DISSERTATION PROSPECTUS

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In this dissertation we will examine the effect microscopic defects have on macroscopic observations of classical many-body systems. This will be accomplished through the introduction, generalization, and exhaustive analysis of simple models with many coupled degrees of freedom, where some of those degrees of freedom can be quenched into a defective or damaged state. These simple models are constructed as cellular automata: a d-dimensional grid of cells, each of which takes on a continuous value, bounded from above and below. In this work, we restrict our attention to spacial dimension $d = 2$. The temporal evolution of the system is specified by a rule which uniquely maps the value of each cell at time t, to some new value at time $t + \delta t$. This rule depends only on the value of the cell and the values of the cell's neighbors. Traditionally, neighbors are defined as abutting cells; however, in this dissertation, the definition is extended to let neighbors include all cells within some distance R of the evolving cell. Damaged or defective cells are then defined as cells that do not follow this evolution rule as the system is advanced in time.

Our work concentrates on two classes of many-body systems: building and support materials, and solid earth. Building and support materials include, for example, structural `I`-beams used as supports in housing foundations and overpasses, support cables used in the construction of suspension bridges, and radiation shields used to block radiation from escaping into uncontaminated areas. Fatigue has been observed in these materials as a result of the constant stresses and strains applied to them; therefore, they become increasingly susceptible to failure over time. To model this type of stress-induced damage, we begin with an automata where the value at each cell is a measure of the local stress or strain and, initially, all the cells obey the same time evolution rule. As the stress on a cell increases, it becomes more prone to a local failure. Should a cell fail, it is then considered damaged and follows a different rule for its temporal evolution than the rest of the system. We construct various models using different update rules for the damaged sites.

The first question we address is whether or not these systems can be described by a metastable equilibrium theory analogous to those used in the description of nucleation. We find that while, in many ways, the catastrophic failure event (i.e. the evolution of the system from partially damaged to completely damaged and therefore no longer able to support its load) resembles nucleation, the system does not remain in a metastable equilibrium state over a sufficient timescale for the techniques developed to describe nucleation from one phase to another to be applicable to failing systems. Nevertheless, we are still able to learn a great deal about the catastrophic event. For example, we determine the geometry of the event is influenced greatly by the range of interaction, R. In particular, for long-range interactions, the catastrophic event grows in the shape of the interaction from a localized region on the lattice. On the other hand, when interactions are restricted to nearest-neighbors, the catastrophic event is non-localized, jagged, and appears to have a fractal geometry at the moment of catastrophic failure.

Our study of damage in solid earth has focused primarily on the phenomena of Gutenberg-Richter scaling. The Gutenberg-Richter law is the empirical observation that the frequency of seismic events with a magnitude of at least M occurring within a fault system is exponentially suppressed in M, that is $f \sim 10^{-bM}$. The magnitude can be swapped for a different observable, the seismic moment, μ, which is logarithmically related to the magnitude as $\mu = \frac{2}{3}\log_{10} M - 10.7$ and thus the Gutenberg-Richter distribution is a power-law in the seismic moment: $f \sim \mu^{-2b/3}$. Various models of single faults have been proposed and many produce Gutenberg-Richter-like statistics; however, seismological observations reveal that it is the fault system that obeys this scaling relation and individual faults, in general, do not scale in this way. In addition, the proposed models have largely been homogenous whereas fault systems are known to be extremely inhomogenous. Our model fault system consists of individual faults with quenched damage (as opposed to the dynamic defects discussed above). We find that the damaged faults do not generate Gutenberg-Richter statistics, but rather appear to have statistics consistent with a Gamma-like probability density function. While the Gamma distribution has two free parameters (α and θ), the various faults produce statistics with the same value of α. The remaining parameter, θ, is a measure of the amount of damage on the fault and can be scaled out so that the distributions generated by different faults collapse to a universal distribution.

Similar models have shown seismic phenomenology beyond Gutenberg-Richter scaling. In particular, the modified and unmodified Omori's law, the inverse Omori law, the increase in the number of aftershocks with increasing mainshock magnitude, aftershock diffusion, foreshock migration, spacial correlations between the aftershocks and the mainshock, the distribution of foreshock magnitudes, and the number of fore-and-aftershocks per mainshock have all been observed, at least qualitatively. This phenomenology is investigated in our models as well.

The scope of the dissertation is outlined below.