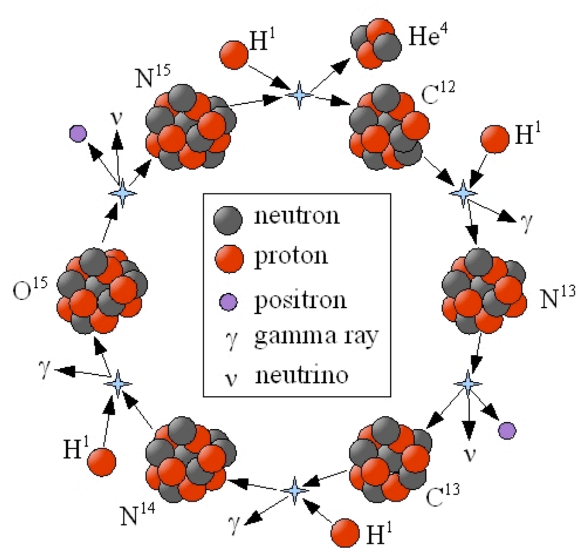

Figure 29.7: The CNO cycle of fusion reactions that produces much of the energy in massive stars. The CNO cycle has the net effect of fusing hydrogen into helium.

31. In relatively low-mass stars, such as our Sun, much of the energy generated by the star comes from the proton-proton chain, which has the net effect of fusing hydrogen into helium. The step that produces the helium-4 atom is: ${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \Rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H}$.
The helium-3 atoms are produced from the fusion of hydrogen isotopes in earlier steps in the process. How much energy is released in the step that produced helium-4? Note that the mass of ${}^{3}_{2}\mathrm{He}$ is 3.01602932 u.

32. Complete the following fission reaction: ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \Rightarrow {}^{236}_{92}\mathrm{U} \Rightarrow {}^{143}_{56}\mathrm{Ba} + \_\_\_ + 3\left({}^{1}_{0}\mathrm{n}\right)$.

33. How much energy is released in the following fission reaction?
$${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \Rightarrow {}^{236}_{92}\mathrm{U} \Rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\left({}^{1}_{0}\mathrm{n}\right).$$

Exercises 34 – 38 involve applications of nuclear physics.

34. A common application of nuclear radiation is **food irradiation**, which is the exposure of food products (such as meat, poultry, and fruit) to ionizing radiation to kill bacteria, prolong shelf life, and delay the ripening of fruit. One example of this treatment is the exposure of a meat product to a dose of 3 kilograys of gamma radiation from a cobalt-60 source. One gray (Gy) represents 1 joule of energy exposure per kilogram of food. Cobalt-60 decays by the beta-minus process to an excited form of nickel-60, which then decays to its ground state through the emission of two gamma rays, with energies of 1.17 MeV and 1.33 MeV. (a) How many nuclei are there in 1 gram of cobalt-60, which has an atomic mass of 59.9338171 u? (b) Cobalt-60 has a half-life of 5.27 years. How many nuclei, in a 1-gram sample of cobalt-60, would decay in 1 second? (c) If the two gamma rays associated with each decaying cobalt-60 nuclei deposit all their energy in a 1 kilogram package of meat, how long would the meat have to be exposed to receive a dose of 3 kGy?

35. When radiocarbon dating was carried out on the Shroud of Turin in 1988, the three labs doing the measurements agreed that the shroud dated to approximately the year 1300 AD, providing evidence against the idea that the shroud (a picture of which is shown in Figure 29.8) was the burial cloth of Jesus Christ. If the ratio of carbon-14 to carbon-12 in the atmosphere has remained constant over time at 1.2×10^{-12}, (a) approximately what ratio did the researchers find for the sample of shroud they measured?, and (b) what ratio would they have found if the cloth was 2000 years old?

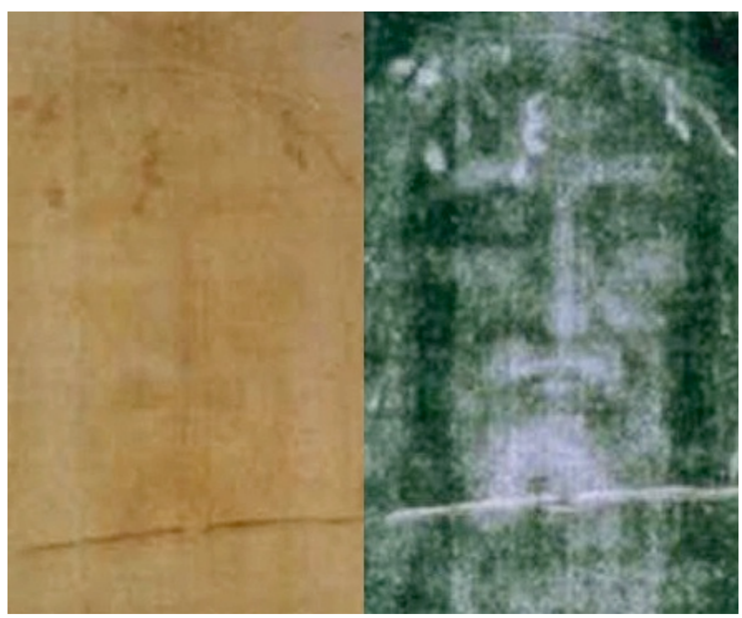

Figure 29.8: A photograph of part of the Shroud of Turin, on the left, and the corresponding negative image, on the right, for Exercise 35. Image credit: Wikimedia Commons.

36. Some smoke detectors have an alpha emitter, americium-241, in them. The alpha particles ionize air molecules in the space between two oppositely charged plates (a parallel-plate capacitor), and the current associated with these ions in the parallel-plate capacitor is measured by the smoke detector. When this current drops, because the ions are blocked by smoke particles, the detector's alarm sounds. (a) When an americium-241 atom decays via alpha decay, what does it decay into? (b) Using the data in Table 29.3 to help you, how much energy (in MeV) is released in the decay process of a single americium-241 atom? For reference, typical ionization energies are tens of electron volts.

37. Single photon emission computed tomography (SPECT) is a medical imaging procedure that makes use of technetium-99m, an excited form of technetium-99 that undergoes gamma decay with a half-life of 6 hours. The gamma rays are relatively low energy, around 140 keV, comparable to standard x-rays. SPECT is thus much like a CT scan, but with photons coming from inside the body rather than passing through the body. Technetium-99m is used in well over half of all medical imaging procedures that use radioactive nuclei, and is especially useful for bone scans and brain scans. By what factor has the activity level of the technetium-99m fallen to 48 hours after it was first administered to the patient?

38. Burning 20 tons of coal provides about 5×10^{11} J, which is approximately the annual energy requirement of an average person in the United States. If this amount of energy was provided by the electricity generated by the fission of uranium-235 in a nuclear reactor, instead, what mass of natural uranium would be required? About 0.7% of natural uranium is uranium-235 (almost all the rest is uranium-238, which does not fission like U-235 does). Assume that the fission of each U-235 atom provides 200 MeV of energy, and that the nuclear power plant has an efficiency of 33% in transforming the energy from the fission reactions into electricity.

General problems and conceptual questions

39. Marie Curie was a pioneer in the field of radioactivity. Do some research about Marie Curie and write a couple of paragraphs about her life and her contributions to nuclear physics.

40. Enrico Fermi made a number of important contributions to our understanding of nuclear physics. Do some research about Fermi and write a couple of paragraphs about his contributions to nuclear physics, describing how his work relates to the principles of physics discussed in this chapter.

41. Equation 29.1 gives the approximate radius of a nucleus, assuming it to be spherical. (a) What is the atomic mass number of a zinc-64 atom? (b) What is the cube root of zinc-64's atomic mass number? (c) Approximately what is the radius of a zinc-64 atom?

42. A particular nucleus has a radius of about 6×10^{-15} m. (a) Approximately what is the atomic mass number of the nucleus? (b) Approximately where would you find such a nucleus on the periodic table, assuming the nucleus is stable?

43. (a) If you convert the mass of an electron entirely to energy, how much energy, in keV, do you get? (b) If you convert the mass of a positron entirely to energy, how much energy, in keV, do you get? When an electron and positron encounter one another, they annihilate one another, and the particles are transformed entirely into two photons. Assume that the electron and positron are initially at rest before the annihilation process. After the annihilation process, what is the (c) energy, and (d) wavelength of each of the two photons?

44. The daily food intake of an average person consists of approximately 10 million joules of food energy. If we could generate this energy internally through the fusion of helium-3 and lithium-6 into two helium-4 atoms plus a proton, which releases about 17 MeV of energy, approximately what mass of helium-3 and lithium-6 would we need to take in every day?

45. Polonium-210 decays via alpha decay into lead-206 and an alpha particle. For the purposes of this exercise, use the approximation that the lead-206 atom has 50 times the mass of the alpha particle. We will also assume that the polonium-210 is at rest before the decay process. (a) Immediately after the decay, how does the momentum of the lead-206 atom compare to the momentum of the alpha particle? (b) Immediately after the decay, what is the ratio of the alpha particle's kinetic energy to the lead-206 atom's kinetic energy?

46. Try this at home. M&M's (the candy) are not radioactive, but a package of M&M's (or something equivalent, like coins) can be used as a model of a system with a half-life. Obtain a package of M&M's and, after first counting how many M&M's there are, do the following. Place all the M&M's in cup (starting with a number of M&M's that is a power of 2, like 64 M&M's, makes this exercise easier), and then shake them out onto a clean surface, like a plate. Remove all the M&M's that have their "m" down – those are the ones that have decayed. Count all the "m" up M&M's, representing undecayed nuclei, and place them back in the cup. Repeat the process until all the M&M's have decayed. (a) Make a table of your results, recording the number of "undecayed nuclei" as a function of the number of throws. (b) Use Equation 29.12 to predict the number of "undecayed nuclei" remaining – add this theoretical data to your table. (c) Plot a graph of your results, and draw the curve representing the theoretical results. (d) Account for any differences between your values and the theoretical values. (e) If we started with a very large number of radioactive nuclei, would we expect to see similar percentage deviations from the theoretical values? Explain. (f) If you repeated the experiment with the M&M's one thousand times, averaged your results, and plotted your averaged results against the theoretical values, would you expect more, less, or the same deviation from the theoretical values compared to the deviation obtained from a single trial? Explain. (g) Eat the M&M's.

47. Much like the M&M's in Exercise 46, a large number of six-sided dice can be used as a model of a system with a half-life. If you shake all the dice, how many throws represent one half-life if you define the "decayed" dice as (a) all those showing an even number, or (b) all those showing a 1?

48. Oxygen-15 has a half-life of 2 minutes. Fluorine-18 has a half-life of 110 minutes. For a sample of oxygen-15 to have the same decay rate as a sample of fluorine-18, how should (a) the number of radioactive nuclei in the two samples compare? (b) the masses of the two samples compare?

49. At $t = 0$, you have two samples of radioactive nuclei. Sample A contains N nuclei of a radioactive isotope that has a half-life of 3 hours. Sample B contains a to-be-determined number of nuclei of a different radioactive isotope, with a half-life of 4 hours. How many nuclei of the second radioactive isotope should sample B contain (at $t = 0$) if, at $t = 12$ hours, (a) the number of undecayed nuclei in the two samples is equal, or (b) the decay rate, measured in nuclei per second, of the two samples is equal.

50. You have two samples of radioactive nuclei. At $t = 0$, sample A contains N nuclei of a particular radioactive isotope, while sample B contains twice as many nuclei of a different radioactive isotope. Also at $t = 0$, the decay rate, measured in nuclei per second, of sample A is three times larger than the decay rate in sample B. How do the half-lives of the two different types of nuclei compare?

51. The graph in Figure 29.9 shows the ratio of the number of undecayed nuclei to the initial number of undecayed nuclei, as a function of time. Use the data in the graph to estimate the half-life of the nuclei.

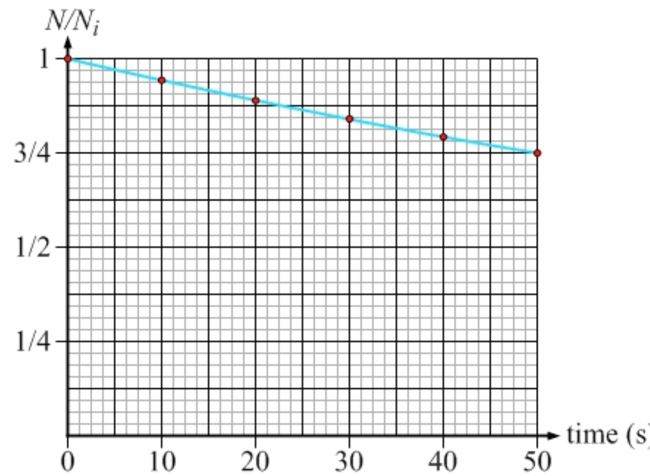

Figure 29.9: A graph of the number of undecayed nuclei to the initial number of undecayed nuclei, as a function of time, for Exercise 51.

52. Instead of defining the half-life for a particular isotope, we could define the "one-tenth-life" (or the life for any other fraction less than 1). After a time of a single one-tenth-life has passed, only 1/10th of the initial number of radioactive nuclei remain. (a) Determine the ratio of a one-tenth-life to a half-life. (b) The half-life of cesium-137 is 30 years. What is the one-tenth-life for cesium-137? (c) After how many one-tenth-lives does at activity level of a sample of radioactive material drop to 1% of its initial value? (d) How much time elapses until the activity level of a sample of cesium-137 drops to 1% of its initial value?

53. After 6 days, the activity level of a sample of radioactive material has fallen to 40% of its initial value. After an additional 6 days elapses, what is the activity level of the sample, compared to its initial value? (Hint: there is a very easy way to do this calculation – it is possible to work out the answer without a calculator.)

54. Right now, a sample contains 1.0×10^{20} undecayed nuclei, and a much larger number of decayed nuclei. If the half-life of the undecayed nuclei is 5 months, how many undecayed nuclei did the sample contain a year ago?

55. At $t = 0$ s, a sample of radioactive nuclei contains 1.0×10^{20} undecayed nuclei, which have a half-life of 10 seconds. First, use equation 29.10, with $N = 1.0 \times 10^{20}$ nuclei, to estimate the number of undecayed nuclei remaining at (a) $t = 1$ s, (b) $t = 10$ s, and (c) $t = 20$ s. Second, use equation 29.12 to estimate the number of undecayed nuclei remaining at (d) $t = 1$ s, (e) $t = 10$ s, and (f) $t = 20$ s. (g) Explain which method is the correct method to use to accurately determine the number of undecayed nuclei remaining, and why the other method gives inaccurate results.

56. When a heavy radioactive nucleus decays, it often decays into a nucleus that is also radioactive. The result can be a sequence of decay reactions that continues until a stable nucleus is reached. The sequence of reactions is known as a **decay chain**. An example of a decay chain is shown in Figure 29.10, displaying the sequence of isotopes that make up the chain that starts with the radioactive isotope thorium-232 and ends with the stable isotope lead-208. Note that bismuth-212 has two possible decay modes, so the chain branches from there, but both branches end up at lead-208. (a) Identify the sequence of decays in this particular chain, including the two branches. (b) By comparing the mass of thorium-232 to the mass of lead-208 plus the mass of all the alpha particles (the beta-minus particles are already included) produced in the decay chain, you can find the total energy released by the chain. What is this total energy? Does it matter which branch the chain takes after reaching bismuth-212?

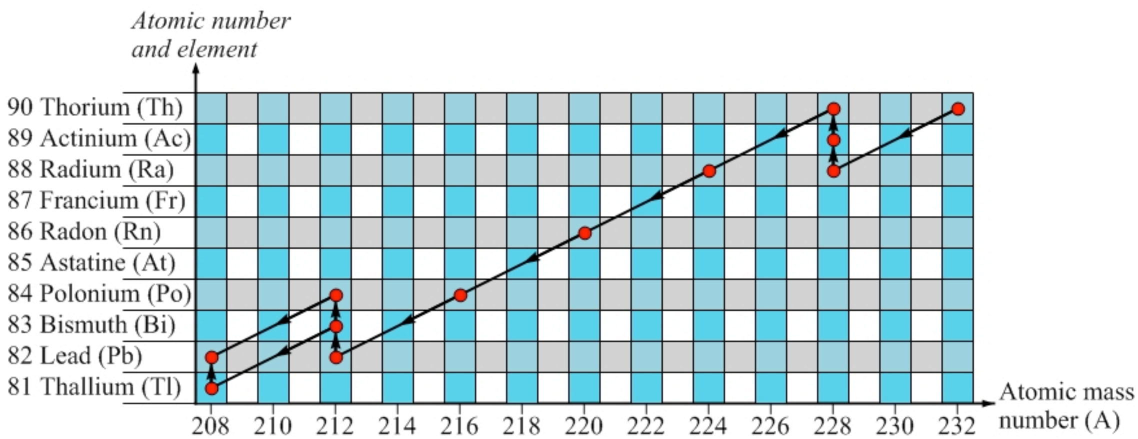

Figure 29.10: The decay chain that starts with the radioactive isotope thorium-232, and ends with the stable isotope lead-208. The radioactive isotopes that the nucleus passes through as it decays are shown with the red circles. Each step in the chain is either an alpha decay, which reduces the atomic number by 2 and the mass number by 4, or a beta-minus decay, which increases the atomic number by 1 without changing the mass number. For Exercise 56.

57. In Exercise 56, we defined the concept of a decay chain. Draw a decay chain diagram, like the one in Figure 29.10, representing the decay chain that starts with radium-226 and ends with lead-206. That sequence proceeds as follows: three alpha decays, followed by two beta-minus decays, followed by one alpha decay, two beta-minus decays, and a final alpha decay. On your decay chain diagram, clearly identify the various isotopes that the chain passes through on the way from the radioactive isotope radium-226 to the stable isotope lead-206.

58. The ratio of carbon-14 to carbon-12 in the shaft of a wooden arrow, unearthed when a foundation was being dug for a new house, is 70% of the same ratio in a growing tree today. Assuming the ratio of carbon-14 to carbon-12 in the atmosphere has been constant, estimate the age of the arrow.

59. In 1991, a body was found by hikers in the Alps, just inside Italy along the border between Italy and Austria. After being removed from the ice that had preserved the body, the body was carefully studied. This individual is now known as Ötzi the Iceman, and one of the methods used to study Ötzi was radiocarbon dating. The radiocarbon dating process carried out at the University of Vienna resulted in a “radiocarbon age” of 4550 years BP. BP stands for “Before Present,” where present is defined to be the year 1950 AD, the year when the radiocarbon dating process was first done. In determining a radiocarbon age, the half-life of carbon-14 is taken to be 5568 years. Assuming the ratio of carbon-14 to carbon-12 has held constant in the atmosphere at a value of 1.2×10^{-12}, what was the ratio in Ötzi’s body in 1950? Note that correcting Ötzi’s age for known historical fluctuations in the carbon-14 to carbon-12 ratio, as well as the correct half-life of carbon-14, led to the conclusion that there was a more than 60% probability that Ötzi died between 3230 and 3100 BC.

60. Nickel has five different stable nuclides, but nickel-66 is not one of them. All of the stable nuclides of nickel have fewer neutrons than nickel-66 has, in fact. (a) What do you think the dominant radioactive decay process is for nickel-66? (b) What is the daughter nuclide produced by this decay process?

61. One of the largest stable nuclides is a nuclide of lead, lead-208, with 82 protons and 126 neutrons. This nuclide can be formed by various radioactive decay processes. (a) If lead-208 is produced by a single beta-plus decay, what was the original nuclide? (b) If lead-208 is produced by a single beta-minus decay, what was the original nuclide? (c) If lead-208 is produced by a single alpha decay, what was the original nuclide? (d) Calculate the amount of energy released in the alpha decay process.

62. Near the low atomic number end of the chart of the nuclides, nuclides with equal numbers of protons and neutrons tend to be stable. As the atomic number increases, however, nuclides need more neutrons than protons to be stable. Consider the nuclide with 50 protons and 50 neutrons. (a) What element is this nuclide? (b) What do you expect the dominant radioactive decay mode to be for this nuclide? Explain your answer.

63. Three students are having a conversation. Comment on how the answers obtained by each student compare to the correct answer to the question they are trying to solve. Explain what, if anything, is wrong with each of their methods.

Mike: OK, the question gives the half-life as 2 years, and it asks for the percentage remaining after just 1 year. Isn’t that easy? If half of it decays in 2 years, then 25% decays in 1 year, so the fraction remaining after 1 year is 75%, right?

Jessica: I think you have to use one of the equations. I used Equation 29.12 ($N = N_i e^{-\lambda t}$***), and got an answer around 70%.***

Debbie: I think Jessica’s right, that you need to use an equation. I used Equation 29.10, though, after I re-arranged it to $\Delta N = -\lambda N(\Delta t)$**. Then I subtracted what was lost from 100% to get the answer. I got more like 65% left, though.**