Predicting oil recovery using percolation theory

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ABSTRACT: In this paper we apply scaling laws from percolation theory to the problem of estimating the time for a fluid injected into an oil field to breakthrough into a production well. The main contribution is to show that, when these previously published results are used on realistic data, they are in good agreement with results calculated in a more conventional way but they can be obtained significantly more quickly. As a result they may be used in practical engineering circumstances and aid decision-making for real field problems.

KEYWORDS: *percolation, breakthrough, geostatistical analysis*

INTRODUCTION

Oil reservoirs are extremely complex, containing geological heterogeneities on all length scales which have a significant impact on hydrocarbon recovery. The conventional approach to estimating recovery is to build a detailed geological model, populate it with flow properties, upscale it and then perform a flow simulation. In order to estimate the uncertainty in production a number of possible geological realizations are constructed (with associated probabilities) and this procedure repeated many times. A simple order of magnitude estimate of computing times (given today's model sizes and computing speeds) indicates that this could take many hundreds of days. Clearly this is completely impractical for many purposes.

Given this practical limitation, a number of steps have been taken, for example improved upscaling methods (King 1989; King *et al.* 1993), fast simulation (Bratvedt *et al.* 1992; Thiele *et al.* 1995*a*, *b*) and so on. In this paper we adopt a different approach. We simplify the geological model and flow physics such that quasi-analytical predictions of uncertainty can be made extremely quickly. The advantage is that the effects of the complex geometry which influence the flow can be readily estimated in fractions of a second on a spreadsheet. Clearly the disadvantage is that much of the flow physics and subtleties of the heterogeneity distribution are missed. Whilst it is the aim of future research to address those issues we show, in this paper, that this simple model can already give reasonable estimates of the production performance when applied to a real dataset.

We start by simplifying the rock heterogeneity by assuming that the permeability can be split into 'good' rock (i.e. finite, non-zero permeability) and 'poor' rock (low or zero permeability). For all practical purposes the flow takes place just in the good rock. For example, the good rock may be ancient river channels that are productive sandbodies with lengths of tens of kilometres, widths tens to hundreds of metres, and thicknesses up to tens of metres. It is the interconnectivity of these channels that controls the flow. The spatial distribution of the sand is governed by the geological process but can frequently be considered as independent or of a short range correlation. Hence, the problem of the connectivity of the sandbodies is precisely a continuum percolation problem. The place of the occupancy probability p of percolation theory is taken by the volume fraction of good sand (the net to gross ratio). This percolation view of sandbody connectivity has been used before (King 1990) but here we look not just at the static connectivity but also at the dynamic displacement on this percolating system.

The second simplification is of the flow physics. Here we assume that the displacement is like passive tracer transport. In other words we have single phase flow from injector to producer (we only consider a single well pair) and we assume that the injected fluid is passively convected along these streamlines. More complex displacement mechanisms along the streamlines could be imagined and handled in a similar way that new reservoir streamline simulation techniques are being developed.

It is recognized that these are gross simplifications. However, we shall be using scaling laws derived from percolation theory (Havlin & Ben-Avrahim 1987) to determine production performance and its associated uncertainty. We shall use the principle of universality that the large-scale behaviour of the percolating system is independent of the fine-scale details (e.g. precise shape of the sandbodies). To be specific we shall consider the time to breakthrough as the measure of performance (post-breakthrough behaviour is being considered in future research).

Percolation theory is an extremely successful model of connectivity and dynamics in complex geometries (Stauffer & Aharony 1994). The simplest model is to consider a lattice of sites which are occupied with a probability p or not with probability $1-p$. Clusters are formed when neighbouring sites are occupied. As this occupancy probability is increased, the clusters grow in size and merge until, at a critical value (the percolation threshold, p_c), one large cluster completely dominates the connectivity (there are also other small clusters which

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get absorbed as p further increases). Around this threshold value all the properties (number of clusters, size distribution of clusters etc.) are controlled by universal scaling (power) laws. They are universal in the sense that the values of the exponents are independent of the details of the model, such as the lattice type (square, triangular etc.) or even if there is a lattice at all (continuum percolation). These exponents only depend on the dimension of space. The percolation threshold does depend on the nature of the lattice but it has been calculated and well documented for all the cases we will consider. More details are given in Havlin & Ben-Avrahim (1987) and Stauffer & Aharony (1994).

FLOW MODEL

To simplify the model we shall assume that the permeability is either zero (shale) or one (sand). The sandbodies are cuboidal (and in the first instance isotropic in shape). They are distributed independently and randomly in space to a volume fraction of p. Further, we shall assume that the displacing fluid has the same viscosity and density as the displaced fluid. This has the advantage that, as the injected fluid displaces the oil, the pressure field is unchanged. This pressure field is determined by the solution of the single phase flow equations ($\nabla . K \nabla P = 0$). The injected flow then just follows the streamlines (normals to the isobars) of this flow. In dimensionless units the permeability (K) is either zero or one as described before. The boundary conditions have fixed pressure of +1 at the injection well and 0 at the production well. In this work we shall only consider a single well pair separated by a Euclidean distance r. The breakthrough time then corresponds to the first passage time for transport between the injector and the producer.

For a given model of the reservoir we can then sample for different realizations of the locations of the wells (or equivalently for the same well locations for different models of the reservoir with the same underlying statistics) and plot the distribution of breakthrough times. This is the conditional probability that the breakthrough time is t_{br}, given that the reservoir size (measured in dimensionless units of sandbody length) is L and the net to gross is p, i.e. $\mathrm{P}(t_{\mathrm{br}} | r, L, p)$. In previous studies (Dokholyan *et al.* 1998; Lee *et al.* 1999) we have shown that this distribution obeys the following scaling

$$\mathrm{P}(t_{\mathrm{br}} | r, L, p) \sim \frac{1}{r^{d_t}} \left(\frac{t_{\mathrm{br}}}{r^{d_t}} \right)^{-g_t} f_1 \left(\frac{t_{\mathrm{br}}}{r^{d_t}} \right) f_2 \left(\frac{t_{\mathrm{br}}}{L^{d_t}} \right) f_3 \left(\frac{t_{\mathrm{br}}}{\xi^{d_t}} \right) \tag{1}$$

$$f_1(\mathrm{x}) = \exp(-\mathrm{a}\,\mathrm{x}^{-\phi})$$
$$f_2(\mathrm{x}) = \exp(-\mathrm{b}\,\mathrm{x}^{-\psi})$$
$$f_3(\mathrm{x}) = \exp(-\mathrm{c}\,\mathrm{x}^{-\pi})$$

Currently the best estimates of the various coefficients and powers (as found from detailed numerical experiments on lattices and theory, see Andrade *et al.* 2000) in this are:

$d_{\mathrm{t}} = 1.33 \pm 0.05$; $g_{\mathrm{t}} = 1.90 \pm 0.03$; $a = 1.1$; $b = 5.0$; $c = 1.6 (p < p_c)$ $2.6 (p > p_c)$
$\varphi = 3.0$; $\psi = 3.0$; $\pi = 1.0$ and $\xi = |p - p_{\mathrm{c}}|^{-\nu}$ $\nu = 4/3$; $p_{\mathrm{c}} = 0.668 \pm 0.003$ (for continuum percolation).

In this paper we will not discuss the background to this scaling relationship but concentrate on how well it succeeds in predicting the breakthrough time for a realistic permeability field. However, it is worth spending some time describing the motivation behind the form of the various functions. The first expression (f_1) is an extension to the expression developed by others (see Havlin & Ben-Avrahim (1987) for a detailed discussion) for the shortest path length in a percolating cluster between two points. One can think in the following way (this is not a derivation but a description of what is happening). In a homogeneous system ($p=1$) the streamlines are simple lines or curves between the points (that is they have dimension 1). In a disordered system the heterogeneities make the streamlines more 'disordered' rather like a random walk, but they cannot cross and, as such, they are more like a self-avoiding random walk. The probability distribution function for the end-to-end distance of a simple random walk are known to be Gaussian. De Gennes (1979) postulated that for a self-avoiding walk the statistics should be given by a 'stretched Gaussian distribution' similar to f_1. We have simply assumed that this functional form also applies to the length shortest streamline and thereby to the shortest transit time with its own parameters that need to be determined by numerical experiment.

To this there are some corrections for real systems. One is related to the size of the system. In a finite size system very large excursions of the streamlines are not permitted by the boundaries so there is a maximum length permitted (and also a maximum to the minimum transit time). This cut-off is given by the expression f_2. Also away from the percolation threshold the clusters of connected bodies have a 'typical' size (given by ξ) which also truncates the excursion of the streamlines. This leads to the cut-off given by the expression f_3. The multiplication together of these three expressions is an assumption that has been tested by Dokholyan *et al.* (1998). A more detailed derivation of this form is given there and the references therein. Here we shall concentrate on using this scaling form to make predictions about the distribution of breakthrough times for a realistic dataset.

APPLICATION TO REAL FIELD DATA

Before we can apply this result to real field data we must convert the units from the dimensionless scaling form given above to real field units. This is not as obvious to do as it may appear because of the non-linear relationship between the various properties (e.g. time and distance) that enters into the scaling law. The first thing to note is that by a relationship like $t \sim r^{\alpha}$ one really means $t/t_{\mathrm{o}} = (r/r_{\mathrm{o}})^{\alpha}$, where t_{o} and r_{o} are 'typical' times and lengths. The typical length is easy to estimate. It should be just the sandbody dimension, r_{s}, in which case the typical time should be the time taken to transit one sandbody. Now, it is easy to show (Andrade *et al.* 2000) that for a pair of wells separated by a distance r (measured in centimetres), in a homogeneous region of permeability K (measured in Darcies) that the transit time (measured in seconds) is given by

$$t = \frac{4 \eta u r^2}{3 K \Delta P} \tag{2}$$

where η is the viscosity of the fluid (in centipoise), $u = ln(r/r_{\mathrm{w}})$ and r_{w} is the wellbore radius (in centimetres) and ΔP is the pressure drop between the wells (in atmospheres). So for each sandbody this is the typical transit time except that we need to know the pressure drop across each body. Here we shall take a very simplistic view that the total pressure drop between the wells is linear in the number of sandbodies between the wells. So that the pressure drop for each body becomes $\Delta P/(r/r_{\mathrm{s}})$. This analysis is rather simple minded in that it ignores the overlaps between the sandbodies, which would reduce this time. However, this should just affect the coefficient in front of the above formula. To determine this we carried out a number of numerical simulations to calibrate this form. Having done this we found that the typical transit time was

$$t_0 = \frac{12 \eta u r_{\mathrm{s}}^2}{15 K \Delta P / (r/r_{\mathrm{s}})} \tag{3}$$

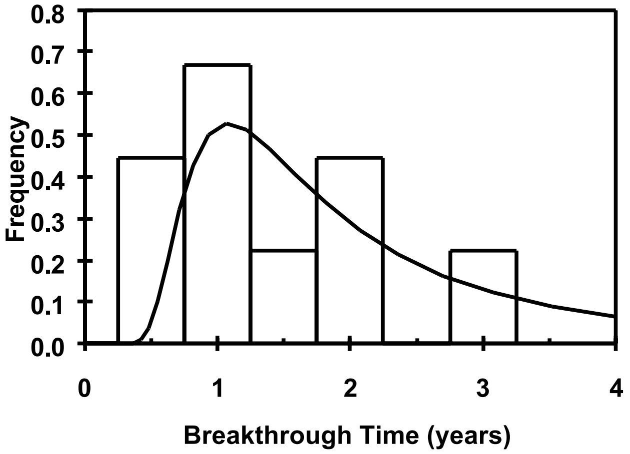

Fig. 1. Comparison of probability distribution of breakthrough times for the example reservoir obtained from percolation theory (smooth curve) and from full field model (histogram).

where *u* here is the logarithm of the dimensionless sandbody size.

We took as an example a deep water turbidite reservoir. The field is approximately 10 km long by 1.5 km wide by 150 m thick. The turbidite channels (which make up most of the net pay in the reservoir) are typically 8 km long by 200 m wide by 15 m thick. These channels have their long axes aligned with the long axis of the reservoir. The net to gross ratio is 50%. The typical well spacing was around 1.5 km either aligned or perpendicular to the long axis of the field. In order to account for the anisotropy in the shape of the sandbodies and the field we first make all length units dimensionless by scaling with the dimension of the sandbody in the appropriate direction (so the field dimensions are then L_x, L_y and L_z in the appropriate directions). Then the scaling law, Equation (1), can be applied with the minimum of these three values ($L=\min(L_x, L_y, L_z)$). Similarly for the well spacing.

Although the real field is rather more complex than this, and a more realistic reservoir description was made and put into a conventional simulator, we assumed (for the purposes of comparison with the percolation model) that the reservoir could be modelled with rectangular 'boxes' of the dimensions given above. We could then enter these dimensions into the scaling formula, Equation (1) (additionally one needs the injection pressure of 340 bar, the production pressure of 260 bar, a sand permeability of 4 D and an oil viscosity of 1 cp).

Using these data we find breakthrough times of around 4 years. However, we have to bear in mind that the actual displacement process is not the same as simple passive tracer transport. To account for this we assumed Buckley–Leverett displacement along the fastest streamline (in other words we ignored the coupling between viscous displacement and the flow pattern). We determined the frontal shock speed from the standard Welge construction which gave a breakthrough time one quarter of that obtained by ignoring this effect. To account for this we could use an effective viscosity in the above formulæ of 0.25 cp. Given this we obtained the distribution of breakthrough times shown in Figure 1.

In addition to this, conventional numerical simulations were carried out for the field. We could then collect the statistics for breakthrough times for the various well pairs to compare with this theoretical prediction. Not all pairs exhibited breakthrough in the timescale over which the simulations were run and there were only three injectors so there were only 9 samples. The histogram of breakthrough times is also shown in Figure 1. Clearly, with such a small sample these results cannot be taken as conclusive, however, certainly they are indicative that the percolation prediction from the simple model is consistent with the results of the numerical simulation of the more complex reservoir model. The agreement with the predictions is certainly good enough for engineering purposes. We would hope that if the simulation had been run for longer and more well pairs had broken through that better statistics could have been collected. The main point being that the scaling predictions took a fraction of a second of cpu time (and could be carried out on a simple spreadsheet) compared with the hours required for the conventional simulation approach. This makes this a practical tool to be used for making engineering and management decisions.

CONCLUSIONS

We have applied results obtained earlier for the scaling law for breakthrough time distributions for oil field recovery to realistic field data. We have shown that by making a number of simplifying assumptions we can readily use previous results from percolation theory to make extremely rapid estimates of the uncertainty in breakthrough time. The agreement between the theory and the conventional simulation approach is accurate enough for engineering purposes and, therefore, makes it a practical tool for supporting decision-making.

It is hoped to be able to extend these encouraging initial results to more general cases where the permeability has a more continuous variation and where the flow is more complex (e.g. waterflood). It is also hoped to be able to look at post-breakthrough behaviour.

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