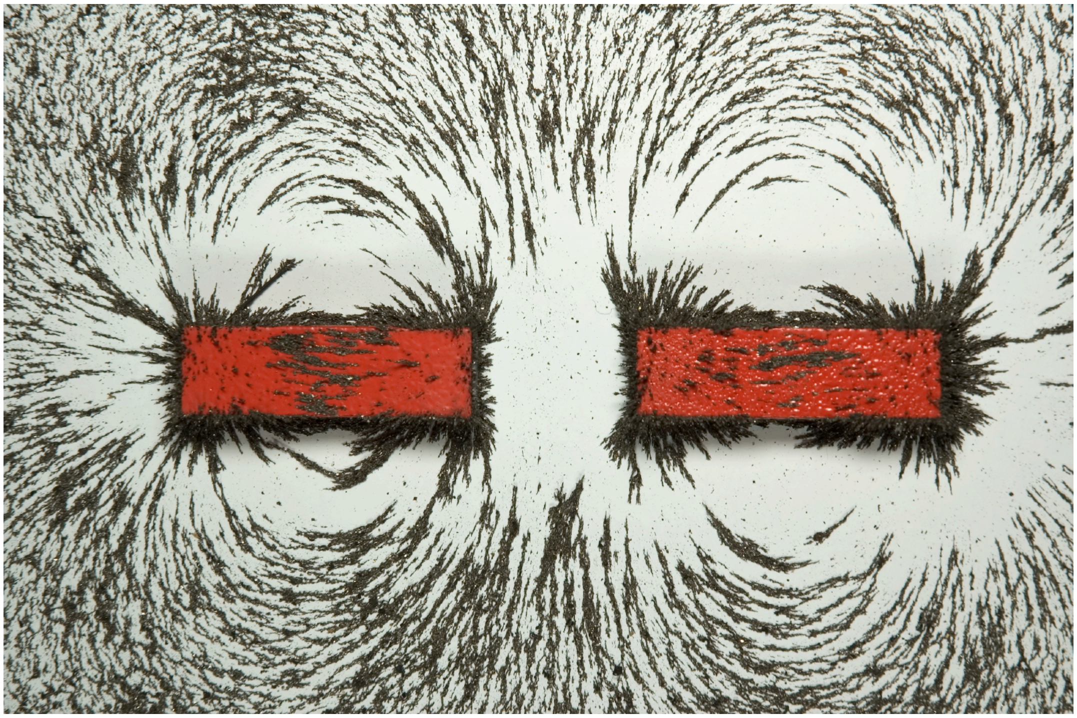


One way to visualize a magnetic field is to use iron filings. Each iron filing aligns with the field. In the photo at right, the iron filings indicate that the magnets repel one another. Also, the field is strongest at the ends (poles) of the magnets, where the density of filings is largest. Photo credit: Thomas Mounsey / iStockphoto.

Chapter 19 – Magnetism

CHAPTER CONTENTS

In this chapter, we will explore magnetism. We will discuss how magnetic fields influence charged particles and currents, and we will also discuss where magnetic fields come from. Magnetic fields are very useful things. Among other uses, the magnetic field from a magnet can interact with the steel door of a refrigerator to hold a child's artwork to the fridge; a hiker can use a compass, which has a magnetic needle that interacts with the Earth's magnetic field, to determine which way to walk to get out of the woods; and the magnetic field generated by a magnetic resonance imaging (MRI) machine in a hospital can interact with molecules in your body to investigate what is causing a particular pain in your knee.

The Earth's magnetic field is tremendously important to us, deflecting a dangerous stream of charged particles, coming at Earth from the Sun, around the Earth. The Earth would be a rather different planet were it not for this field.

19-1 The Magnetic Field

Challenge yourself. Write down any similarities you see between magnetic fields and electric fields, as well as any differences you see. Then check your list against the list below. As we will learn later, there are two ways to generate a magnetic field. One way is to use a current, and the similarities and differences below apply to magnetic fields generated by currents. The second way to produce a magnetic field is by changing an electric field, which we will investigate later in the book.

Some Similarities between Electric Fields and Magnetic Fields

- Electric fields are produced by two kinds of charges, positive and negative. Magnetic fields are associated with two magnetic poles, north and south. As we will learn in this chapter, magnetic fields are also produced by charges (but moving charges).
- Like electric charges repel, while unlike charges attract. Like magnetic poles repel, while unlike poles attract.
- The electric field points in the direction of the force experienced by a positive charge. The magnetic field points in the direction of the force experienced by a north pole.

Some Differences between Electric Fields and Magnetic Fields

- Positive and negative charges can exist separately. North and south poles always come together. Single magnetic poles, known as magnetic monopoles, have been proposed theoretically, but a magnetic monopole has never been observed.
- Electric field lines have definite starting and ending points. Magnetic field lines are continuous loops. Outside a magnet, the magnetic field is directed from the north pole to the south pole. Inside a magnet, the magnetic field runs from south to north.

The magnetic fields we will deal with in this chapter are associated with currents (this includes fields from a refrigerator magnet, the Earth, and an MRI machine). The symbol we use for magnetic field is B.

The SI unit of magnetic field is the tesla (T).

Visualizing Magnetic Fields

As with electric fields, we use field lines and field vectors to visualize magnetic fields. Figure 19.1(a) shows magnetic field lines near a bar magnet, where we can see that the field lines are continuous. Recall that the field is strongest where the field lines are densest, which is inside the magnet itself. The field-vector view, in Figure 19.1(b), emphasizes that the magnetic field exists at all points in space. The field vectors are darkest where the field is strongest.

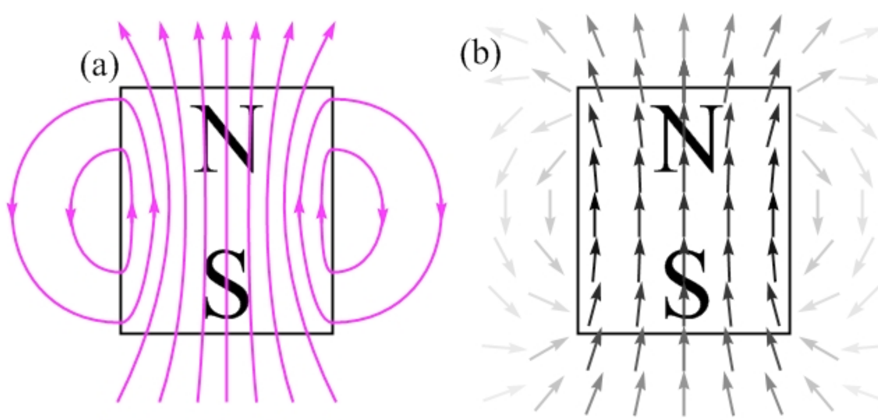


Figure 19.1: Two views of the magnetic field near a bar magnet. Figure (a) shows the magnetic field lines, while figure (b) shows the field vectors. Each of the field vectors can be thought of as compass needles.

The Magnetic Field of the Earth

The Earth's magnetic field is weak compared to a magnet on your refrigerator, but the Earth's field is strong enough to be used by humans, birds, and bacteria for navigation, and to act as a protective shield for the Earth. The Earth's field has a strength of about 5×10^{-5} T at the surface of the Earth, depending on the location. Presently, the Earth's field is gradually decreasing in strength. For comparison, a typical refrigerator magnet has a magnetic field in the millitesla range, while a strong magnet in a research lab has a field of about 10 T. At the surface of the Earth, the magnetic field is stronger near the poles, where the field lines are vertical, and weaker near the equator, where the field lines are horizontal.

The Earth's field is similar in form to that from a bar magnet, and thus resembles the field in Figure 19.1. However, the Earth's magnetic field is not nearly as symmetric as the field from a bar magnet. As shown in Figure 19.2, the form of the Earth's magnetic field is strongly influenced by the solar wind, which consists of charged particles that are emitted by the Sun. Fortunately for us, the Earth's field protects us from much of the effects of the solar wind.

What produces the Earth's magnetic field? This question is one that scientists are working to answer completely, but the basic mechanism is that is associated with electric charge carried by swirling flows of molten iron deep within the Earth's core.

The location of the Earth's magnetic poles changes over time, as the currents within the Earth change. Mostly, the magnetic poles wander around gradually, but roughly every 250 000 years or so, on average, the Earth's magnetic field flips direction (taking a couple of thousand years to flip). The last flip was about 780 000 years ago, so we are overdue for a change. A reversal of the Earth's field direction requires a major change in the flow patterns of the molten iron within the Earth. Understanding such major changes is an area of cutting-edge research.

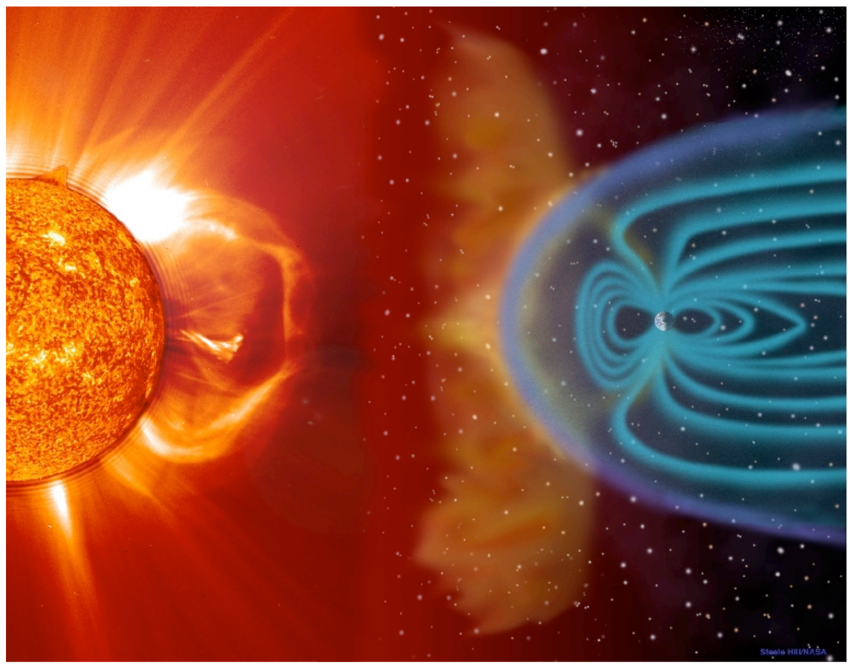


Figure 19.2: In this illustration, the Sun is shown at the left. The solar wind, which streams outward from the Sun, causes the Earth's magnetic field (at the right) to be highly asymmetric, with the field lines extending a significant distance to the right, beyond the extent of this illustration. The Earth's field acts as a rather effective shield against the energetic charged particles in the solar wind. Illustration from SOHO (ESA & NASA).

Related End-of-Chapter Exercises: 10, 33, 36.

Essential Question 19.1: If the north pole of a compass needle points toward the south pole of a magnet, why does the north pole of a compass point north on the Earth?

Answer to Essential Question 19.1: The magnetic south pole of the Earth is actually near the geographic north pole of the Earth (the geographic poles correspond to the Earth's axis of rotation), and the Earth's magnetic north pole is near the geographic south pole.

19-2 The Magnetic Force on a Charged Object

In Chapter 16, we investigated the force experienced by a charged object in an electric field. This force is given by equation 16.3, $\vec{F}_E = q\vec{E}$. The relationship between the magnetic force exerted on a charged particle and the magnetic field is a little more complicated than that between the electric force and the electric field. Let's try to understand the relationship, in the magnetic situation, by making some observations (see Table 19.1) involving various charged particles in a uniform magnetic field (a magnetic field with constant magnitude and direction).

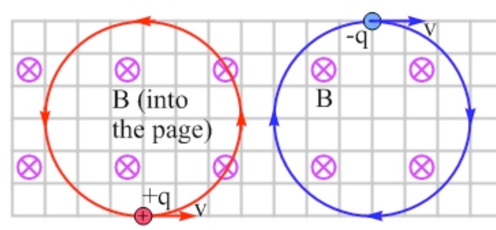
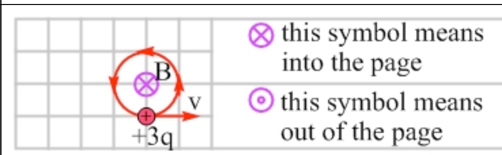
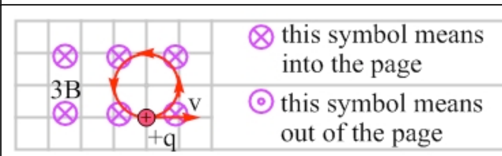
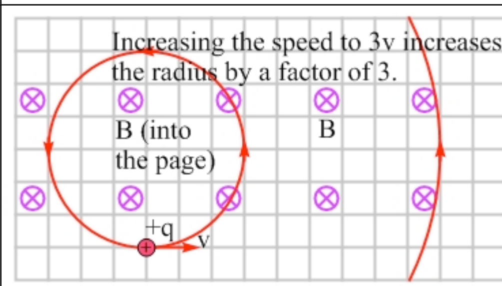
Experiment	Result	Comment or pictorial representation
1. The charged particle is released from rest in the magnetic field.	The particle remains at rest.	The particle experiences no force. (Note: if the particle was in an electric field instead, it would have a constant acceleration.)
2. The particle has an initial velocity parallel to the magnetic field.	The particle moves at constant velocity.	The particle experiences no force! The magnetic field has no influence on a charged particle moving parallel to the field.
3. The particle is given an initial velocity in a direction perpendicular to the magnetic field. The field is directed into the page.	**Figure 19.3**: The particle moves in a circular path at constant speed in a plane perpendicular to the magnetic field. Particles with charges of opposite sign orbit in opposite senses.	
4. The charge of the positively charged particle in experiment 3 is tripled to $+3q$.	**Figure 19.4**: The radius of the circular path is reduced by a factor of 3, indicating that the force on the particle has tripled.	
5. The strength of the magnetic field for the particle in situation 3 is tripled.	**Figure 19.5**: The radius of the circular path is reduced by a factor of 3, indicating that the force on the particle has tripled.	
6. The magnitude of the initial velocity for the particle in situation 3 is tripled.	**Figure 19.6**: The radius of the circular path is increased by a factor of 3, indicating that the force on the particle has tripled. Also, the period of the circular orbit is observed to be independent of the speed of the particle.	

Table 19.1: The behavior of various charged particles in a uniform magnetic field. All other influences (such as gravity) are neglected.

Related End-of-Chapter Exercises: 6 and 38.

For experiments 3 – 6 in Table 19.1, the particles experience uniform circular motion, so we can apply the analysis methods we used in Chapter 5. The net force on the particle, which is directed toward the center of the circle, comes from the magnetic field. Thus, the magnitude of the magnetic force on a particle of mass *m* and speed *v* is: $F_M = mv^2 / r$. Solving for the radius, *r*:

$$r = \frac{mv^2}{F_M}. \qquad \text{(Equation 19.1)}$$

In experiments 4 and 5, the mass and speed of the particle are constant. Thus, any change in radius comes from a change in the force. In Experiment 4, tripling the charge decreases the radius by a factor of 3, so the magnetic force must increase by a factor of 3. Thus, we conclude that the magnetic force is proportional to the charge. Analyzing experiment 5 in the same way, we conclude that the magnetic force is proportional to the magnitude of the magnetic field.

The results of experiment 6 are interesting. Based on Equation 19.1, we might expect that tripling the speed would lead to an increase in radius by a factor of 9. Because we observe that the radius increases by a factor of 3, we conclude that there must be a factor of *v* hidden in F_M . In other words, the magnetic force on a charged particle is proportional to the speed of the particle. Let's write a compact equation for the force exerted by a magnetic field on a charged particle.

$F_M = qvB \sin\theta$, (Eq. 19.2: **The magnitude of the magnetic force on a charge *q***)

where θ is the angle between the velocity, $\vec{v}$, and the magnetic field, $\vec{B}$. The direction of the force is perpendicular to the plane defined by $\vec{v}$ and $\vec{B}$, and given by the right-hand rule.

The right-hand rule for determining the direction of the magnetic force on a moving charge

First, make sure you use your right hand! Also, refer to Figure 19.7.

- Point the fingers on your right hand in the direction of the charge's velocity.
- While keeping your fingers aligned with the velocity, rotate your hand so that, when you curl your fingers, you can curl them into the direction of the magnetic field.
- Hold out your thumb so it is perpendicular to your fingers. Your thumb points in the direction of the force experienced by a positively charged particle.
- If the particle has a negative charge, your right hand lies to you. Just reverse the direction of the force. The magnetic force on a negatively charged particle is opposite in direction to that of a positively charged particle if the particles are traveling in the same direction.

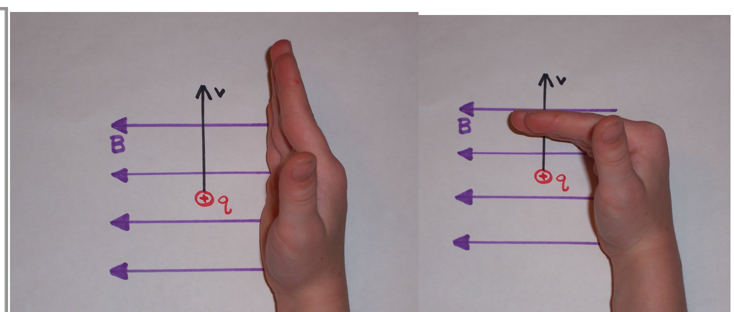


Figure 19.7: To find the magnetic force on a charged particle moving in a magnetic field, point the fingers on your right hand in the direction of the velocity, as in the photo on the left. Orient your hand so you can curl your fingers into the magnetic field, as in the photo on the right. Keep your thumb perpendicular to your fingers, and your thumb points in the direction of the force experienced by a positive charge (out of the page, in this case). A negative charge experiences a force in the opposite direction. Photo courtesy of A. Duffy.

Essential Question 19.2: A charge with an initial velocity directed parallel to a magnetic field experiences no magnetic force, and travels in a straight line. If the initial velocity is perpendicular to the field, the particle travels in a circle. Predict the shape of the path followed by a charge with a velocity component parallel to the field, and a velocity component perpendicular to the field.

Answer to Essential Question 19.2: Try this – walk in a straight line while whirling your hand in a circle around an axis parallel to your velocity. Your hand traces out a spiral, the shape of the path followed by the particle. For the particle, the spiral's axis is parallel to the magnetic field.

19-3 Using the Right-hand Rule

Let's practice using the right-hand rule (described at the end of section 19-2).

EXAMPLE 19.3 – Applying the right-hand rule

Draw a picture of each situation below, and use the picture to help answer the question.

(a) In what direction is the magnetic force on a particle that has a positive charge, and which has a velocity directed to the right in a uniform magnetic field directed up the page?

(b) In what direction is the magnetic force on a negatively charged particle, and which has a velocity directed down the page in a uniform magnetic field that has one component directed out of the page, and the other component directed down the page?

(c) In what direction is the velocity of a particle that has a positive charge, and which experiences a magnetic force directed into the page in a uniform magnetic field directed right?

SOLUTION

(a) Consider Figure 19.8. The magnetic force is perpendicular to the plane defined by the velocity and the magnetic field, which is the plane of the page. Thus, the force is either directed into or out of the page. Our right hands can distinguish between these directions. Place your right hand on the page, with the fingers pointing right. If the plane of the hand is perpendicular to the page, with the thumb sticking up out of the page, we can curl the fingers into the direction of the field. Thus, the force on the positive charge is out of the page.

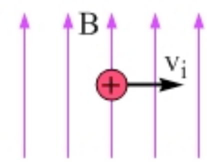


Figure 19.8: A positively charged particle is directed right, in a field directed up.

(b) This situation is shown in Figure 19.9. A magnetic field that is parallel to the particle's velocity exerts no force. Thus, we can focus on the perpendicular component of the field. Place your right hand flat on the page with the palm up and the fingers directed to the bottom of the page, so you can curl the fingers up, out of the page. The thumb points to the left, in the direction of the force on a positive charge. Our charge is negative, so the right hand lies to us and the force is directed to the right.

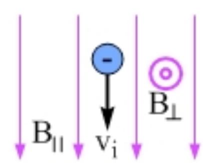


Figure 19.9: A negatively charged particle is moving down, in a field directed both down the page, parallel to the velocity, ($B_{||}$) and out of the page ($B_\perp$), perpendicular to the velocity.

(c) Consider the situation shown in Figure 19.10. First, remember that the velocity is perpendicular to the force, so, if the force is perpendicular to the page, the velocity must be in the plane of the page. Let's apply the right-hand rule in reverse order. Curl the fingers of your right hand, and hold your hand with the thumb pointing down, into the page, and with your curled fingers directed right. When you un-curl your fingers 90°, they point up the page in the direction of the velocity component that is perpendicular to the magnetic field. However, there may or may not be a velocity component directed left or right, parallel to the magnetic field lines. This case is ambiguous – we can't say for certain which direction the velocity is in because the angle between $\vec{v}$ and $\vec{B}$ does not have to be 90°.

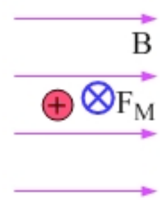


Figure 19.10: A moving positively charged particle experiences a force directed into the page, in a field directed right.

Related End-of-Chapter Exercises: 13 – 16.

A special case: a charged particle with a velocity perpendicular to the magnetic field

When the velocity of a charged particle is perpendicular to the field, the particle follows a circular path. Applying the form of Newton's second law for circular motion, $\sum F = mv^2 / r$:

$$qvB\sin\theta = \frac{mv^2}{r}.$$

The velocity and magnetic field are perpendicular, so we have $\sin(90°) = 1$. Solving for r:

$$r = \frac{mv}{qB}.$$ (Equation 19.3: **Radius of the path, when $\vec{v}$ and $\vec{B}$ are perpendicular**)

We can find the time it takes the particle to go around the circle once, which we call the period of the orbit, by dividing the circumference of the orbit by the particle's speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{vqB} = \frac{2\pi m}{qB}.$$ (Equation 19.4: **The period of the circular orbit**)

Note that ***the period is independent of the speed*** of the particle.

EXPLORATION 19.3 – Identifying the particles

As shown in Figure 19.11, four particles pass through a square region of uniform magnetic field directed perpendicular to the page.

Particle	Charge	Mass	Speed	Path taken
1	0	$2m$	$6v$	
2	$+q$	m	$4v$	
3	$+2q$	$4m$	v	
4	$-q$		$2v$	

Table 19.2: The charge, mass, and speed for four particles passing through a magnetic field.

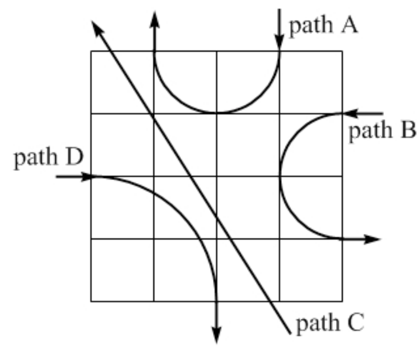


Figure 19.11: The paths followed by the particles.

Step 1 – ***Identify the path taken by particle 1.*** Because particle 1 has no charge, the field exerts no force on it. Particle 1 travels through the field in a straight line – so, it follows path C.

Step 2 – ***Are the paths clockwise or counterclockwise? Use your answer to determine the direction of the magnetic field.*** Positive charges travel in one sense, while negative charges travel in the opposite sense. Paths A and D both show clockwise motion, so these are the paths taken by the two positive charges. Path B shows a counterclockwise motion, so path B is taken by particle 4, the only particle with a negative charge. Applying the right-hand rule to the positive particle traveling along path D, we find that the magnetic field is directed out of the page.

Step 3 – ***Apply Equation 19.3 to see which particle follows path A and which follows path D.*** Equation 19.3 tells us that the radius of the path is proportional to the mass multiplied by speed divided by the charge (we neglect the factor of the magnetic field, which is the same for all the charges). This combination is $4mv/q$ for particle 2, and $2mv/q$ for particle 3. Thus, the path followed by particle 2 has twice the radius (path D) as that followed by particle 3 (path A).

Key idea: The radius of curvature, and the direction of curvature, provides information about the speed, charge, and/or the mass of a particle. **Related End-of-Chapter Exercises: 7, 8, 17, 18.**

Essential Question 19.3: Rank the four particles in Exploration 19.3 in terms of the magnitude of the force applied to them by the magnetic field, from largest to smallest.

Answer to Essential Question 19.3: The direct way to solve this problem is to use Equation 19.2, $F_M = qvB \sin\theta$. The particles are in the same magnetic field, and $\sin\theta = 1$ because each velocity is perpendicular to the magnetic field. Thus, in this situation the force magnitude is proportional to the magnitude of the charge multiplied by the speed. This combination of factors is 0 for particle 1, 4qv for particle 2, and 2qv for particles 3 and 4, so the ranking is 2 > 3 = 4 > 1.

19-4 Mass Spectrometer: An Application of Force on a Charge

There are a number of practical devices that exploit the force that a magnetic field applies to a charged particle. Let's investigate one of these devices, the mass spectrometer.

EXPLORATION 19.4 – How to make a mass spectrometer

Mass spectrometers, which separate ions based on mass, are often used by chemists to determine the composition of a sample. Let's explore one type of mass spectrometer, which uses electric and magnetic fields. For each step below, sketch a diagram to help you with the analysis.

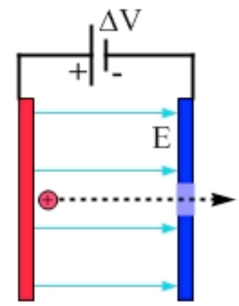


Figure 19.12: Charged particles are released from rest near the left-hand plate of a parallel-plate capacitor, accelerate across the gap, and emerge via a hole cut in the right-hand plate.

Step 1 – The accelerator. ***Release a charged particle from rest near one plate of a charged parallel-plate capacitor, so that the particle accelerates toward the other plate of the capacitor. Apply energy conservation to obtain a relation between ΔV, the potential difference across the capacitor, and the speed of the particle when it emerges from a small hole in the second plate.*** The particle shown in Figure 19.12 has a positive charge, but the accelerator can also work for negatively charged particles if we reverse the battery attached to the capacitor. Let's apply what we learned in Chapter 17. If we define the particle's electric potential energy to be zero at the negative plate ($U_f = 0$), the potential energy is $U_i = q(\Delta V)$ when the particle is next to the positive plate. The particle has no initial kinetic energy ($K_i = 0$), but as the particle accelerates across the gap between the plates the electric potential energy is transformed into kinetic energy. There is no work done by non-conservative forces ($W_{nc} = 0$). With three terms being zero, our five-term conservation of energy equation, $K_i + U_i + W_{nc} = K_f + U_f$, becomes $U_i = K_f$. Using our expressions for potential and kinetic energy gives:

$$q(\Delta V) = \frac{1}{2}mv^2 \text{, so the particle's speed when it emerges is } v = \sqrt{\frac{2q(\Delta V)}{m}}.$$

Step 2 – The velocity selector. ***The particle passes through a second parallel-plate capacitor in which the plates are parallel to the particle's velocity. In addition to the electric field inside the capacitor there is also a magnetic field, directed perpendicular to both the electric field and the velocity of the particle. The combined effect of the two fields is that a particle with just the right speed experiences no net force and passes undeflected through the velocity selector. Determine how this speed relates to the magnitudes of the fields.*** In the situation shown in Figure 19.13, the top plate is positively charged, and the bottom plate is negatively charged, so the electric field in the capacitor is directed down. The electric force on the positive charge is also directed down, because $\vec{F}_E = q\vec{E}$. For the particle to experience no net force, the magnetic force must exactly balance the electric force. The magnitude of the magnetic force is given by $F_M = qvB\sin\theta$. Because the velocity and magnetic field are at right angles, $\theta = 90°$ and $\sin(90°) = 1$.

Setting the magnitudes of the two forces equal, $F_M = F_E$, gives $qvB = qE$. The factors of q cancel. Solving for the speed of the undeflected particles gives:

$v = \frac{E}{B}$.   (Eq. 19.5: **The speed of undeflected particles in a velocity selector**)

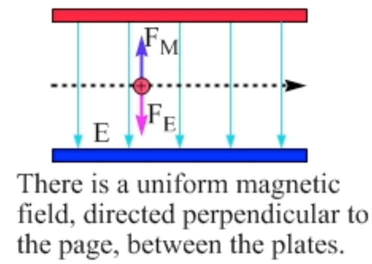


Figure 19.13: A charged particle with just the right velocity passes undeflected through the velocity selector because the magnetic force balances the electric force.

Question: Is the magnetic field in Figure 19.13 directed into or out of the page?

Answer: By the right-hand rule, to obtain a magnetic force directed up on a positive charge with a velocity to the right, the magnetic field is into the page.

Question: What happens to particles traveling faster than the undeflected particles? What happens to particles traveling slower than the undeflected particles?

Answer: The magnetic force depends on speed, while the electric force does not. For particles going faster than the selected speed, the magnetic force exceeds the electric force. The net upward force deflects the fast particles up out of the beam (see Figure 19.14). For relatively slow particles, the magnetic force is less than the electric force. The net downward force deflects the slow particles down out of the beam.

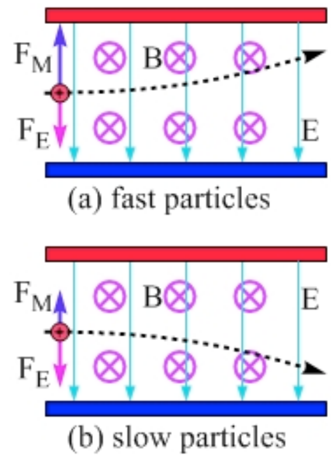


Figure 19.14: (a) A velocity selector deflects fast particles in one direction (up, in this case) and (b) slow particles in the opposite direction (down, here).

Step 3 – The mass separator. ***Particles that pass undeflected through the velocity selector are sent into the mass separator, which consists of a uniform magnetic field that is perpendicular to the velocity of the particles. If the particles are collected after traveling through half-circles in the field, find an expression for the separation between particles of mass m_1 and particles of mass m_2.*** Consider equation 19.3 ($r = mv/qB$), which gives the radius of the path followed by a charged particle in a uniform magnetic field. We give the particles the same charge in step 1, and they are in the same magnetic field, so the radius depends on mass and speed. The velocity selector ensures that all the particles reaching the mass separator have the same speed, so the radii differ only because the masses differ. As shown in Figure 19.15, the particles enter the field at the same point. After passing through a half-circle, the separation between the particles is the difference between the diameters of the circular paths. Thus, the separation is given by:

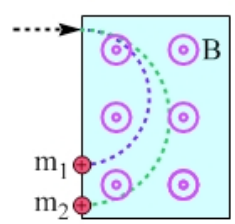


Figure 19.15: After traveling through a half-circle in the mass separator, particles of different mass are separated by a distance equal to the difference between the diameters of their paths.

$$\Delta s = 2r_1 - 2r_2 = 2\left(\frac{m_1 v}{qB} - \frac{m_2 v}{qB}\right) = \frac{2v}{qB}(m_1 - m_2)$$ (Eq. 19.6: **Separation in the mass spec.**)

Key Ideas for the mass spectrometer: The mass spectrometer is an excellent example of how we exploit uniform electric and magnetic fields, both individually and in combination, when working with charged particles.
Related End-of-Chapter Exercises: 45 – 47.

Essential Question 19.4: If the particles in step 2 of Exploration 19.4 were negatively charged, would we need to reverse the direction of either field (or both fields) for the velocity selector to function properly, or would it work for negatively charged particles without any changes?

Answer to Essential Question 19.4: The velocity selector works for negatively charged particles without any changes. When the particle has a negative charge, the electric force and the magnetic force both reverse direction. For particles traveling at a speed of $v = E/B$, however, the two forces balance and the particles are undeflected. Faster particles would be deflected down, given the orientation of the fields in Figure 19.14, while slower particles would be deflected up.

19-5 The Magnetic Force on a Current-Carrying Wire

Starting with the magnetic force on a moving charge, $F_M = qvB \sin\theta$, let's derive the equation for the magnetic force on a current-carrying wire. In the equation below, we write the velocity as a length L divided by a time interval Δt. We also use the definition of current, as the amount of charge q passing a point in a certain time interval Δt. This gives:

$$F_M = qvB\sin\theta = q\frac{L}{\Delta t}B\sin\theta = \frac{q}{\Delta t}LB\sin\theta = ILB\sin\theta .$$

The magnitude of the magnetic force exerted on a wire of length L, carrying a current I, by a magnetic field of magnitude B is:

$$F_M = ILB\sin\theta ,$$ (Equation 19.7: **The magnetic force on a current-carrying wire**)

where θ is the angle between the current direction and the magnetic field.

The direction of the force is given by a right-hand rule that follows the rule for charges. Point the fingers on your right hand in the direction of the current. Align your hand so that when you curl your fingers, they point in the direction of the magnetic field. Stick out your thumb, and it points in the direction of the magnetic force. The current direction is defined to be the direction of flow of positive charge, so the direction given by your hand never needs to be reversed, as it does when we find the direction of the magnetic force on a negatively charged particle.

EXPLORATION 19.5A – Three paths from a to b

As shown in Figure 19.16, three wires carry equal currents from point a to point b. The wires are in a uniform magnetic field directed to the right.

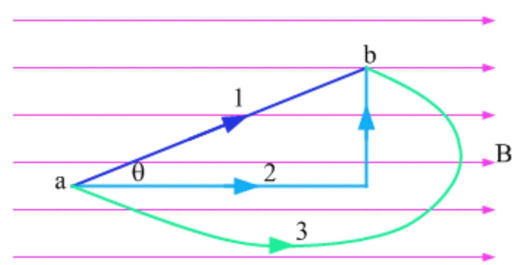


Figure 19.16: Three wires carry currents of the same magnitude from point a to point b in a uniform magnetic field.

Step 1 – ***Apply the right-hand rule to find the direction of the magnetic force exerted on each wire by the field.*** For wire 1, pointing the fingers of the right hand in the direction of the current and then curling them into the direction of the magnetic field gives a force directed into the page. For wire 2, there is no force on the part that is parallel to the field. Apply the right-hand rule to the part of wire 2 that is perpendicular to the field – the force is directed into the page. For wire 3, we ignore the parts of the wire that are going left or right, parallel to the field, and focus on the sections that are going up or down, perpendicular to the field. Wire 3 goes up more than it goes down. Applying the right-hand rule with the current directed up gives a force into the page.

Step 2 – ***Rank the wires based on the magnitude of the force they experience. To help you, re-draw wire 3 as short segments that are either parallel or perpendicular to the magnetic field.*** Applying Equation 19.7, we find for wire 1 that $F_{M1} = ILB\sin\theta$, where wire 1 has a length L. For wire 2, only the part directed up, perpendicular to the field, experiences a force. This section of wire 2 has a length of $L\sin\theta$, so applying equation 19.7 for wire 2 gives $F_{M2} = I(L\sin\theta)B\sin(90°)$. The factor of $\sin(90°)$ equals 1, giving: $F_{M1} = F_{M2} = ILB\sin\theta$.

Let's re-draw wire 3 using line segments that are either parallel to or perpendicular to the magnetic field, as in Figure 19.17. We could use shorter segments, so the wire would not look so jagged, but the argument that follows would still apply. The segments that are parallel to the field experience no force. For the segments perpendicular to the field, subtract the total length of segments that carry current down from the total length of segments that carry current up, because their forces are in opposite directions. The net displacement is $L\sin\theta$ up, the same as wire 2. Thus, all three wires experience the same force and the ranking is 1 = 2 = 3. The same argument applies for any wire: all wires carrying equal currents from a to b in a uniform magnetic field experience equal forces.

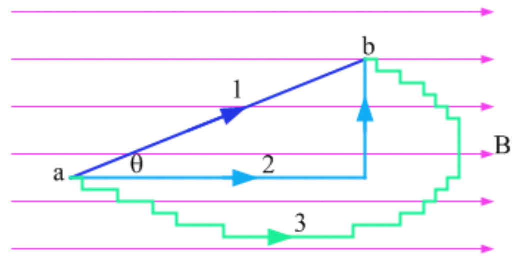


Figure 19.17: Wire 3 is replaced by a joined set of line segments, parallel and perpendicular to the magnetic field.

Key idea: All wires carrying equal currents from one point to another in a uniform magnetic field experience the same magnetic force. **Related End-of-Chapter Exercises: 51, 52.**

EXPLORATION 19.5B – The force on a current-carrying loop

Step 1 – ***In what direction is the net magnetic force on a rectangular wire loop that carries a clockwise current I in a uniform magnetic field that is directed out of the page? The loop is in the plane of the page.*** A diagram is shown in Figure 19.18 (a). To find the net magnetic force on the loop, we add, as vectors, the forces on each side of the loop. As shown in Figure 19.18 (b), the force on the left side cancels the force on the right side, and the force at the top cancels the force at the bottom. Thus, the net force on the loop is zero.

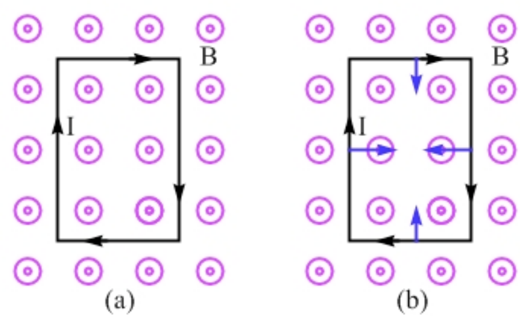


Figure 19.18: (a) A rectangular loop in a uniform magnetic field directed out of the page. (b) The magnetic force on each side of the loop is shown with an inward-directed arrow. The net force on the loop is zero.

Step 2 – ***In what direction is the net force on the loop if the field is directed to the left?*** In this special case, the forces on the top and the bottom sides are each zero. As Figure 19.19 (b) shows, the remaining forces cancel, and again the net force on the loop is zero. This is always true - as long as the magnetic field is uniform, the net magnetic force on any complete current-carrying loop is zero.

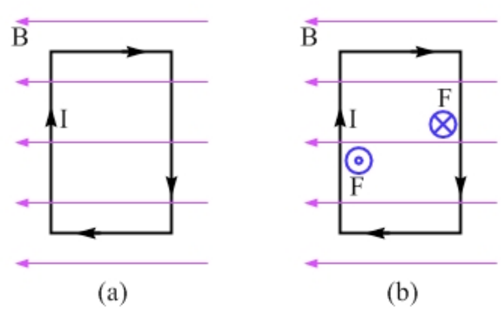


Figure 19.19: (a) A rectangular loop in a uniform magnetic field directed to the left. (b) In this case, the top and bottom sides experience no force because they are parallel to the magnetic field. The force on the left side is out of the page, while the force on the right side is into the page. These forces are of equal magnitude, so the net force is zero.

Key idea for current-carrying loops: In a uniform magnetic field, the net magnetic force acting on a current-carrying loop is zero. **Related End-of-Chapter Exercise: 53.**

Essential Question 19.5: Consider figures 19.18 and 19.19. The net magnetic force acting on a complete current-carrying loop is zero, but what about the net magnetic torque on the loop?

Answer to Essential Question 19.5: Figure 19.18, with the magnetic field perpendicular to the plane of the loop, is a special case, in which both the net force and the net torque are zero. In Figure 19.19, the forces acting on the loop would cause the loop to rotate, with the left side of the loop coming out of the page and the right side going into the page. In this case there is a net torque acting on the loop, which is generally the case.

19-6 The Magnetic Torque on a Current Loop

In a uniform magnetic field, a wire loop carrying current experiences no net force, but there is generally a net torque acting that tends to make the loop rotate. As we will see, we can exploit the interaction between the loop and the field to make a motor.

Let's begin by drawing a number of views of a rectangular current-carrying loop in a uniform magnetic field. The loop can rotate without friction about an axis parallel to its long sides, passing through the center of the loop. In the front view, imagine viewing the loop as if your eye is at the bottom of the page, looking along the page at the loop. The angles specified in Figure 19.20 are the angles between the loop's area vector, which is perpendicular to the plane of the loop, and the magnetic field.

Specifying area as a vector is something new. The magnitude of the area vector is the loop's length multiplied by its width. The direction of the area vector is perpendicular to the plane of the loop, but there are two directions that are perpendicular to this plane. The orientation of the area vector can be found by applying a right-hand rule. Curl the fingers on your right hand in the direction of the current flow. Your thumb, when you stick it out, gives the direction of the area vector. The angles specified in Figure 19.20 are the angles between the area vector and the magnetic field.

If the loop starts from rest in Figure 19.20(a), it will begin to rotate. The angular velocity increases in a clockwise direction, as observed from the front view. The clockwise torque continues until Figure 19.20(d), at which point there is no torque. In the absence of friction, the loop's angular momentum keeps it moving forward, with the counter-clockwise torque in Figures 19.20(e) – (g) slowing the loop down and bringing it instantaneously to rest in Figure 19.20(g). The loop then reverses direction, and goes through the pictures in reverse order.

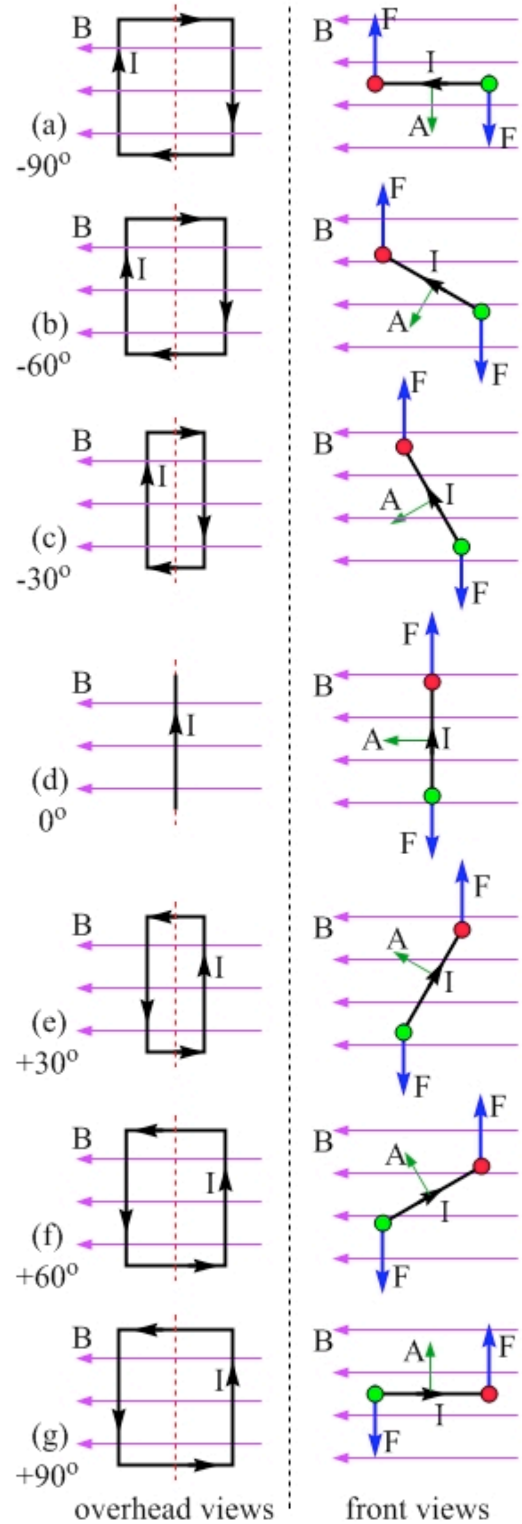


Figure 19.20: The arrow, labeled A in each front view, is the loop's area vector, which is directed perpendicular to the plane of the loop. When the area vector is perpendicular to the magnetic field, the loop experiences maximum torque. If the loop starts from rest in (a), the clockwise torque it experiences in orientations (a) – (d) cause the loop to rotate clockwise. After (d), the torque reverses direction, bringing the loop instantaneously to rest at (g), at which point the motion reverses.

The situation from Figure 19.20(a) is re-drawn in Figure 19.21, showing the height H and width W of the loop. Apply equation 10.9, $\tau = r F \sin\theta$, to find the torque applied by the field on the loop.

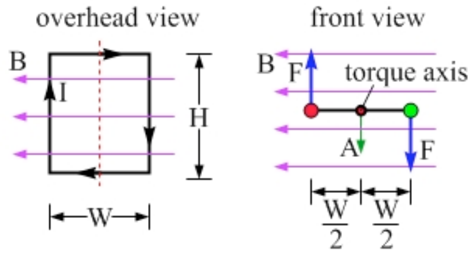


Figure 19.21: The situation from Figure 19.20(a), with dimensions given for the loop.

Take torques about the axis through the center of the loop, shown as a dashed red line in Figure 19.21 (overhead view). The distance from each force to this axis is $W / 2$, each force has a magnitude of $F = IHB$ (from equation 19.7), and the angle between the line we're measuring the distance along and the line of the force is 90°. A factor of 2 accounts for the two identical torques, which are both clockwise. The magnitude of the net torque on the loop in this orientation is:

$$\tau = 2\frac{W}{2} IHB = IAB$$, where $A = HW$ is the area of the loop.

As the loop rotates, the torque is reduced as the angle between each force and the plane of the loop changes from 90°. This angle is equal to the angle between the area vector and field, so we can write a general expression for the torque, and generalize to coils with more than one loop.

The magnitude of the torque applied by a uniform magnetic field on a coil of N loops carrying a current I, with an angle θ between the area vector $\vec{A}$ and the magnetic field $\vec{B}$, is:

$\tau = NIAB\sin\theta$. (Equation 19.8: **Torque on a current-carrying coil in a magnetic field**)

The DC (Direct Current) Motor – an application of the torque on a current-carrying loop

If the loop starts from rest in the orientation in Figure 19.20(a), then the loop will flop back and forth as the torque alternates between clockwise and counterclockwise. Friction will eventually bring the loop to rest in the zero-torque orientation of Figure 19.20(d). With one change, however, the loop becomes an electric motor – a device for transforming electrical energy into mechanical energy. We simply reverse the direction of the current each time the plane of the loop is perpendicular to the field (as in Figure 19.20(d)). With this change, the torque on the loop, and therefore the rotation of the loop, is always in the same direction.

An easy way to accomplish the current reversal is to use a split-ring commutator, as shown in Figure 19.22. The split-ring commutator is a cylinder divided into two halves that are electrically insulated from one another. The cylinder rotates with the loop, and its left side rubs against a fixed wire connected to the positive terminal of a battery while its right side rubs against a fixed wire connected to the battery's negative terminal. As shown in Figure 19.22, the current reverses direction in the loop every half-rotation. From our perspective, however, the current always goes clockwise around the loop, which is why the torque is always in the same direction.

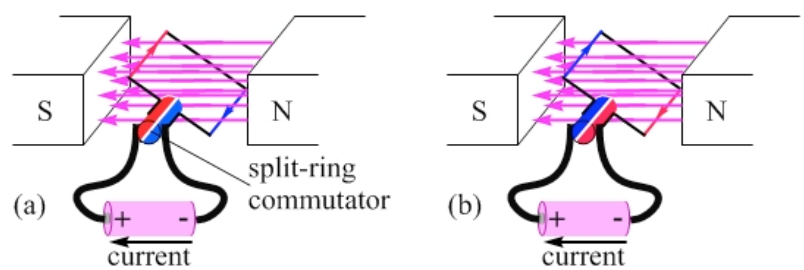


Figure 19.22: The split-ring commutator (the split cylinder that the dark wires from the battery are connect to) reverses the current in the loop every half rotation. From our perspective, the current is always clockwise, but from the loop's perspective the current has reversed between (a) and 180° later in (b).

Related End-of-Chapter Exercises: 54, 59.

Essential Question 19.6: In the situation shown in Figure 19.20, we ignored the magnetic forces acting on the shorter sides of the loop. Why can we do this?

Answer to Essential Question 19.6: In this situation, the forces acting on the two short sides of the loop produce forces that cancel one another. These forces are either zero or are directed along the axis we take torques around, giving no torque about that axis.

19-7 Magnetic Field from a Long Straight Wire

Let's now turn to investigating how to produce a magnetic field. Similar to the way that electric fields can be set up by charged particles and act on charged particles, magnetic fields can be set up by moving charges (or currents) and act on moving charges. The analog of the point charge for magnetism is the long straight current-carrying wire. Figure 19.23 shows the magnetic field from a long straight wire. The magnetic field from a wire decreases with distance from the wire. Instead of the field being proportional to the inverse square of the distance, as is the electric field from a point charge, the magnetic field is inversely proportional to the distance from the wire. Another difference between the electric field situation and the magnetic field situation is that the magnetic field lines are complete loops.

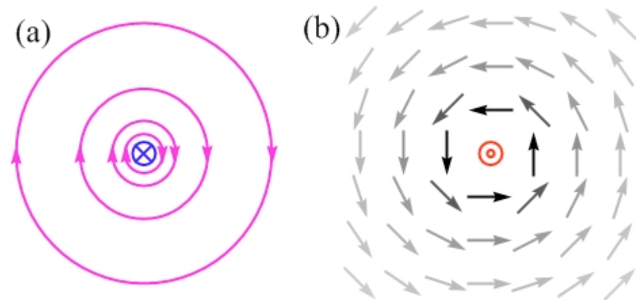


Figure 19.23: The magnetic field lines from a long straight wire wrap around the wire in circular loops. In (a), we see magnetic field lines near a wire that carries current into the page. In (b), field vectors are plotted. The vectors circulate counterclockwise, because the current in (b) is out of the page, opposite to what it is in (a). Both views show the strength of the magnetic field decreasing as the distance from the wire increases.

The magnetic field at a distance r from a long straight wire carrying a current I is:

$$B = \frac{\mu_0 I}{2\pi r}.$$ (Eq. 19.9: **The magnetic field from a long-straight wire**)

The direction of the magnetic field is given by a right-hand rule. In this rule, point the thumb on your right hand in the direction of the current in the wire. When you curl your fingers, they curl the same way that the magnetic field curls around the wire. The constant μ_0 in equation 19.9 is known as the **permeability of free space,** and has a value of $\mu_0 = 4\pi \times 10^{-7}$ T m / A.

In Chapter 8, we analyzed situations involving objects with mass interacting with each other via the force of gravity. In Chapter 16, we investigated situations involving interacting charged particles. Let's investigate analogous magnetic situations involving long straight wires.

EXPLORATION 19.7 – The magnetic force between two parallel wires

A long straight wire (wire 1) carries a current of I_1 into the page. A second long straight wire (wire 2) is located a distance d to the right of wire 1, and carries a current of I_2 into the page. Let's determine the force per unit length experienced by wire 2 because of wire 1.

Step 1 – ***Find the magnitude and direction of the magnetic field set up by wire 1 at the location of wire 2.*** The magnitude of the field is given by equation 19.9: $B_1 = \mu_0 I_1 /(2\pi d)$. To find the direction of this field at the location of wire 2, recall that the field lines are circular loops centered on wire 1. Applying the right-hand rule (see the previous page), we find that these field lines go clockwise. The field at any point is tangent to the field line, so the field at the location of wire 2 is directed straight down (see Figure 19.24).

Step 2 – ***Apply equation 19.7 to find the force per unit length that wire 2 experiences because of the magnetic field of wire 1.*** Equation 19.7 ($F_M = ILB\sin\theta$) gives us the force a wire of length L experiences in a magnetic field. However, we do not have a length to use for wire 2, so we bring the factor of length to the left side. Substituting the expression for B_1 from step 1 gives:

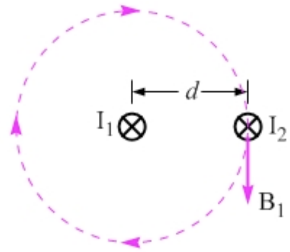


Figure 19.24: The magnetic field set up by wire 1 at the location of wire 2 is directed down the page, tangent to the direction of wire 1's field line at that point.

$$\frac{F_{12}}{L} = I_2 B_1 \sin(90°) = \frac{\mu_0 I_1 I_2}{2\pi d}.$$ (Eq. 19.10: **The force between two parallel wires**)

Applying the right-hand rule associated with equation 19.7, we find that the force experienced by wire 2 is to the left. In other words, when the currents are in the same direction the wires attract. If the currents are in opposite directions, they repel.

Step 3 – ***Which wire exerts more force on the other, if $I_1 = 3I_2$?*** No matter how the currents compare, the wires experience forces of equal magnitude in opposite directions – Newton's third law applies. Another way to see this is that equation 19.10 applies equally well to either wire.

Key ideas: Two long straight wires that are parallel to one another exert forces on one another. If the currents are in the same direction, the wires attract one another. If the currents are in the opposite direction, the wires repel one another. **Related End-of-Chapter Exercises: 49 and 57.**

EXAMPLE 19.7 – Finding the net magnetic field

Four long straight parallel wires pass through the *x*-*y* plane at a distance of 4.0 cm from the origin. Figure 19.25 shows the location of each wire, and its current. Find the net magnetic field at the origin because of these wires.

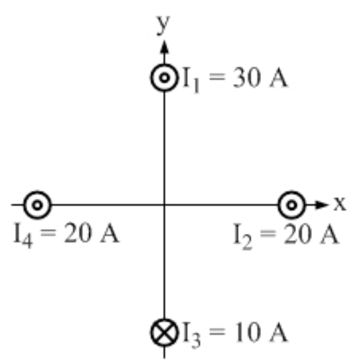


Figure 19.25: The currents carried by four long straight wires.

SOLUTION

Let's apply the principle of superposition to find the net magnetic field at the origin. The four fields, one from each wire, are shown in Figure 19.26. The two fields that are along the *y*-axis, from the two wires on the *x*-axis, cancel one another. The two fields along the *x*-axis, from the wires on the *y*-axis, add together. Thus, the net magnetic field at the origin is directed along the positive *x*-axis, and has a magnitude of:

$$B_{net} = B_1 + B_3 = \frac{\mu_0 I_1}{2\pi r} + \frac{\mu_0 I_3}{2\pi r} = \frac{\mu_0}{2\pi r}(I_1 + I_3) = \frac{4\pi \times 10^{-7} \text{ T m/A}}{2\pi (0.040 \text{ m})}(30 \text{ A} + 10 \text{ A}) = 2.0\times 10^{-4} \text{ T} .$$

Note that magnetic field is a vector. We can add the magnitudes of the two individual fields to find the magnitude of the net field only because the two fields are in the same direction.

Related End-of-Chapter Exercises: 11, 24 – 27.

Essential Question 19.7: For the situation in Exploration 19.7, let's say that the two wires are 40 cm apart and that $I_1 = 3I_2$. Where is the net magnetic field equal to zero near these wires?

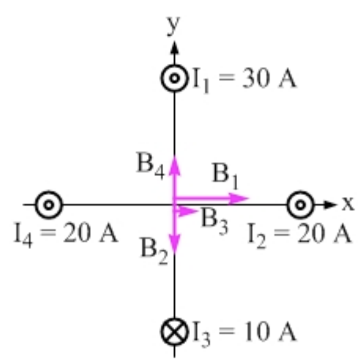


Figure 19.26: The four fields that add as vectors to give the net field.

Answer to Essential Question 19.7: To get a net magnetic field of zero, the two fields must point in opposite directions. This happens only along the straight line that passes through the wires, in between the wires. The current in wire 1 is three times larger than that in wire 2, so the point where the net magnetic field is zero is three times farther from wire 1 than from wire 2. This point is 30 cm to the right of wire 1 and 10 cm to the left of wire 2.

19-8 Magnetic Field from Loops and Coils

The Magnetic Field from a Current Loop

Let's take a straight current-carrying wire and bend it into a complete circle. As shown in Figure 19.27, the field lines pass through the loop in one direction and wrap around outside the loop so the lines are continuous. The field is strongest near the wire. For a loop of radius R and current I, the magnetic field in the exact center of the loop has a magnitude of

$$B = \frac{\mu_0 I}{2R} .$$ (Equation 19.11: **The magnetic field at the center of a current loop**)

The direction of the loop's magnetic field can be found by the same right-hand rule we used for the long straight wire. Point the thumb of your right hand in the direction of the current flow along a particular segment of the loop. When you curl your fingers, they curl the way the magnetic field lines curl near that segment. The roles of the fingers and thumb can be reversed: if you curl the fingers on your right hand in the way the current goes around the loop, your thumb, when you stick it out, shows the way the field line points inside the loop.

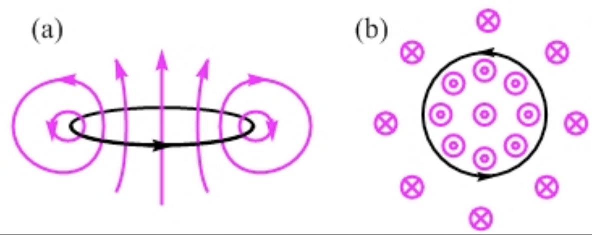


Figure 19.27: (a) A side view of the magnetic field from a current loop. (b) An overhead view of the same loop, showing the field in the plane of the loop.

The magnetic field from a current loop is similar to from a thin disk magnet that you might find on your fridge (as long as the north and south poles of the disk magnet are on opposite faces of the disk, which is generally the case). This similarity between the fields is no coincidence. The disk magnet is made from **ferromagnetic** material – ferromagnetic means having magnetic properties similar to that of iron. In any material, each electron in an atom has an associated angular momentum. In many materials these angular momenta either cancel out or are randomly aligned, giving rise to little or no magnetic field. In ferromagnetic materials, however, the angular momenta of neighboring atoms line up, producing a substantial magnetic field.

A model of a ferromagnetic material is shown in Figure 19.28. Each atom acts like a tiny current loop, with the loops carrying currents that circulate in the same direction. In the inner part of the magnet, nearby currents point in opposite directions and cancel one another out. Around the edge of the magnet, however, there is no cancellation, and the net effect of the currents at the outside is like that of a single current that goes all the way around the outer edge of the disk.

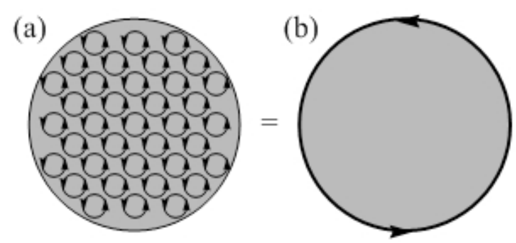


Figure 19.28: A model of a ferromagnetic material. Each atom acts like a tiny current loop, with neighboring current loops aligned with one another. Except around the outer edge of the magnet, nearby currents are directed in opposite directions, and cancel one another. The net effect is that the disk magnet produces a magnetic field similar to that from a current loop of the same radius as the disk.

The Magnetic Field from a Current-Carrying Coil

A cylindrical current-carrying coil, or **solenoid**, is like a stack of current loops. In the ideal case, the solenoid is infinitely long, producing a uniform magnetic field inside the solenoid and negligible field outside the solenoid. The solenoid is the magnetic equivalent of the parallel-plate capacitor – when both devices extend to infinity, the field produced (magnetic field for the solenoid, electric field for the capacitor) is uniform. Just as the electric field from the capacitor depends on the capacitor geometry, the magnetic field produced by the solenoid depends on the solenoid geometry. For a solenoid of length L, with a total of N turns (or $n = N/L$ turns per unit length), and carrying a current I, the magnitude of the magnetic field inside the solenoid is:

$$B = \frac{\mu_0 N I}{L} = \mu_0 n I \,.$$ (Equation 19.12: **The magnetic field for an ideal solenoid**)

As shown in Figure 19.29, the magnetic field in an ideal solenoid is parallel to the axis of the solenoid. If you curl the fingers of your right hand in the direction of the current, your thumb points in the direction of the field inside the solenoid.

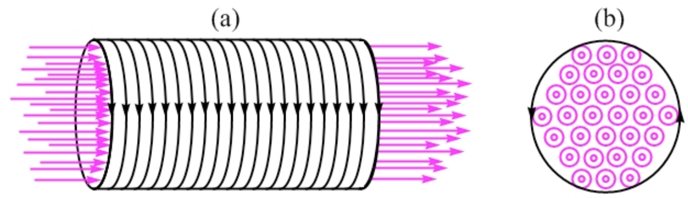


Figure 19.29: The magnetic field from an ideal (infinitely long) solenoid. A three-dimensional view is shown in (a), while (b) shows the field from the perspective of someone looking along the axis of the solenoid from the right.

Equation 19.12 applies to an ideal solenoid, of infinite length. Figure 19.30 shows that for a real solenoid, of finite length, the magnetic field is strongest in the center, and reduces in magnitude toward the ends as field lines leak out the sides of the solenoid. The field from a real solenoid has the same form as the field from a typical bar magnet. Like a disk magnet, the bar magnet can be modeled as a number of tiny current loops, all aligned, associated with the angular momentum of electrons in atoms. The net effect of these current loops is a current that circles the outside of the bar magnet, like the current in a solenoid.

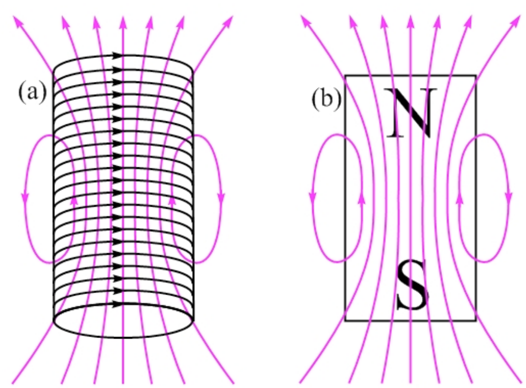


Figure 19.30: The magnetic field from a solenoid of finite length (a), has the same form as the field from a bar magnet (b).

The pictures in Figure 19.30 could be labeled (a) an electromagnet, and (b) a permanent magnet. In a permanent magnet, the magnetic field is always on. An electromagnet can be turned on or off as a current is turned on or off. An electromagnet made by connecting a coil of wire to a battery generally has a weak magnetic field. If a ferromagnetic core, like an iron nail, is placed in the coil, however, the magnetic field can be increased by a factor of several hundred. The core should consist of "soft" ferromagnetic material, as opposed to the "hard" material that permanent magnets are made from. In hard ferromagnetic materials, neighboring atoms remain aligned when an external magnetic field is removed. In soft materials, the alignment mostly disappears when the external field is removed, so the magnetic field turns off when the current turns off.

Related End-of-Chapter Exercises: 28, 29, 31, 32.

Essential Question 19.8: Starting at a point on the axis of a solenoid, which has 800 turns per meter, an electron is given an initial velocity of 500 m/s in a direction perpendicular to the axis. The electron takes 75 ns to go through a complete circle. What is the current in the solenoid?

Answer to Essential Question 19.8: This situation involves much of what we learned in this chapter. First, what is the connection between all this information? We can connect everything via the magnetic field. With the information about the electron, we can use equation 19.4 to solve for the magnetic field. Equation 19.4 gives the relation between the period of the circular orbit as: $T = 2\pi m/(qB)$. Solving for the magnetic field gives:

$$B = \frac{2\pi m}{qT} = \frac{2\pi\ (9.11\times10^{-31}\ \text{kg})}{(1.60\times10^{-19}\ \text{C})\,(75\times10^{-9}\ \text{s})} = 4.77\times10^{-4}\ \text{T}.$$

We can then use equation 19.12, $B = \mu_0 nI$, to solve for the current in the solenoid. This gives: $I = B/(\mu_0 n) = (4.77\times10^{-4}\ \text{T})/(4\pi\times10^{-7}\ \text{T m/A}\times800\ \text{m}^{-1})/ = 0.47\ \text{A}$.

Chapter Summary

Essential Idea: Magnetism and Magnetic Fields.

Magnetism and magnetic fields have many applications, ranging from the Earth's magnetic field, which we use for navigation and which protects us from harmful radiation, to the powerful magnetic field in an MRI machine, which helps doctors diagnose medical problems, to the basic magnets that hold pictures on a refrigerator.

Magnetic Field

- Magnetic fields are associated with two magnetic poles, north and south. Magnetic fields are produced by moving charges.
- Like magnetic poles repel, while unlike poles attract.
- The magnetic field points in the direction of the force experienced by a north pole.
- North and south poles always come in pairs.
- Magnetic field lines are continuous loops. Outside a magnet, the magnetic field is directed from the north pole to the south pole. Inside a magnet, the magnetic field runs from south to north.

The Magnetic Force on a Charged Particle Moving in a Magnetic Field

A charged particle moving through a magnetic field experiences a magnetic force that is perpendicular to both the magnetic field and the velocity of the particle:

$F_M = qvB\ \sin\theta$, (Eq. 19.2: **The magnitude of the magnetic force on a charge *q***)

where θ is the angle between the velocity, $\vec{v}$, and the magnetic field, $\vec{B}$. The direction of the force is perpendicular to the plane defined by $\vec{v}$ and $\vec{B}$, and given by the right-hand rule. The steps in the right-hand rule are:

- Point the fingers on your right hand in the direction of the charge's velocity.
- While keeping your fingers aligned with the velocity, rotate your hand so that, when you curl your fingers, you can curl them into the direction of the magnetic field.
- Hold out your thumb so it is perpendicular to your fingers. Your thumb points in the direction of the force experienced by a positively charged particle.
- If the particle has a negative charge, your right hand lies to you. Just reverse the direction of the force. The magnetic force on a negatively charged particle is opposite in direction to that of a positively charged particle if the particles are traveling in the same direction.

The path followed by a moving charge in a magnetic field

A charged particle moving parallel to a magnetic field experiences no force, and moves in a straight line. A charged particle moving perpendicular to a magnetic field experiences a force that makes it travel in a circle, with a radius and period of:

$r = \frac{mv}{qB}$, (Equation 19.3: **Radius of the path, when $\vec{v}$ and $\vec{B}$ are perpendicular**)

$T = \frac{2\pi m}{qB}$. (Equation 19.4: **The period of the circular orbit**)

If the velocity of the particle is neither parallel to, not perpendicular to, the magnetic field, the particle follows a spiral path, spiraling around the magnetic field.

Forces and torques on current-carrying wires and loops

The magnitude of the magnetic force exerted on a wire of length L, carrying a current I, by a magnetic field of magnitude B is:

$F_M = ILB\sin\theta$, (Equation 19.7: **The magnetic force on a current-carrying wire**)

where θ is the angle between the current direction and the magnetic field.

The direction of the force is given by a right-hand rule that follows the rule for charges. Point the fingers on your right hand in the direction of the current. Align your hand so that when you curl your fingers, they point in the direction of the magnetic field. Stick out your thumb, and it points in the direction of the magnetic force.

A loop of current in a uniform magnetic field experiences no net magnetic force. The magnitude of the magnetic torque on a coil consisting of N loops carrying a current I is:

$\tau = NIAB\sin\theta$, (Equation 19.8: **Torque on a current-carrying coil in a magnetic field**)

where θ is the angle between the area vector $\vec{A}$ of the loop, which is perpendicular to the plane of the loop, and the magnetic field $\vec{B}$.

Currents produce magnetic fields

Even in permanent magnets, the magnetic field comes from currents, being associated with the angular momentum of electrons in atoms, and the alignment of these angular momenta.

The magnetic field a distance r from a long straight wire carrying a current I is:

$B = \frac{\mu_0 I}{2\pi r}$. (Eq. 19.9: **The magnetic field from a long-straight wire**)

The **permeability of free space** has a value of $\mu_0 = 4\pi \times 10^{-7}$ T m / A .

$B = \frac{\mu_0 I}{2R}$. (Eq. 19.11: **The magnetic field at the center of a current loop of radius R**)

$B = \frac{\mu_0 NI}{L} = \mu_0 nI$, (Equation 19.12: **The magnetic field for an ideal solenoid**)

where N is the number of turns, L is the length of the solenoid, and $n = N/L$ is the number of turns per unit length.

End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions, designed to see whether you understand the main concepts of the chapter.

1. A charged particle is moving with a constant velocity directed up the page when it enters a region of uniform magnetic field. (a) If the particle is deflected into a circular path, but it remains in the plane of the page, what does this tells us about the magnetic field? (b) If the particle has a positive charge and it is deflected to the left when it enters the field, what does this tell us about the magnetic field?

2. What is the direction of the magnetic force experienced by the charged particle in each situation shown in Figure 19.31?

3. A negatively charged particle with an initial velocity directed east experiences a magnetic force directed north. (a) If the initial velocity is perpendicular to a uniform magnetic field, in which direction is the field? (b) If the initial velocity and the magnetic field are not necessarily perpendicular, what can you say about the direction of the field?

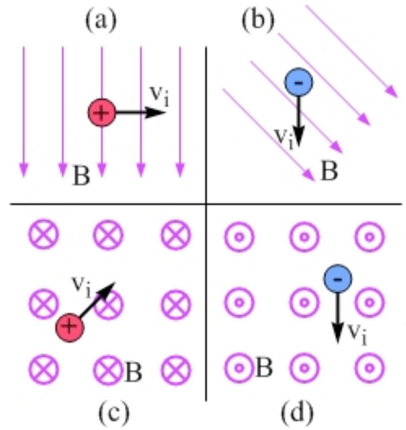


Figure 19.31: Four situations involving a charged particle in a uniform magnetic field, for Exercises 2 and 6.

4. As shown in Figure 19.32, a positively charged particle has an initial velocity directed straight up the page. The initial velocity is perpendicular to a uniform field in the region. (a) If the into-the-page symbol shows the direction of the magnetic force on the particle, in which direction is the magnetic field? (b) If, instead, the into-the-page symbol shows the direction of the magnetic field, in which direction is the force on the particle?

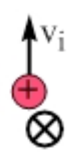


Figure 19.32: A positively charged particle has an initial velocity directed up. Either the magnetic field or the magnetic force is directed into the page. For Exercise 4.

5. Two vectors, which are clearly not perpendicular to one another, are shown in Figure 19.33. (a) Could these two vectors represent the velocity of a negatively charged particle (1) and the direction of a uniform magnetic field (2)? If not, explain why not. If so, in which direction is the magnetic force on the particle? (b) Could the two vectors represent the velocity of a positively charged particle (1), and the direction of the magnetic force acting on the particle (2)? If not, explain why not. If so, in which direction is the magnetic field that exerts the force on the particle?

Figure 19.33: Two vectors, for Exercise 5.

6. Return to the situations shown in Figure 19.31. If the particles have charges of the same magnitude, are in uniform fields of the same magnitude, and have the same initial speed, rank the four situations based on the magnitude of the force experienced by the particle.

7. Figure 19.34 shows the paths followed by three charged particles through a region of uniform magnetic field that is directed perpendicular to the page. If particle 1 has a positive charge, what is the sign of the charge of (a) particle 2? (b) particle 3? (c) In which direction is the magnetic field?

8. Return to the situation described in the previous exercise and shown in Figure 19.34. If the particles have the same mass, and their charges have the same magnitude, rank the particles based on (a) their speed, and (b) the magnitude of the magnetic force they each experience. (c) Which particle spends the most time in this square region? Explain.

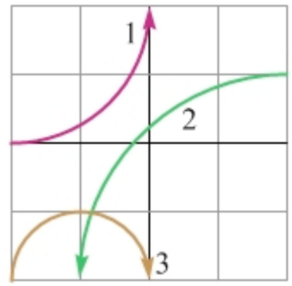


Figure 19.34: The paths followed by three charged particles through a region of uniform magnetic field directed perpendicular to the page, for Exercises 7 and 8.

9. The complete picture of field lines in a particular region is blocked by the square screen in Figure 19.35. (a) Could these field lines be electric field lines? If not, explain. If so, what is behind the screen? (b) Could these field lines be magnetic field lines? If not, explain. If so, what is behind the screen?

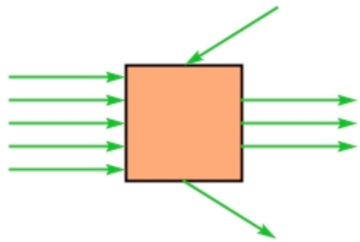


Figure 19.35: Our view of the field lines in a particular region is blocked by a square screen, for Exercise 9.

10. Consider the picture of magnetic field lines in a particular region, shown in Figure 19.36. (a) Points a, b, and c are located on the same field line. Rank these points based on the magnetic field at these points. (b) At which point, a or d, is the magnetic field larger in magnitude? Explain.

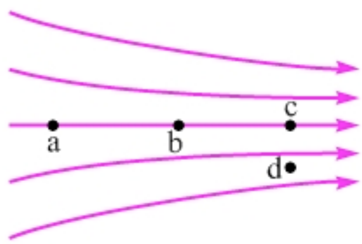


Figure 19.36: The magnetic field in a particular region, for Exercise 10.

11. Two long straight parallel wires are located so that one wire passes through each of the lower corners of an equilateral triangle, as shown in Figure 19.37. The wires carry currents that are directed perpendicular to the page. The net magnetic field at the top vertex of the triangle is also shown in the figure. What is the direction of the current in (a) wire 1? (b) wire 2? (c) Which wire carries more current?

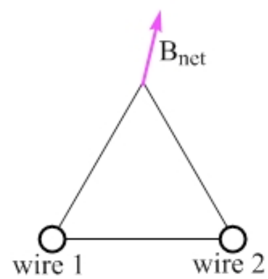


Figure 19.37: Two long straight current-carrying wires pass through the lower corners of an equilateral triangle. For Exercise 11.

12. Four long straight parallel wires pass through the corners of a square, as shown in Figure 19.38. The currents in each wire have the same magnitude, but their directions (either into or out of the page) are unknown. How many possible combinations of current directions are there if the net magnetic field due to these wires at the center of the square is (a) zero? (b) directed to the right? (c) directed toward the top left corner of the square. Draw the various configurations in each case.

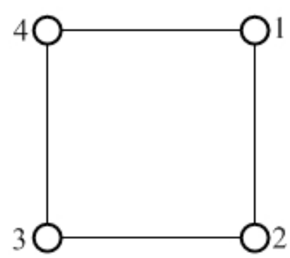


Figure 19.38: Long straight parallel current-carrying wires pass through the corners of a square, for Exercise 12.

Exercises 13 – 16 are designed to give you practice applying the equation for the magnetic force experienced by a charged particle in a magnetic field, $F_M = qvB \sin\theta$.

13. The particle in Figure 19.39 has a charge of 5.0 μC, a speed of 1.4×10^3 m/s, and it is in a uniform magnetic field, directed into the page, of 2.5×10^{-2} T. As the figure shows, the initial velocity of the particle is directed at 30° below the positive *x*-direction, assuming the *x*-direction is toward the right. What is the magnitude and direction of the magnetic force acting on the particle?

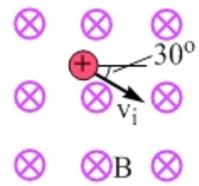


Figure 19.39: A particle with a positive charge has a velocity directed at 30° below the positive *x*-direction, in a magnetic field directed into the page. For Exercise 13.

14. A charged particle is in a region in which the only force acting on it comes from a uniform magnetic field. When the particle's initial velocity is directed in the positive *x*-direction (to the right), the particle experiences no magnetic force. When the particle's initial velocity is directed in the positive *y*-direction (up the page), the particle experiences a magnetic force in the positive *z*-direction (out of the page). (a) If the particle has a negative charge, in which direction is the magnetic field? In which direction is the magnetic force on the particle if the particle's initial velocity is directed in the (b) +*z*-direction? (c) –*z*-direction? (d) –*x*-direction? (e) –*y*-direction?

15. In a particular region, there is a uniform magnetic field with a magnitude of B = 2.0 T. You take a particle with a charge of +5.0 μC and give it an initial speed of 4.0×10^5 m/s in this field. (a) Under these conditions, what is the magnitude of the maximum magnetic force experienced by the particle? (b) What is the magnitude of the minimum magnetic force experienced by the particle? (c) If the particle experiences a force with a magnitude that is 25% of the magnitude of the maximum force, what is the angle between the particle's velocity and the magnetic field?

16. A particle with a charge of –5.0 μC travels in a circular path of radius 30 m at a speed of 15 m/s. The particle's path is circular because it is traveling in a region of uniform magnetic field. If the particle's mass is 25 μg, what is the magnitude of the magnetic field acting on the particle?

Exercises 17 – 19 involve charged particles moving in circular paths in uniform magnetic fields.

17. Figure 19.40 shows the paths followed by three charged particles through a region of uniform magnetic field that is directed perpendicular to the page. If the particles have the same speed, and their charges have the same magnitude, rank the particles based on (a) their mass, and (b) the magnitude of the magnetic force they each experience.

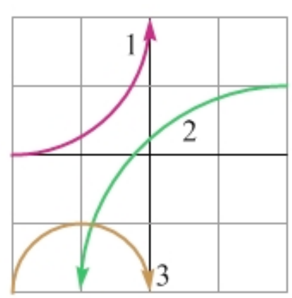


Figure 19.40: The paths followed by three charged particles through a region of uniform magnetic field directed perpendicular to the page, for Exercise 17.

18. Figure 19.41 shows the paths followed by three charged particles through a region of uniform magnetic field that is directed out of the page. The particles have the same magnitude charge. Complete table 19.3, filling in the six pieces of missing data.

Particle	Sign of Charge	Mass	Speed	Path taken
1		$2m$	$4v$	
2		$4m$		
3		m	$2v$	L

Table 19.3: The charge, mass, and speed for three particles passing through a magnetic field.

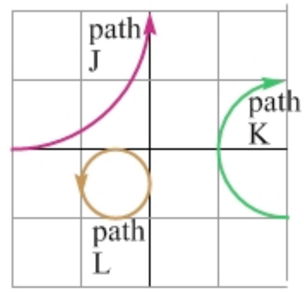


Figure 19.41: The paths of the three particles.

19. A particle with a charge of –8.0 μC and a mass of 9.0×10^{-9} kg is given an initial velocity of 5.0×10^4 m/s in the positive x-direction. The particle is in a uniform magnetic field, with $B = 2.0$ T, that is directed in the negative z-direction, and the particle starts from the origin. Assume the particle is affected only by the magnetic field. (a) How long after leaving the origin does the particle first return to the origin? (b) What is the maximum distance the particle gets from the origin? (c) Where is the particle when it achieves its maximum distance from the origin?

Exercises 20 – 24 deal with long straight wires in situations that are analogous to situations involving masses, from Chapter 8, and charged particles, from Chapter 16.

20. Figure 19.42 shows three long straight parallel wires that are carrying currents perpendicular to the page. Wire 1 experiences a force per unit length of 4 N/m to the left because of the combined effects of wires 2 and 3. Wire 2 experiences a force per unit length of 12 N/m to the right because of the combined effects of wires 1 and 3. Is there enough information given to find the force per unit length experienced by wire 3 because of the combined effects of wires 1 and 2? If so, find it. If not, explain why not.

Figure 19.42: Three long straight wires carry currents perpendicular to the page. The net forces per unit length on two of the wires are shown. For Exercise 20.

21. Figure 19.43 shows two long straight parallel wires that are separated by 12 cm. Wire 1, on the left, has a current of 3.0 A directed out of the page. Wire 2, on the right, has a current of 2.0 A directed into the page. (a) What is the magnitude and direction of the net magnetic field from these two wires at the point midway between them? (b) Are there any points a finite distance from the wires at which the net magnetic field from the wires is zero? If not, explain why not. If so, state the location of all such points.

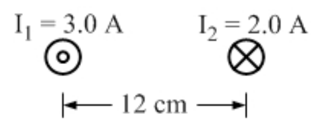


Figure 19.43: Two long straight parallel wires carry currents perpendicular to the page. For Exercise 21.

22. Three long straight parallel wires pass through three corners of a square measuring 20 cm on each side, as shown in Figure 19.44. Find the magnitude and direction of the net magnetic field at (a) the center of the square. (b) the top left corner of the square.

23. Three long straight parallel wires pass through three corners of a square measuring 20 cm on each side, as shown in Figure 19.44. Find the magnitude and direction of the magnetic force per unit length experienced by wire 2, in the bottom right corner, due to the other two wires.

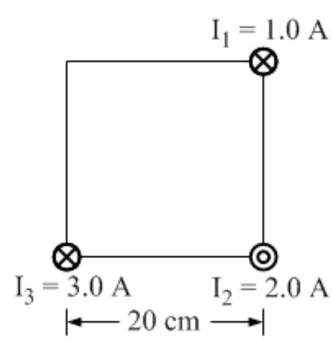


Figure 19.44: Three long straight current-carrying wires pass through three corners of a square. For Exercises 22 and 23.

Exercises 24 – 27 involve adding magnetic fields as vectors.

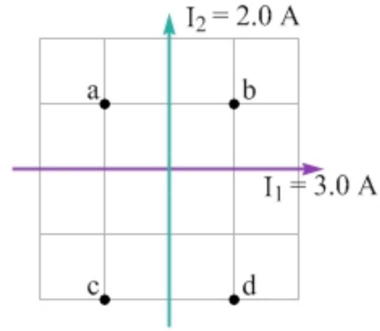


Figure 19.45: Two long straight perpendicular wires carry currents in the plane of the page. For Exercises 24 and 25.

24. Two long straight wires are in the plane of the page, and are perpendicular to one another. As shown in Figure 19.45, wire 2 is placed on top of wire 1, but the wires are electrically insulated from one another. Wire 1 carries a current of 3.0 A to the right, while wire 2 carries a current of 2.0 A up. Rank the four labeled points based on the magnitude of the net magnetic field due to the wires, from largest to smallest.

25. Return to the situation described in Exercise 24, and shown in Figure 19.45. Each of the four points is 20 cm from wire 2. Points a and b are also 20 cm from wire 1, while points c and d are 40 cm from wire 1. Determine the magnitude and direction of the net magnetic field, due to the two wires, at (a) point a, (b) point b, (c) point c, and (d) point d.

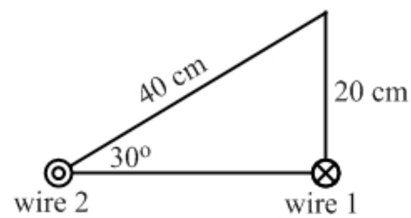


Figure 19.46: Two long straight parallel wires, carrying currents in opposite directions, pass through two corners of a right-angled triangle, for Exercises 26 and 27.

26. Two long straight parallel wires pass through two corners of a right-angled triangle, as shown in Figure 19.46. The currents in the wires are directed in opposite directions. Wire 1 produces a magnetic field of 4.0×10^{-5} T at the unoccupied corner of the triangle, which is 20 cm from wire 1. If the current in wire 2 has the same magnitude as that in wire 1, determine the magnitude and direction of the net magnetic field from the two wires at the unoccupied corner of the triangle.

27. Return to the situation described in Exercise 26, and shown in Figure 19.46. Now the magnitude of the current in wire 2 is adjusted so that the net magnetic field at the unoccupied corner of the square, due to the wires, is directed straight up. Find the ratio of the magnitude of the current in wire 2 to that in wire 1.

Exercises 28 – 32 deal with current-carrying loops and solenoids.

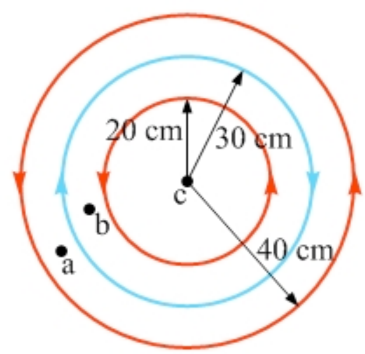


Figure 19.47: Three concentric loops, for Exercise 28, or a cross-section through three concentric solenoids, for Exercise 29.

28. Figure 19.47 shows three concentric current-carrying loops. The inner loop, with a radius of 20 cm, carries a current of 2.0 A counterclockwise. The middle loop, with a radius of 30 cm, carries a current of 3.0 A clockwise. The outer loop, with a radius of 40 cm, carries a current of 4.0 A counterclockwise. Determine the magnitude and direction of the net magnetic field at point c, the center of each loop, due to the loops.

29. Return to the situation described in Exercise 28, but now Figure 19.47 represents a cross-sectional slice through a set of three long current-carrying solenoids. (a) Assuming the solenoids are ideal and have the same number of turns per unit length, and using the currents specified in the previous exercise, rank points a, b, and c based on the magnitude of the net magnetic field at their location. (b) Determine the magnitude and direction of the net magnetic field at point b, if each solenoid has 800 turns/m.

30. As shown in Figure 19.48, a long straight wire, carrying a current I up the page, lies in the plane of a loop. The long straight wire is 40 cm from the center of the loop, which has a radius of 20 cm. Initially, the magnetic field at the center of the loop is due only to the current in the long straight wire. When a current in the loop is turned on, however, the net field at the center of the loop has a magnitude two times larger than the field at that point from the long straight wire. What is the magnitude (in terms of I) and direction of the current in the loop? Find all possible solutions.

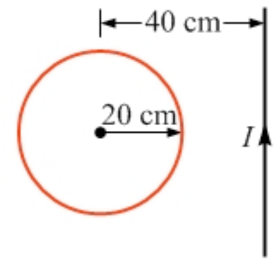


Figure 19.48: A long straight current-carrying wire passes 40 cm from the center of a loop that has a radius of 20 cm, for Exercise 30.

31. A particular ideal solenoid has 1200 turns/m, a radius of 10 cm, and a current of 4.0 A. An electron is fired from the axis of the solenoid in a direction perpendicular to the axis. What is the maximum speed the electron can have if it is not to run into the side of the solenoid?

32. A very strong neodymium-iron-boron (NdFeB) magnet with a magnetic field of around 0.1 T can be purchased for a few dollars. (a) Compare this magnetic field to the field you can get by making your own electromagnet, assuming that you wind some wire at 800 turns/m around an aluminum nail, and the current in the coil is 1.0 A. (b) If you wind the wire around an iron or steel nail instead, you can amplify the magnetic field by a factor of several hundred. How does this field compare to the field from the NdFeB magnet? (c) Name a key advantage that your electromagnet offers over the NdFeB magnet.

General problems and conceptual questions

33. Consider the photograph of two magnets, shown on the opening page of this chapter. (a) Are the magnets attracting one another or repelling? Explain. (b) Can you say which end of the magnet on the right is a north pole and which end is a south pole? Explain.

34. The SI unit of the tesla is named after a very interesting person. Do some research on this individual, and write two or three paragraphs describing his/her contributions to science.

35. We know quite a lot about the historical behavior of the Earth's magnetic field, through a field of study called **paleomagnetism**. Investigate the methods used by researchers in this field, and write 2-3 paragraphs about these methods.

36. What is the predominant direction of the Earth's magnetic field if you are standing on the surface of the Earth (a) at a point on the Earth's equator? (b) in northern Canada, directly above one of the Earth's magnetic poles?

37. The northern lights (*aurora borealis*) and southern lights (*aurora australis*) are colorful displays of light associated with fast-moving charged particles entering the Earth's atmosphere. (a) Recalling that moving charges tend to spiral around magnetic field lines, explain why these light shows are generally confined to Earth's polar regions. (b) In the event that a positively charged particle enters Earth's atmosphere above the equator, with a velocity directed straight down toward the ground, in which direction is the particle initially deflected by the Earth's magnetic field?

38. A charged particle is passing through a particular region of space at constant velocity. One possible explanation for the fact that the particle is undeflected while it is in this region is that there is neither an electric field nor a magnetic field present. Let's consider other possibilities. Explain your answers to the following. (a) Could there be a uniform electric field in the region, but no magnetic field? (b) Could there be a uniform magnetic field in the region, but no electric field? (c) Could there be both a uniform electric field and a uniform magnetic field present?

39. An electron is in a uniform magnetic field. Assume that the electron interacts only with the magnetic field, and that all other interactions, including gravity, can be neglected. 2.0 s after being released, what is the electron's speed if it is (a) released from rest? (b) released with a velocity of 800 m/s in a direction parallel to the magnetic field? (c) released with a velocity of 800 m/s in a direction perpendicular to the magnetic field.

40. Figure 19.49 shows, to scale, the initial velocities of four identical charged particles in a uniform magnetic field. Rank the particles based on the magnitude of the magnetic force they experience from the field if the field is directed (a) up the page (b) to the right (c) out of the page.

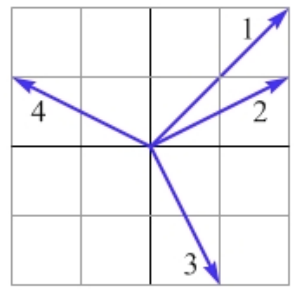


Figure 19.49: The initial velocities of four identical charged particles in a uniform magnetic field, for Exercise 40.

41. When a particle with a charge of +3.0 μC and a mass of 5.0×10^{-8} kg passes through the origin, its velocity components are $\vec{v}_x = +3.0\times10^4$ m/s in the x-direction and $\vec{v}_y = +4.0\times10^4$ m/s in the y-direction. The particle is traveling through a region of uniform magnetic field which is directed in the negative x-direction, with a magnitude of 2.0×10^{-2} T. The particle follows a spiral path in the field. (a) What is the radius of the spiral path, and in which direction does the axis of the spiral point? (b) How long does it take the particle to make one complete loop on the spiral? (c) How far from the origin is the particle when it has made one complete loop on the spiral?

42. Return to the situation described in Exercise 42, but now we will add a uniform electric field of 1000 N/C in the +x-direction, parallel to the magnetic field. (a) With the addition of the electric field, which of the answers to the previous exercise would change and which would stay the same? Explain. (b) Re-calculate all the answers to the previous exercises that are changed by the addition of the electric field.

43. Four particles pass through a square region of uniform magnetic field, as shown in Figure 19.50. The magnetic field inside the square region is perpendicular to the page, and the field outside the region is zero. The paths followed by particles 1 and 2 are shown; while for particles 3 and 4 the direction of their velocities and the points at which they enter the region are shown. The paths of all particles lie in the plane of the paper. Particle 1 has a mass m, a speed v, and a positive charge $+q$. Assume that the only thing acting on the particles as they move through the magnetic field is the field. (a) In which direction is the uniform magnetic field in the square region? (b) As particle 1 moves through the field, does its kinetic energy increase, decrease, or stay the same? Explain. (c) What is the sign of particle 2's charge? (d) If particle 2's mass is m and its charge has a magnitude of q, what is its speed? (e) Particle 3 has a mass $2m$, a speed $2v$, and a charge $-2q$. Re-draw Figure 19.50, and sketch on this diagram, as precisely as you can, the path followed by particle 3 through the region of magnetic field. (f) Particle 4 has a mass $2m$, a speed $2v$, and no charge. Sketch its path through the region of magnetic field. (g) Which particle feels the largest magnitude force as it passes through the field?

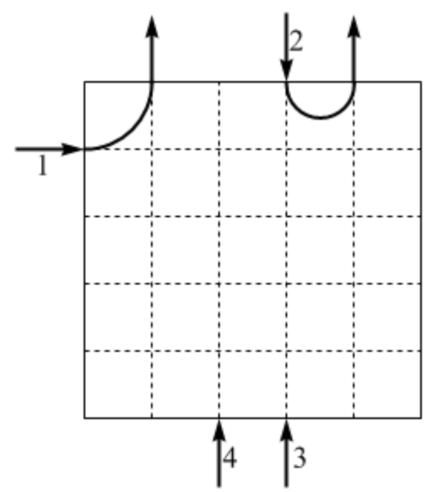


Figure 19.50: Four particles pass through a square region of uniform magnetic field directed perpendicular to the page. Only the paths followed by particles 1 and 2 are shown. For Exercise 43.

44. Two particles are sent into a square region in which there is a uniform magnetic field directed into the page, as shown in Figure 19.51. There is no magnetic field outside of the region. The velocity of the particles is perpendicular to the magnetic field at all times. Particle 1 is sent through the field twice, entering at the same point both times. The first time it has an initial speed v and the second time it has an initial speed of $2v$. (a) Which path, P or Q, corresponds to when the initial speed of particle 1 is $2v$? (b) For which path is the magnitude of the force experienced by particle 1 larger? (c) Is the charge on particle 1 positive or negative? (d) Particle 2 has the same mass and the same magnitude charge as particle 1, but the sign of its charge is opposite to that of particle 1. Particle 2 has an initial speed of v when it enters the field. Re-draw the diagram and sketch the path followed by particle 2.

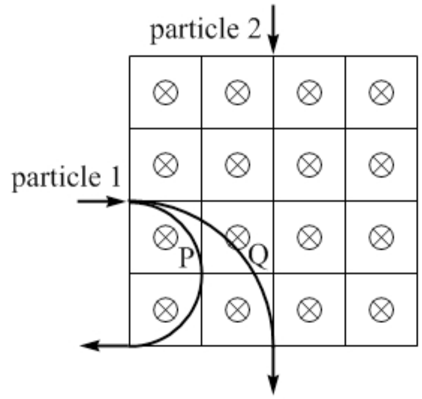


Figure 19.51: Two particles pass through a square region of uniform magnetic field directed into the page. Particle 1 is sent through the field twice, following path *P* on one occasion and path *Q* on the other. For Exercise 44.

45. The velocity of the electrons in an electron beam is 1.0×10^5 m/s directed right. The electrons pass through a velocity selector without being deflected in any way, as shown in Figure 19.52. The velocity selector consists of a set of parallel plates with a uniform electric field with $\vec{E} = 2000\ V/m$ directed down, and a uniform magnetic field directed perpendicular to the velocity of the electrons. (a) In which direction is the magnetic field? (b) What is the magnitude of the magnetic field inside the velocity selector? (c) What will happen to electrons traveling faster than those that are undeflected?

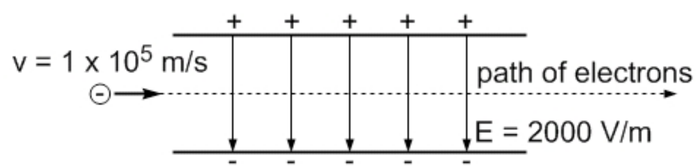


Figure 19.52: Electrons with just the right speed pass undeflected through a velocity selector, for Exercise 45.

46. A charged particle enters a velocity selector with a speed of 2.0×10^4 m/s. A graph of the net force acting on the particle when it enters the velocity selector, as a function of the magnetic field in the velocity selector, is shown in Figure 19.53. A positive net force means that the particle is deflected up, while a negative net force means that it is deflected down. The geometry of the velocity selector is similar to that shown in Figure 19.52, with the initial velocity of the particle directed parallel to the plates of the velocity selector, and perpendicular to both the electric field and the magnetic field, which are perpendicular to one another. Using the graph, determine (a) the magnitude of the electric field in the velocity selector, and (b) the magnitude of the charge on the particle.

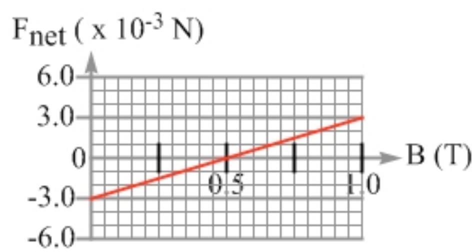


Figure 19.53: A graph of the net force acting on a charged particle when it enters a velocity selector, as a function of the magnetic field in the velocity selector, for Exercise 46.

47. You have a sample of chlorine, and you are trying to determine what fraction of your sample consists of chlorine-35 atoms (with an atomic mass of about 35 atomic mass units) and what fraction is made up of chlorine-37 atoms (with an atomic mass of about 37 atomic mass units). To do this, you use a mass spectrometer, as described in section 19–4, ionizing the atoms so that each atom is singly-ionized, with a charge of $+e$. (a) In

the velocity selector, you use a uniform electric field of 2000 N/C, and a uniform magnetic field of 5.0×10^{-2} T. These two fields are mutually perpendicular, and are each perpendicular to the velocity of the chlorine ions when they enter the velocity selector. What is the speed of chlorine ions that pass undeflected through the velocity selector? (b) The undeflected ions continue to the uniform magnetic field in the mass separator. If you want the ions of different mass to be separated by 1.0 mm after going through a half-circle in the mass separator, to what value should you set the magnitude of the magnetic field in the mass separator?

48. A graph of the magnetic field from a long straight wire, as a function of the inverse of the distance from the wire, is shown in Figure 19.54. What is the magnitude of the current in the wire?

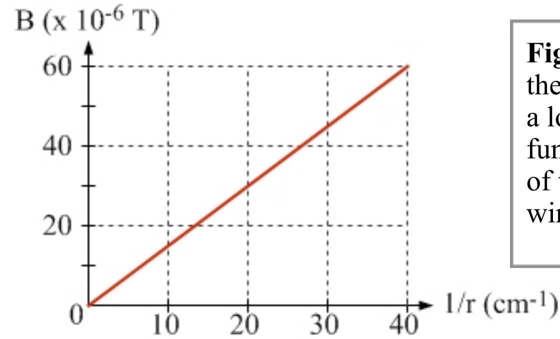


Figure 19.54: A graph of the magnetic field from a long straight wire, as a function of the inverse of the distance from the wire, for Exercise 48.

49. Three long straight parallel wires are placed in a line, with 40 cm between neighboring wires, as shown in Figure 19.53. Wires 1 and 2 carry currents directed into the page, while wire 3 carries a current directed out of the page. The currents all have the same magnitude. (a) Rank the three wires based on the magnitude of the net force per unit length experienced by each wire because of the other two. (b) If the current in each wire is 4.0 A, confirm your ranking from part (a) by determining the magnitude and direction of the force experienced by each wire.

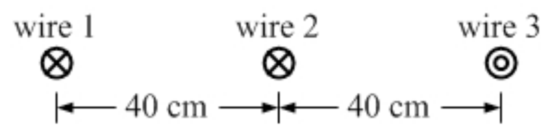


Figure 19.55: Three long straight parallel current-carrying wires, for Exercises 49 and 50.

50. Three long straight parallel wires are placed in a line, with 40 cm between neighboring wires, as shown in Figure 19.55. Wires 1 and 2 carry currents directed into the page, while wire 3 carries a current directed out of the page. Wire 1 experiences no net force due to the other two wires. (a) How does the magnitude of the current in wire 3 compare to that in wire 2? (b) How does the net force per unit length experienced by wire 3 compare to that of wire 2?

51. Two points, a and b, are 5.0 m apart. There is a uniform magnetic field of 2.0 T present that is directed parallel to the line connecting a and b. How much force does the field exert on a wire carrying 3.0 A of current from a to b, if the wire is (a) 5 m long? (b) 10 m long, and follows a circuitous path from a to b?

52. Repeat Exercise 51, but now the magnetic field is directed perpendicular to the line connecting a and b.

53. A rectangular loop with a current of 2.0 A directed clockwise measures 30 cm long by 10 cm wide, as shown in Figure 19.56. The loop is placed 10 cm from a long straight wire that carries a current of 3.0 A to the right. The goal of this exercise is to determine the magnitude and direction of the net magnetic force exerted on the loop by the long straight wire. (a) In section 19-5, we discussed the fact that a current-carrying loop in a uniform magnetic field experiences no net force. Why does the loop in this situation experience a non-zero net force? (b) Draw a diagram showing the force experienced by

each side of the loop. (c) Explain why the forces on the left and right sides of the loop are difficult to calculate, but why we do not need to calculate them to find the net force on the loop. (d) Find the force exerted on the upper side of the loop by the long straight wire. (e) Find the force exerted on the lower side of the loop by the long straight wire. (f) Combine your previous results to find the net force acting on the loop.

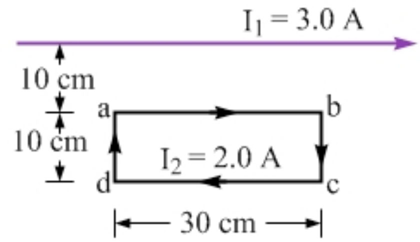


Figure 19.56: A current-carrying loop near a current-carrying long straight wire, for Exercises 53 and 54.

54. Return to the situation described in Exercise 53, and shown in Figure 19.56. Does the rectangular loop experience a net torque, about an axis that passes through the center of the loop, because of the magnetic field from the long straight wire? Explain.

55. Two very long straight wires carry currents perpendicular to the page, as shown in Figure 19.57. The x-axis is in the plane of the page. Wire 1, which carries a current I_1 into the page, passes through the x-axis at $x = +a$. Wire 2, located at $x = -2a$, carries an unknown current. The net field at the origin ($x = 0$), due to the current-carrying wires, has a magnitude of $B = 2\mu_0 I_1/(2\pi a)$. What is the magnitude and direction of the current in wire 2? Find all possible solutions.

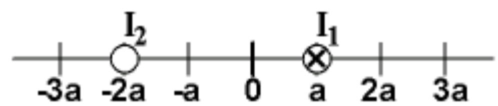


Figure 19.57: Two long straight parallel wires pass through $x = +a$ and $x = -2a$, respectively. For Exercise 55.

56. Two long straight wires, that are parallel to the y-axis, pass through the x-axis. One wire carries a current of $2I$ in the $+y$-direction, and passes through the x-axis at the point $x = -d$. The second wire carries an unknown current, and passes through an unknown location on the positive x-axis. The net magnetic field at all points on the y-axis is zero due to the wires, and the force per unit length experienced by each wire has a magnitude of $3\mu_0 I^2/(2\pi d)$. Find the current in the second wire, and determine at which point it passes through the x-axis.

57. Three equally spaced long straight wires are arranged in a line, as shown in Figure 19.58. The currents in the wires are as follows: wire 1 carries a current I into the page; wire 2 carries a current of $2I$ into the page; wire 3 carries a current of $3I$ out of the page. (a) Rank the three wires based on the magnitude of the net force per unit length they experience, from largest to smallest. (b) In which direction is the net force experienced by wire 1 due to the other two wires?

Figure 19.58: Three long straight parallel wires carry currents perpendicular to the page, for Exercise 57.

58. As shown in Figure 19.59, a long straight wire, supported by light strings tied to the wire at regular intervals, hangs at equilibrium at 30° from the vertical in a uniform magnetic field directed down. The current is directed perpendicular to the page. The wire has a mass per unit length of 0.12 kg/m, and the magnetic field has a magnitude of 0.36 T. What is the magnitude and direction of the current in the wire?

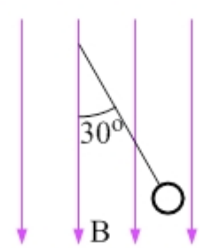


Figure 19.59: A long straight current-carrying wire, supported by strings, is in equilibrium at an angle of 30° from the vertical. For Exercise 58.

59. A rectangular wire loop measures 4.0 cm wide by 8.0 cm long. The loop carries a current of 5.0 A. The loop is in a uniform magnetic field with $B = 2.5 \times 10^{-3}$ T. What is the magnitude of the torque exerted by the field on the loop if the direction of the magnetic field is (a) parallel to the short sides of the loop? (b) parallel to the long sides of the loop? (c) perpendicular to the plane of the loop? In each case, take torques about an axis that maximizes the torque from the field.

60. Return to the situation described in Exercise 59. Rank the three cases, (a), (b), and (c), based on the magnitude of the angular acceleration experienced by the loop, assuming the loop is made from a uniform wire.

61. Return to the situation described in Exercise 59. If each centimeter of the wire loop has a mass of 5.0 grams, determine the angular acceleration of the loop in the situation when the magnetic field is parallel to the short sides of the loop.

62. An electron with a speed of 4.0×10^5 m/s travels in a circular path in the Earth's magnetic field. (a) Assuming that the magnitude of the Earth's field is 5.0×10^{-5} T, what is the radius of the electron's path? (b) Is it reasonable to neglect the force of gravity acting on the electron in this situation? Justify your answer. (c) An electron moving in a circle acts like a current loop, producing a magnetic field of its own. How large is the magnetic field at the center of the circular path, arising from the electron's motion?

63. Comment on each statement made by two students, who are discussing a situation in which a charged object experiences uniform circular motion because it is moving in a uniform magnetic field.

Kailey: The problem says that the object travels at constant speed in a circular path – the motion is confined to a plane. That tells us that the field must be directed perpendicular to that plane, right?

Isaac: I agree with you. Then it says, how does the radius of the path followed by the first object compare to that followed by the second object. Everything about the two objects is the same except that the second one has twice the speed as the first. Well, the force is proportional to the speed, so, if you increase the force, the radius must decrease.

Kailey: I'm not sure about that. Even if the force was the same on the two particles, if you change the speed the faster one is going to travel in a larger circle. That, by itself, would suggest that the radius goes up. Which effect wins, or do they balance?