A Cellular Automaton Model of Catastrophic Failure

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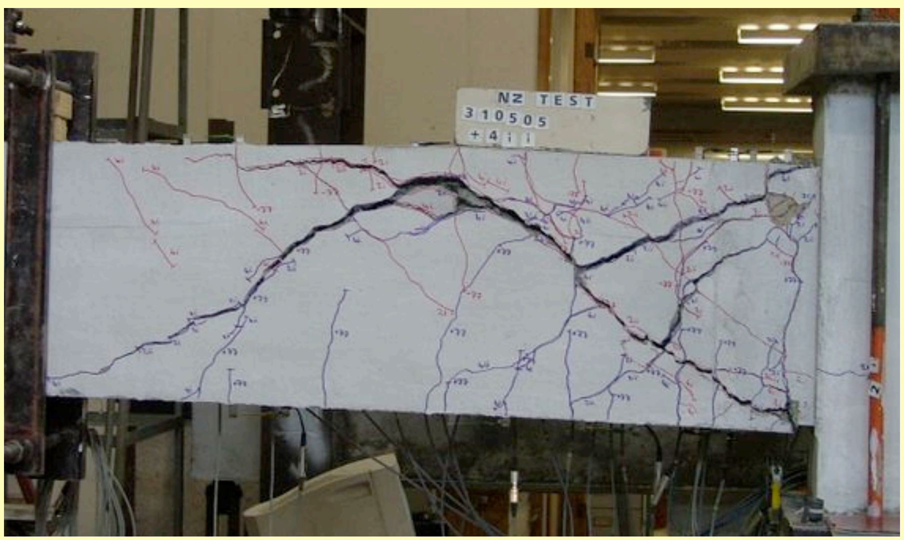
Overview

- Motivation
- Fiber Bundle Models
- Introduction of CA Damage Model
- Definitions of Catastrophic Failure
- Failure Modes in the Model
- Ergodicity
- Conclusion

Motivation

- To understand the mechanisms which lead from damage to catastrophic failure in materials.



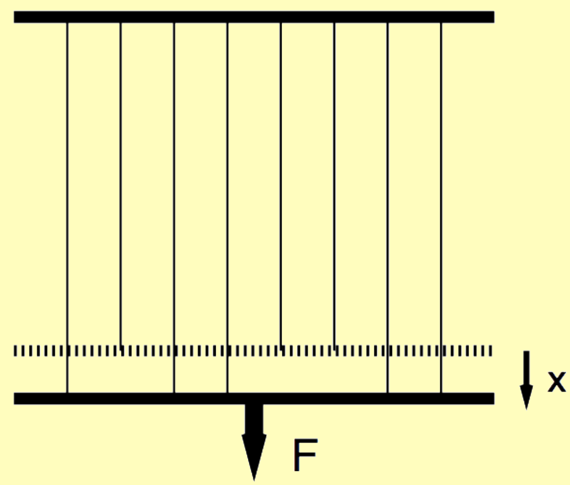
Minnesota Public Radio, 2009

The Fiber Bundle Model

- Peirce, F. T., *J. Text. Ind.* **17**, 355 (1926)
- Daniels, H. E., *Proc. Roy. Soc. London A* **183,** 405 (1945)
- arXiv:0808.1375v2 [cond-mat.stat-mech]

1. Limited in scope.
2. Equilibrium treatment.

$$\sigma_j = \kappa\epsilon_j$$



Olami-Feder-Christensen Model

- We base our model on the Olami-Feder-Christensen (OFC) model used to model earthquake faults.

[Olami *et. al.,* PRL **68**, 1244-1248 (1992)]

(1) $\sigma_i \rightarrow \sigma_i + \delta, \quad \forall \; i$

(2) If $\sigma_i > \sigma^f$

(a) $\sigma_j \rightarrow \sigma_j + (1-\alpha)\left(\frac{\sigma_i - \sigma^r}{q}\right)$

(b) $\sigma_i \rightarrow \sigma^r$

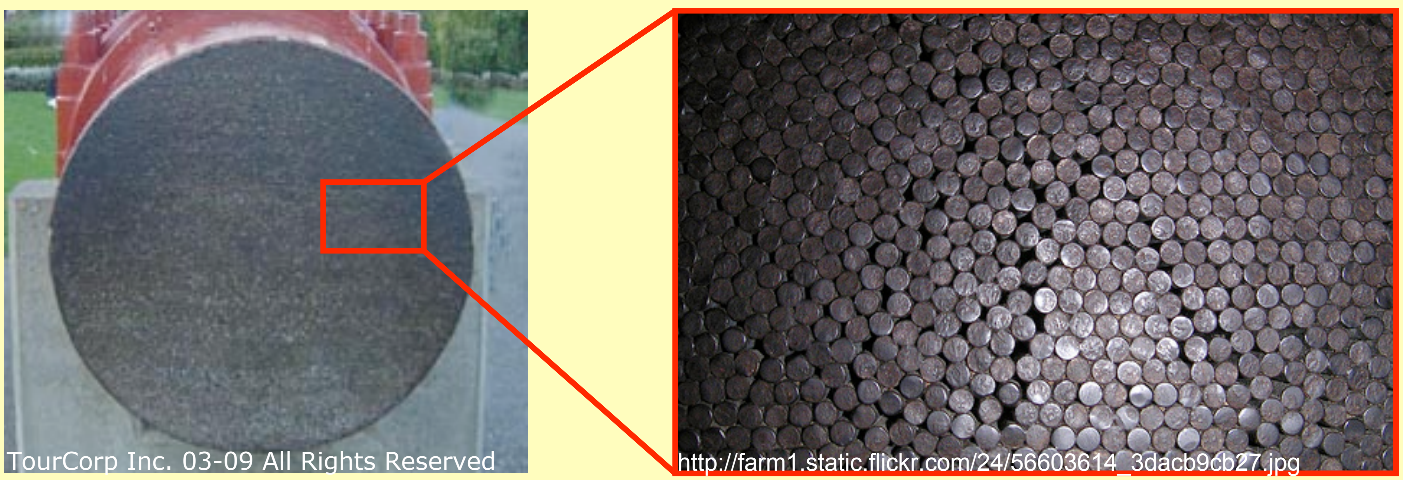
(3) Return to (2)

(4) Return to (1)

σ_i	stress on site i
σ^f	failure threshold
σ^r	residual stress
δ	increase in stress from increasing load
j	"neighbor" site of site i
α	dissipation coefficient
q	number of neighbors of site i

From Earthquake Model to Damage Model

- Once a site fails *f* times, the site is considered dead and no longer interacts with the system.
- Rather than nearest-neighbor interactions, sites interact equally with all other sites within a radius, *R*. (The limit $R \to \infty$ reproduces the FB model.)

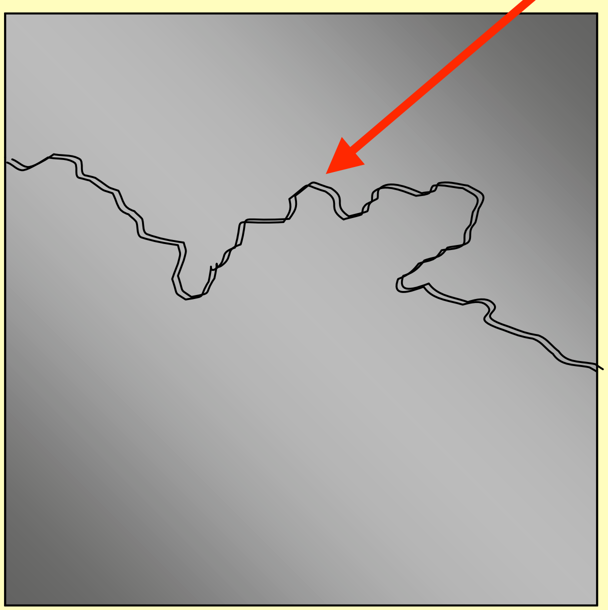


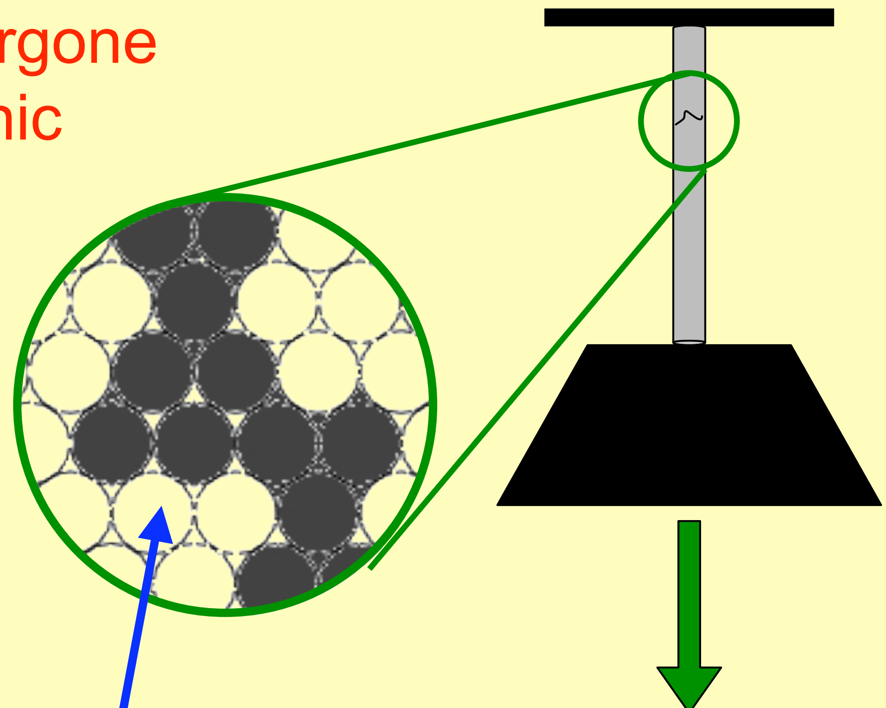
Catastrophic Failure

Percolating Cluster of Dead Sites (e.g. chip boards)

All Sites are Dead (e.g. fiber bundles)

Has undergone catastrophic failure

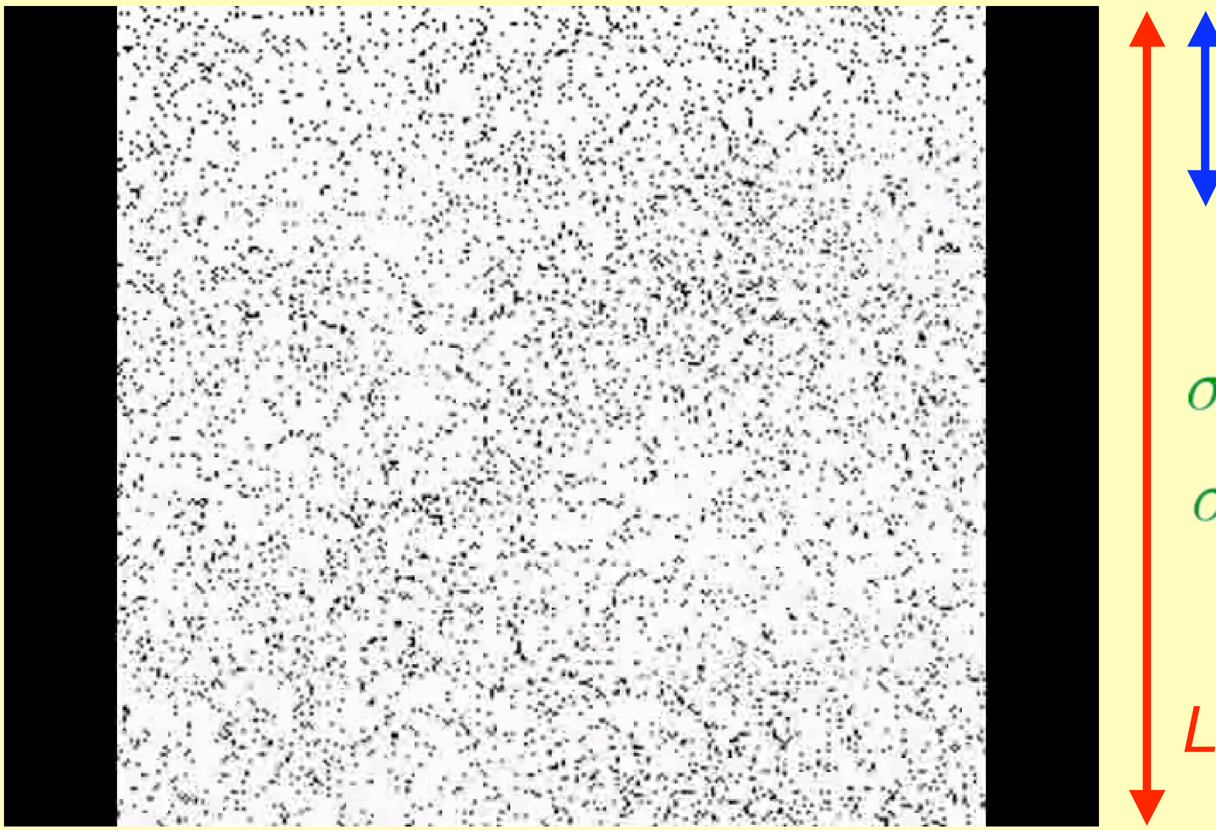




Has not catastrophically failed

Long Range Failure Mode

- For long range interactions, we observe a nucleation-like event which results in catastrophic failure.



$$f = 1$$
$$\sigma^f = 2$$
$$\sigma^r = 1 \pm 1/4$$
$$\alpha = 3/10$$

Nearest-Neighbor Failure Mode

- For nearest-neighbor interactions, we observe a percolating cluster of dead sites while a finite fraction of the sites are still alive.

$R = 1$

$f = 1$

$\sigma^f = 2$

$\sigma^r = 1 \pm 1/4$

$\alpha = 3/10$

$L = 256$

Ergodicity

- To date, damage models have been treated with equilibrium methods.
- We generalize the Thirmulai and Moutain fluctuation metric, $\Omega(t)$, a measure of "effective ergodicity", by choosing the observable as stress rather than energy. [PRA **42**, 4574-4587 (1990)]

$$s_j(t) = \frac{1}{t}\int_0^t \mathrm{d}t' \, \sigma_j(t')$$

time average

$$\Omega(t) = \frac{1}{N}\sum_{j=1}^{N}\left[s_j(t) - \bar{s}(t)\right]^2$$

ensemble average*

$$* \; t \to \infty, \; N \gg 1$$

$$\bar{s}(t) = \frac{1}{N}\sum_{j=1}^{N} s_j(t)$$

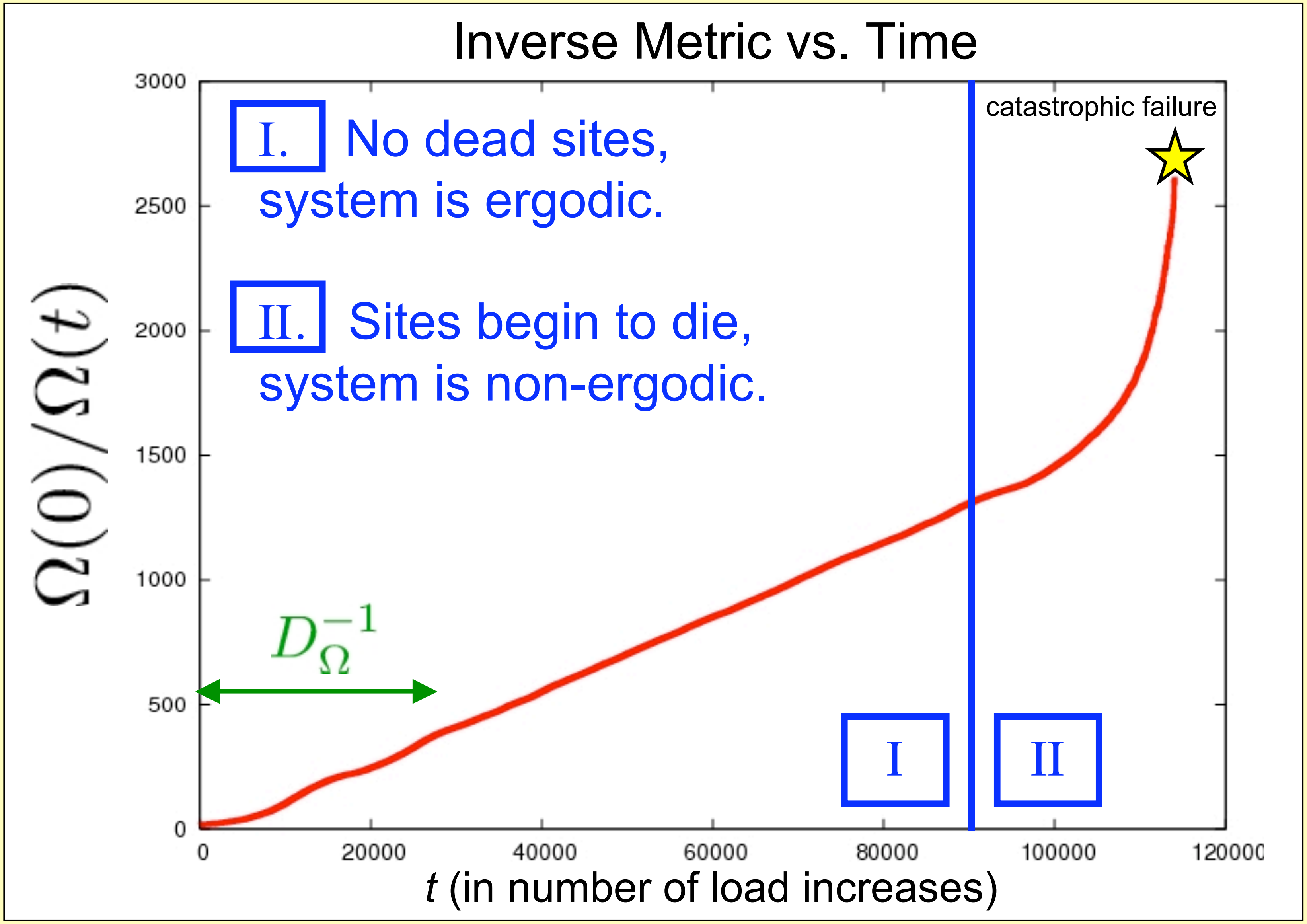
Properties of the TM Metric

- For "effectively ergodic" systems,

$$\Omega(0)/\Omega(t) \sim D_\Omega t$$

where the slope, D_Ω, is a measure of the mixing time.

TM Metric for Long Range Damage Model



Summary

- We introduced a cellular automaton model for damage which reproduced the Fiber Bundle Model in the mean field ($R \to \infty$) limit.
- Our model has two distinct failure modes depending on the interaction range.
- Using the TM metric, we determined that the system is *not* in equilibrium once damage begins to occur and thus equilibrium descriptions are invalid.

Thank You

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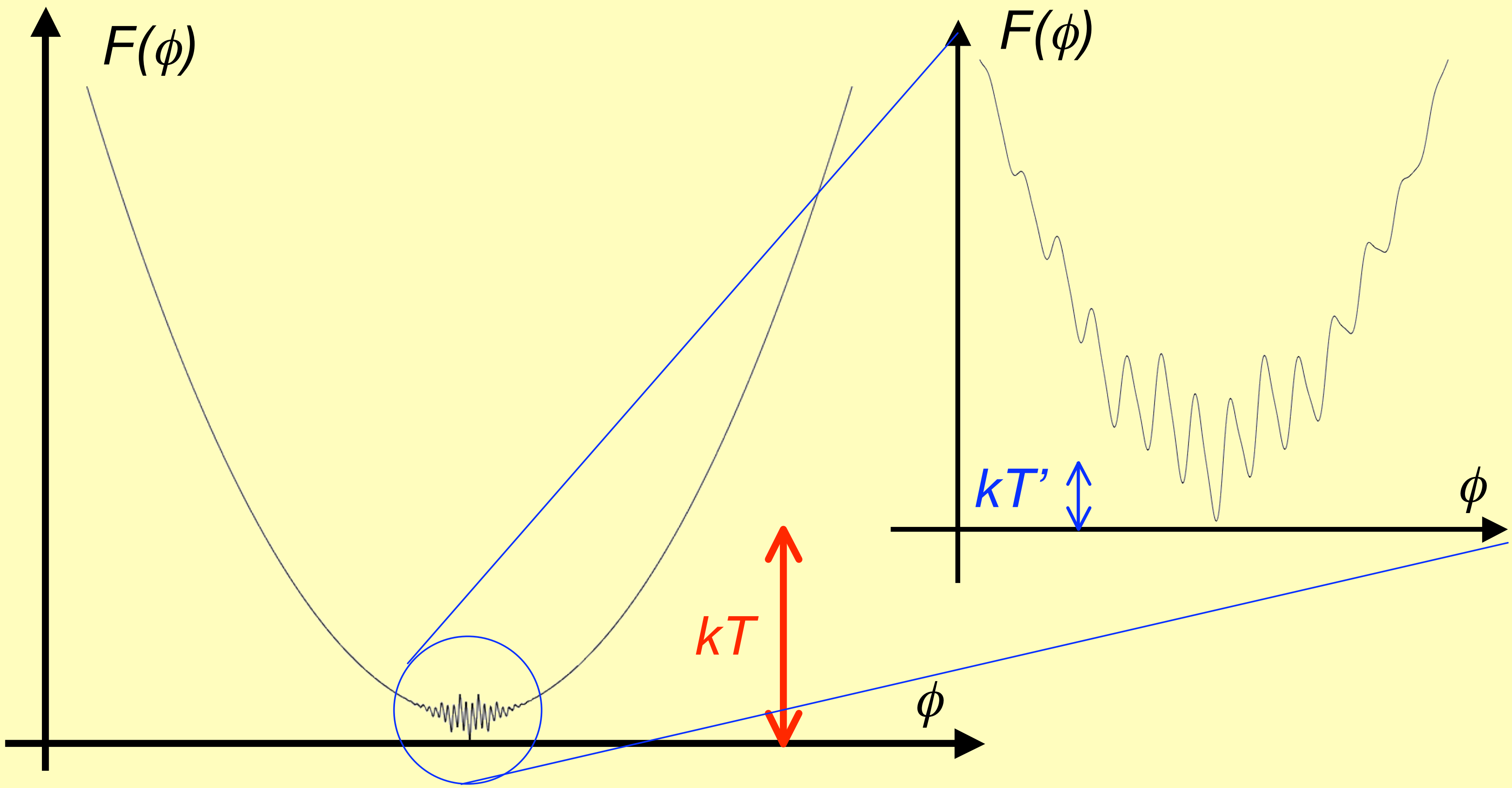
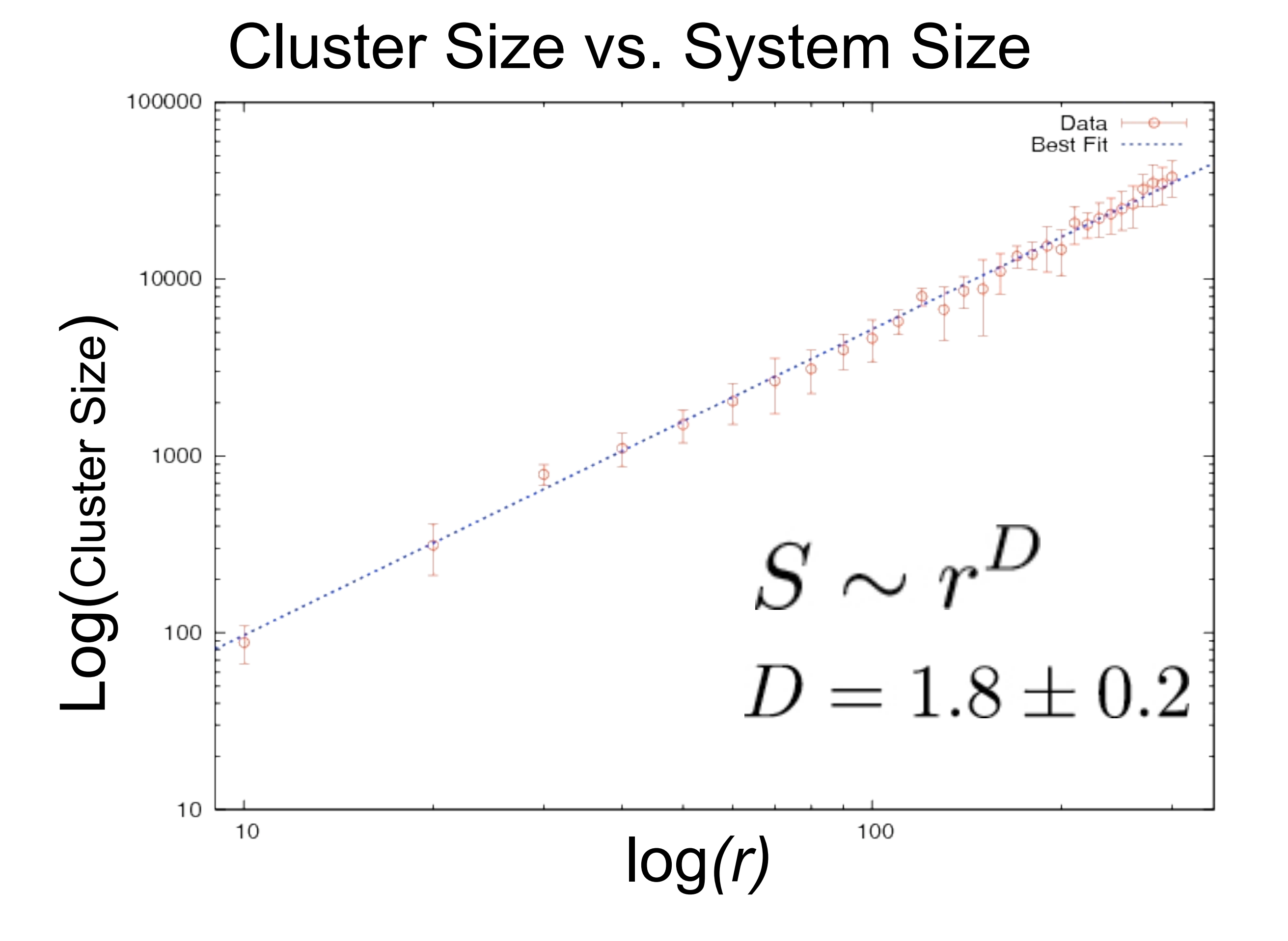
“Effectively Ergodic”



Fractal Dimension of the Percolating Cluster

- We use finite-size scaling to determine the fractal dimension of the cluster.



FB Model Studies

Failure Processes in Elastic Fiber Bundles

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The fiber bundle model describes a collection of elastic fibers under load. The fibers fail sucessively and for each failure, the load distribution among the surviving fibers changes. Even though very simple, this model captures the essentials of failure processes in a large number of materials and settings. We present here a review of the fiber bundle model with different load redistribution mechanisms from the point of view of statistics and statistical physics rather than materials science, with a focus on concepts such as criticality, universality and fluctuations. We discuss the fiber bundle model as a tool for understanding phenomena such as creep, and fatigue, how it is used to describe the behavior of fiber reinforced composites as well as modelling e.g. network failure, traffic jams and earthquake dynamics.